This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Box, No. Coll. T 118.06.814

ESSEX INSTITUTE.

CANCELLED

PRESENTED BY

Geo. R. Lord.

The Library Committee shall divide the books and other articles belonging to the Library into three classes, namely: (a) those which are not to be removed from the building; (b) those which may be taken from the halls only by written permission of three members of the committee, who shall take a receipt of the same and be responsible for their safe return; (c) those which may circulate under the following rules:—

Members shall be entitled to take from the Library one folio, or two quarto volumes, or four volumes of any lesser fold, with the plates belonging to the same, upon having them recorded by the Librarian, or assistant Librarian, and promising to make good any damage they sustain while in their possession, and to replace the same, if lost, by the sum fixed by the Library Committee.

No person shall lend any book belonging to the Institute, excepting to a member, under a penalty of one dollar for every such offence.

The Library Committee may allow members to take more than the allowed number of books upon a written application, and under such [illegible] persons than members to use the Library, [illegible] conditions as they may impose.

No person shall [illegible] book longer than four weeks from the time of its being taken from the Library, if notified that the same is wanted by another member, under a penalty of five cents per day, and no volume shall be detained longer than three months at one time, under the same penalty.

The Librarian shall have power, by order of the Library Committee, to call in any volume after it has been retained by a member ten days.

On or before the first Wednesday in May, all books shall be returned to the Library, and a penalty of five cents per day shall be imposed for each volume detained.

Labels designating the class to which each book belongs shall be placed upon its cover.

No book shall be allowed to circulate until one month after its reception.

HARVARD COLLEGE LIBRARY

3 2044 096 989 793

KEY

TO THE

TUTOR's GUIDE;

OR,

THE ARITHMETICIAN's REPOSITORY;

CONTAINING

THE SOLUTIONS OF THE QUESTIONS, &c.

IN THE

TUTOR's GUIDE,

WITH REFERENCES TO THE PAGES WHERE THEY STAND.

To which are added (where necessary),

SOME USEFUL RULES, &c.

LIKEWISE,

AN APPENDIX;

SHOWING THE COMBINATION OF QUANTITIES;

The different Ways they may be varied;

WITH THE METHOD OF FILLING THE MAGIC SQUARES, &c.

The whole being principally designed for the Ease of Schoolmasters; and, with the GUIDE, furnishes a more complete and extensive SYSTEM of ARITHMETIC than any extant; and will enable all those who are acquainted with the first Principles to attain a complete Knowledge of the several Rules with Ease and Precision.

BY CHARLES VYSE.

NINTH EDITION;

Carefully revised, corrected, and augmented.

PHILADELPHIA:

PRINTED BY JOSEPH CRUKSHANK,

AND SOLD BY P. BYRNE, M. CAREY, T. & W. BRADFORD, JAMES CRUKSHANK, T. DOBSON, KIMBER, CONRAD, & Co. AND JOHN M'CULLOCH.

1806.

Educ T 118.06.874

HARVARD COLLEGE LIBRARY
GIFT OF
GEORGE ARTHUR PLIMPTON
JANUARY 25, 1926

THE

PREFACE.

WHAT gave rise to the following Sheets, was my receiving Letters from several eminent Mathematicians and Schoolmasters, wherein they expressed great Desire for such a Performance, mentioning the Utility such a Work would be to Schoolmasters in general; as the TUTOR'S GUIDE contained such a Variety of Questions, suitable to all Capacities, and adapted for the Use of the Gentleman and Scholar, as well as for the Man of Business—But as many of the Questions are long and difficult, made it impossible for Schools in general to make use of the GUIDE, without the KEY, the Time in Schools not admitting; therefore they gave me great Hopes of Encouragement, by promising to make Use of the TUTOR'S GUIDE in their own Schools; beside recommending it to all Schoolmasters in Great Britain, as the most complete Epitome of Arithmetic, or Question Book, extant; and, with the KEY, would enable the Tutor to instruct ten Pupils with greater Ease, more correctly, and with less Perplexity both to himself and Scholars, than one by any other Book. Thus encouraged, and at the same Time being sensible of the favourable Reception the GUIDE has met with, gives me great Hopes that the following Pages will meet with that En-

couragement due to so useful and laborious a Work.

The Title-page gives a short Account of what the following Pages contain, which I think needless to enlarge upon, therefore shall leave the Book to speak for itself; and if it does not give Satisfaction to the Reader, I am sure all I can say in its Behalf will never recommend it: but this may be justly said; whoever reads it over will find the Solutions of a greater Variety of Questions than in any other Treatise on the same Subject; and are performed in as intelligible and comprehensive a Manner as my Bounds would admit of, or even as is necessary.

How well I have performed this Work must be left to proper Judges; and as I am not sensible of any fundamental Error in the following Pages, yet I cannot pretend to say it is without Imperfections; which I hope the good-natured Reader will excuse, and pass over with the like Candour and Good-will with which it was composed for his Use by

CHARLES VYSE.

15, *Walnut-tree Walk*,
Lambeth, 1802.

THE

KEY

TO THE

TUTOR'S GUIDE;

OR, THE

Arithmetician's Repository.

BOOK I. PART I.

1. NUMERATION.

NUMBERS expressed in WORDS.

1. NINETY-FOUR.—2. Seven hundred and sixty-two.—3. Three thousand and twenty-four.—4. Thirty-seven thousand four hundred and sixty.—5. One hundred forty-two thousand, six hundred and thirteen.—6. Six million, forty thousand, three hundred and ninety.—7. Forty-seven million, six hundred thirty-nine thousand, one hundred and twenty-one.—8. Seven hundred ninety million, four hundred one thousand, nine hundred and fifty.—9. Seventy-nine million, forty-one thousand, nine hundred and fifty-five.

WORDS expressed in FIGURES.

(1) 77, (2) 490, (3) 6055, (4) 17709, (5) 800002, (6) 7044074, (7) 694000060.

Roman Numerical LETTERS expressed in FIGURES.

19, 200, 600, 560, 1001, 1750, 70000, 110000, 1500000, 1600000.

Common FIGURES expressed in Numerical LETTERS.

XXIX. CIV. CCCCXIX. MDCCXLI. MMVII.
XVIIDCLXXVIII. XIV. DCLXXIVLXXXIV.

INTEGERS.

2. ADDITION.

To add these Examples I begin with the first, and say 6 and 7 is 13, and 6 is 19, and 4 is 23, and 2 is 25, and 9 is 34; this is the Amount of the first Row, or Unit's Place, which contains three Tens and 4 over: this 4 I set under the Row, and carry 3 to the next Row, or Ten's Place, saying 3 that I carry and 4 is 7, and 1 is 8, and 9 is 17, and 1 is 18, and 4 is 22, and 7 is 29; this being 2 Tens and 9 over, therefore I set down 9 and carry 2 to the next Row or Hundred's Place, saying 2 and 7 is 9, and 4 is 13, and 1 is 14, and 9 is 23, and 2 is 25, which is 5 to set down, and I carry 2 to the fourth Row, or Place of Thousands, saying 2 and 4 is 6, and 7 is 13, and 2 is 15, and 6 is 21, and 4 is 25, and 7 is 32, which is 2 to set down, and I carry 3 to the fifth Row, or Place of Tens of Thousands, and find that it also amounts to 32, when I set down the odd 2, and carry 3 to the sixth and last Row, or Place of Hundreds of Thousands, and find that it amounts to 22; and as this is the last Row, I set down the whole 22, that is, the 2 under the Row, and the two Tens I carry to the left Hand; and in the same manner proceed with the rest of the Examples.

(1) 2222594 (2) 1696237 (3) 433968 (4) 225879

(5) 2181162 (6) 326034 (7) 341780 (8) 88622054

(9) 1148103

3. SUBTRACTION.

When the upper Figure is greater than the lower, as in the first Example, I say 0 from 2 there remains 2; then 2 from 4 there remains 2, and 6 from 7 there remains 1; in this Manner proceed to the End of this Example. But when the lower Figure is larger than the top one, as in the second Example, I say 7 out of 1 I cannot have, therefore I take 7 out of 10, (which I borrow) and there remains 3, and the top Figure 1, makes 4, which I place under the Unit's Place, and carry 1 to the next lower Figure, saying that 1 I carry to 6 makes 7, then 7 from 4 I cannot, but 7 from 10 there remains 3, and the top Figure 4 makes 7 to set down, and 1 to carry to the next lower Figure, which is 4, makes 5; then say 5 from 9 there

remains 4; but now I do not carry any to the next Figure, because I do not borrow, but only say 7 from 4 I cannot, but 7 from 10 there remains 3, and the top Figure 4 makes 7 to set down, and 1 I carry to 0 is 1, from 0 I cannot, but 1 from 10 there remains 9, and carry 1 to 8 is 9, from 7 I cannot, but 9 from 10 there remains 1, and the top Figure 7 makes 8; lastly, 1 I carry from 1, and there remains 0, which being the last Figure, I do not set it down; and in this manner proceed with the other Examples.

(1) 221122 (2) 897474 (3) 689796 (4) 3160990

(5) 313992 (6) 2708970 (7) 8960066 (8) 3320801

(9) 1069897 (10) 31981794 (11) 106799280

4. MULTIPLICATION.

CASE 1.

I begin with the first Example, and say, twice 3, or 2 Times 3, is 6 to set down; then twice 5 is 10, that is 0, to set down, and carry 1; then twice 8 is 16, and 1 I carry is 17, that is 7 to set down, and I carry 1; then twice 9 is 18, and 1 I carry is 19, that is 9 to set down, and one to carry to the next Figure; and so I proceed to the End of this Example, and likewise with all the rest, always remembering to carry 1 for every 10, to the next Figure on the left Hand, as in Addition

(1) 835219706 (2) 1028157258 (3) 1030723656

(4) 871028415 (5) 1025792286 (6) 5244299221

(7) 2102064752 (8) 3386617605

CASE II.

(9)	(10)	(11)
856424376	172859776	84537943
142737396	43214944	108691641
2283798336	605009216	1171454353

(12) 1237835
1732969
990268
117594325

(13) 2858841
2541192
1905894
218860161

(14) 16241490
10827660
18948405
8120745
10140103590

(15) 295356
1181424
886068
738390
839106396

(16) 47269
378152
236345
141807
330883
3478100289

(17) 662229
441486
147162
515067
294324
3478100289

(18) 4585548
6878322
2292774
5349806
764258
3057032
3189982321 68

(19) 3339168
2086980
834792
1669584
2504376
2921772
3189982321 68

(20) 16314384
8157192
19033448
21752512
10876256
16314384
13595320
153592747031 04

CASE III.

(21) 85968376
10746047
53730235
42984188
43521606407 3076

(22) 7242305625
6437605000
5632904375
1609401250
16657947422305625

CASE IV.

(23) 102263
116872
12709830000

(24) 16608
11072
5536
68092800000

CASE V.

(25) 24674
4×4=16
98696
4
394784

(26) 340764
7×4=28
2385348
4
9541392

(27) 142395
8×7=56
1139160
7
7974120

(28) 176848
9×9=81
1591632
9
14324688

(29) 420746
9×8=72
3786714
8
30293712

(30) 17093
9×7=63
153837
7
1076859

(31) 43074
12×12=144
516888
12
6202656

(32) 14068
12×11=132
168816
11
1856976

CASE VI.

To perform the Examples in this Case, I begin with the last, and say, 9 times 8 is 72, put down 2 and carry 7; then 9 times 6 is 54, and 7 I carry is 61, and the right Hand (or back) Figure 8 is 69, put down 9 and carry 6; then 9 times 7 is 63, and 6 I carry is 69, and 6 the back Figure is 75, put down 5 and carry 7; then 9 times 2 is 18, and 7 I carry is 25, and 7 the back Figure is 32, put down 2 and carry 3; then 9 times 4 is 36, and 3 I carry is 39, and 2 the back Figure is 41, put down 1 and carry 4; then 9 times 1 is 9, and 4 I carry is 13, and 4 the back Figure is 17, put down 7 and carry 1. Now as the Multiplication by the 9 (the Unit's Figure) is ended, I add the 1 I carry to the last Figure in the Multiplicand, and it makes 2, which I put down, and the Work is ended; and in this manner proceed with all the rest of the Examples in this Case.

(33) 1569876 (34) 171312 (35) 1899222 (36) 338464

(37) 542130 (38) 2822784 (39) 248353 (40) 335286

(41) 2712592

5. SECT. V. DIVISION.

CASE I.

In the first Example I begin and say, how oft 2 in 17? Answer, 8 times 2 is 16, and 1 over, which is 10, added to 4 the next Figure, makes 14; then I say, how oft 2 in 14; Ans. 7 times 2 is 14, and 0 over; how oft 2 in 2? Ans. 1, and 0 over; how oft 2 in 6? Ans. 3 times 2 is 6, and 0 over; how oft 2 in 3? Ans. 1, and 1 over which is 10, and the next Figure 6 is 16; how oft 2 in 16? Ans. 8 times 2 is 16, and nothing remains; and in this Manner proceed with all the rest of the Examples in this Case.

(1) Quotient 871318 2

Proof 1742636

(2) 921354—2

(3) 540185—2

(4) 215185—1

(5) 11903492—2

(6) 667751—6

(7) 346012

(8) 752010—4

(9) 251160—4

(10) 23040—4

CASE II.

To perform this Case, I begin with the first Example, and say, how oft is 25 in 73? or, which is better, how oft 2, the first Figure in the Divisor, in 7, the first in the Dividend? and I find (after the Allowance is made for what I shall have to carry) it will only go 2 times; wherefore I place 2 in the Quotient, and multiply 25 the Divisor thereby, the Product viz. 50) set under 73, and subtract; then to the Remainder 23 I bring down the next Figure 6 in the Dividend; then I say (as before) how oft 2 in 23? Ans. 9 times, which I place in the Quotient, and multiply 25, the Divisor thereby: the Product, viz. 225 subtracted from 236 leaves 11, to which bring the next Figure in the Dividend, viz. 4, then proceed as before, till you have brought down all the Figures in the Dividend, and the Work will be finished.

Divisor.	Divid. (11) Quot.	(12)
25)736473575(29458943	5	84)35730972(425368$\frac{60}{84}$
50		336
236	147294715	213
225	5	168
114	Pr. 736473575	450
100		420
147		309
125		252
223		577
200		504
235		732
225		672
107		Remains 60
100		
75		
75		

(13)

```
648 272357640(420305
    2592
    ----
     1315
     1296
    ----
      1976
      1944
    ----
       3240
       3240
    ----
       ....
```

(14)

```
759 3089182967640780697
    3036
    ----
      5318
      5313
    ----
       5296
       4554
    ----
        7427
        6831
    ----
        5966
        5313
    ----
Remains 653
```

(15)

```
3065 63463902247(20706003
     6130
     ----
      21639
      21455
     ----
       18402
       18390
     ----
        12247
         9195
     ----
Remains 3052
```

(16)

```
7489 1204530760(160840
     7489
     ----
      45563
      44934
     ----
       62907
       59912
     ----
       29956
       29956
     ----
```

(17)

```
42163 11273328(2673
      84326
      -----
       284113
       252978
      ------
        311352
        295141
       ------
         162118
         126489
       ------
Remains  35629
```

(18)

```
61745 392628787(6358
      370470
      ------
       221587
       185235
      ------
        363528
        308725
       ------
         548037
         493960
       ------
Remains  54077
```

(19)

684573)3233238699(4723
2738292

4949466
4792011

· 1574559
1369146

· 20541 39
2053719

Remains 420

(20)

476085)9883905478 0(207608
952170

· 3622054
3332595

· 2894597
2856510

· · 3808780
3808680

Remains 100

(21)

4728395)27750950255(5869
23641975

· 41089752
37827160

· 32625925
28370370

· 42555555
42555555

. . . . . .

CASE III.

(22)

28,00)119282,48(4260
112

72
56

168
168

Rem. . . . 248

7 × 400 = 2800

11928248 Proof.

(23)

172,000)247004,674(1436 $\frac{12674}{172000}$
172

· 750
· 688

620
516

1044
1032

Remains . . 12674

CASE V.

(34)
17)690489(40617
104
28
119

(35)
86)5343698(62136
183
116
509
518
Remains 2

(36)
467)2148686(4601
2806
486
Remains 19

(37)
6074)24939844(4106
6438
36444
.....

6. REDUCTION.

MONEY.

(1) 27 £
20
540 Shillings.
12
6480 Pence.

(2) 12)6480 Pence.
2,0)540 Shillings.
27 £.

(3) £. s.
40 10
20
810 Shillings.
12
9720 Pence.
4
38880 Farthings.

(4) 4)38880 qrs.
12)9720
2,0)810
£. 40 10

(5)

£.	s.	d.
104	17	6¾

20

2097 s.

12

25170 d.

4

100683 Farthings.

(6) 4)100683 qrs.

12)251703

2,0)209,7—6

£. 104 17 6¾

(7) 21 Guineas.

7×3=21s.=1 Guin.

147

3

441 Shillings.

12

5292 Pence.

4

21168 Farthings.

(8) 4)21168 qrs.

12)5292

21 { 3)441 / 7)147

21 Guineas.

(9) 42 *Moidores.*

9×3=27s.=1 Moid.

378

3

1134 Shillings.

12

13608 Pence.

4

54432 Farthings.

(10) 4)54432 qrs.

12)13608 d.

27 { 3)1134 s. / 9) 378

42 Moid.

WEIGHTS, MEASURES, &c.

TROY WEIGHT.

(1) 24*lb.*
12
288 oz.
20
5760dwts.
6×4=24
34560
4
138240 grs.

(2) 24 { 4)138240*grs.*
6) 34560
2,0)576,0 dwts.
12)288 oz.
24 lb.

(3) *lb. oz.dwt.gr.*
12 10 0 22
12
154
20
3080
6×4=24
18480
4
73942 grs.

(4) *grs.*
24 { 4)73942
6)18485—2
2,0)308,0—5 { 20 dwts. 5 grs.
12)154
lb. 12 10 0 22

APOTHECARIES' WEIGHT.

(5) 14 lbs.
12
168 ℥
8
1344 ʒ
3
4032 ℈
20
80640 grs.

(6) 2,0)8064,0 grs.
3)4032 ℈
8)1344 ʒ
12)168 ℥
14 lb.

(7)
lb. ℥ 3 ℈ *grs.*
4 11 0 2 17
12
59 ℥
8
472 3
3
1418 ℈
20
28377 grs.

(8) 2,0)2837,7 grs.
3)1418—17 grs.
8)472—2 ℈
12)59
lb. 4 11 0 2 17

AVOIRDUPOIS WEIGHT.

(9)
20 *C.*
4
80 qrs.
28
2240 lb.
16
35840 oz.
16
573440 drs.

(10)
16 { 4)573440 *drs.*
4)143360
16 { 4)35840 oz.
4) 8960
28 { 4) 2240 lb.
7) 560
4)80 qrs.
20 C.

(11)
lb. oz. drs
27 12 11
16
444
16
7115 drs.

(12)
drs.
16 { 4)7115
4)1778—3 } 11 drs.
16 { 4) 444—2
4) 111
lb. 27 12 11

T. C. qrs. lb. oz. drs.

(13) 12 10 0 14 11 15

20

250 C.

4

1000 qrs.

28

28014 lb.

16

448235 oz.

16

7171775

(14)

4)7171775 drs.

4)1792943—3 } 15 drs.

4 448235—3 } 11 oz.

4 112058—3 } 14 lb.

4)28014—2

7) 7003—2

4 1000—3

2,0)25,0

Tons 12 10 0 14 11 15

CLOTH MEASURE.

(15) 24 yds.

4

96 qr.

4

384 nails.

(16) 4 384 nails

4)96

24 yds.

E.E. qr. na.

(17) 72 4 2

5

364 qrs.

4

1458 nails.

(18) 4)1458 nails.

5 364—2

E. ells 72 4 2

(19) 121 Fl. ells.

3

363

4

1452 nails.

(20) 4 1452 nails.

3)363

121 Fl. E.

Fl. E. qrs.

(21) 42 5

6

257 qr.

4

1028 nails.

(22) 4 1028 nails.

6 257 qrs.

Fl. E. 42 5 0

LONG MEASURE.

M. fu. p.

(23) 176 o 30

8

1408 Fur.

40

56350 Poles.

(25) 200 Miles.

1760

352000 Yards.

3

1056000 Feet.

12

12672000 Inches.

24) 4,0)5635,0 Poles.

8)1408—30 F.

Miles 176 o 30 P.

(26) 12)12672000 Inches.

3)1056000 Feet.

176,0)35200,0(200 Miles.

352

...00

Lea.m.fur.p. y.

(27) 12 1 6 29 4

3

37 Miles.

8

302 Fur.

40

12109 Poles.

11 half yds. in a Pole.

133207 Half Yds.

18 Inches = ½ Yd.

2397726 Inches.

3

7193178 Barley Corns.

B. C.

(28) 3)7193178

18 { 3 2397726 Inches.

6) 799242

11)133207 half yds.

4,0)1210,9—8=4 yds.

8)302—29 P.

3)37—6 Fur.

Lea. 12 1 6 29 4

LAND MEASURE.

(29) 42 Acres.
4
168 Rooda.
40
6720 Poles.

(30) 4,0)672,0 Poles.
4 168 Roods.
42 Acres.

A. r. p.
(31) 12 3 29
4
51 Roods,
40
2069 Poles.

(32) 4,0)206,9 Poles.
4 51—29 P.
A. 12 3 29

WINE MEASURE.

(33) 4 Anchors.
10
40 Gal.
4
160 Qts.

(34) 4)160 Qts.
1,0)4,0 Gal.
4 Anchors.

(35) 4 Hhds.
63
252 Gal.
8
2016

(36) 8)2016 Pints.
63 { 7)252 / 9) 36
4 Hhds.

Tierce. gal.
(37) 42 24
7×6=42
294
6
1788 Gal.
8
14304 Pints.

(38) 8)14304 Pints.
42 { 6)1788 / 7) 298
Tierce 42 24

T. p. hd. gal. pts.

(39) 4 1 1 42 6

2

9 Pipes.

2

19 Hhds.

63

1239 Gal.

8

9918 Pints.

(40) 8)9918 Pints.

7)1239—6 Pints.

63

9) 177

2)19—42 Gal.

2)9—1 Hhd.

Tuns 4 1 1 42 6

WINCHESTER MEASURE.

(41) 12 *B. A*

32

384 Gal.

4

1536 Qt.

(42) 4)1536 Qts.

4)384 Gal.

32

8) 96

12 Barrels.

(43) 42 *B. B.*

9×4=36

378

4

1512 Gal.

8

12096 Pints.

(44) 8)12096 Pints.

4)1512 Gal.

36

9)378

42 Barrels.

A. hds. gal. pts.

(45) 6 27 6

48

315 Gal.

8

2526 Pints.

(46) 8)2526 Pints.

6)315—6 Pints.

48

8) 52—3

6—4

27 Gal.

Hhds. 6 27 6

B. hhd. gal.

(47) 14 47
54
803 Gal.
8
6424 Pints.

(48) 8)6424 Pints.

54 { 6)803 Gal.
9)133—5
14—7 } 47 Gal.

Answ. 14 Hhds. 47 Gal.

T. b. hhd. gal.

(49) 6 1 0 42
2
13 B.
2
26 Hhds.
9×6=54
233
6
1446 Gal.
4
5784 Qts.

(50) 4)5784 Qts.

54 { 6)1446
9) 241
6)26—7 } 42 Gal.
2)13
T. 6 1 0 42

DRY MEASURE.

(51) 24 Qrs.
8
192 Bush.
4
768 Pecks.
2
1536 Gal.
4
6144 Qts.

(52) 4)6144 Qts.
2)1536 Gal.
4)768 Pecks.
8)192 Bush.
24 Qrs.

(53) *Cha.* *bu.*
36 26
6×6=36
216
6
1322 Bush.
4
5288 Pecks.

(54) 4)5288 Pecks.
6)1322 Bush.
36 { 6) 220—2 } 26 Bush.
36—4
Ans. 36 Ch. 26 Bush.

(55) 64 Lasts.
2×5=10
128 Weys.
5
640 Qrs.
8
5120 Bush.
4
20480 Pecks.

(56) 4)20480 Pecks.
8 5120
1,0)64,0 Qrs.
64 Lasts.

TIME.

(57) 7 Days.
24
168 Hours.
60
10080 Min.
60
604800 Seconds.

(58) 6,0)60480,0 Seconds.
6,0)1008,0 Min.
24 { 4)168 Hours. / 6)42 }
7 Days.

M. d.

(59) 6 4
28

172 Days.
6×4=24

1032
4

4128 H.
60

247680 M.
60

14860800 Seç.

(60) 6,0)1486080,0 Seconds.

6,0)24768,0 M.

24 { 4)4128 H.
6) 1032

7)172 D.

4) 24—4

M. 6 4 D.

D. H.

(61) 365 6
6×4=24

2190
4

8766 H.
60

52596 M.
60

31557600 Seconds.

(62) 6,0)31557600,0 Sec.

6,0)52596,0 M.

24 { 4)8766
6)2190—6

365 D. 6H.

(63) D. h. m. ″ ‴
365 5 48 57 39
6 × 4 = 24

2190
4

8765 H.
60

525948 M.
60

31556937 Sec.
60

1893416259 Thds.

(64) 6,0)189341625,9 Thirds.

6,0)3155693,7—39‴

6,0)52594,8—57″

24 { 4)8765—48′
6)219—5 H.

Days 365 5 48 57 39

SQUARE or SUPERFICIAL MEASURE.

(65) 42 Yards.
9

378 Feet.
12 × 12 = 144

4536
12

54432 Inches.

(66) 144 { 12)54432 Inches.
12)4536

9 378 Feet.

42 Square Yds.

(67) Sp. f. in.
3 42 64
100

342 Feet.
12 × 12 = 144

5104
12

49312 Inches.

(68) Sqr. in.
144 { 12)49312
12)4109—4 } 64 Inch.
1,00)3,42—5

3 Sq. 42 F. 64 Inch.

CUBIC or SOLID MEASURE.

(69) 27 Yards.
9×3=27

243
3

729 Feet.
1728 Inches=1 Foot.

15552
3456
12096

1259712 Inches.

(70) *Inches.* 1728)1259712(729 Feet.
12096

5011
3456

15552
15552

.....

27 { 3)729*ft.*
9)243

27 Solid Yards.

T.ft.
(71) 4 24
50

224 Feet.
1728

6912
3456
3456

387072 Inches.

Inches. *Feet.*
(72) 1728)387072(5,0 | 24
3456

4147
3456

6912
6912

.....

4 T. 24 Ft.

END OF BOOK I.

THE

KEY

TO THE

TUTOR's GUIDE,

OR, THE

Arithmetician's Repository.

BOOK II. PART I.

NUMERATION.

(1) 1000000+500000=1500000 South-Sea Bonds.
(2) 60000+12000+1300=73300 Lead.
(3) 150000000000+80000000=150800000000 Stivers.
(4) 120206070707 Rials of Plate.
(5) 3033030 Pieces of Eight.
(6) £.404000 34 $14\frac{1}{2}$=£.404001 15 $3\frac{1}{4}$.

ADDITION.

INTEGERS.

(1)		(2)		
	140724		27460	867
	296		176	317
	42		2900	69
	6740		274	1720
	64167		1004	276842
	20		64	49
	2687		596	426074
	2684		41	60
	217360		6104	705998
			38619	

7. MONEY.

To add these Examples, I begin with the First, the Farthings, going up, saying 1 and 3 is 4, and 2 is 6, and 3 is 9, and 1 is 10 Farthings, that is 2½d. or 4 in 10 is 2 times and 2 over, which is ¼ or ½ to put down under the Row of Farthings, and carry 2 Pence to the Unit's Place of Pence, saying 2 I carry and 1 is 3, and 6 is 9, and 1 is 10, and 9 is 19, and 6 is 25, and 4 is 29, and 1 is 30, then coming down the place of Tens, saying, and 10 is 40, and 10 is 50, and 10 is 60, and 10 is 70 Pence, (which by the Pence Table are 5s. 10d. or the 12's in 70 are 5 Times and 10 over, which is 10 Pence to set down under the Row of Pence, and carry 5 Shillings to the Unit's Place of Shillings, saying 5 I carry and 8 is 13, and 2 is 15, and 4 is 19, and 7 is 26, and 1 is 27, and 6 is 33, and 7 is 40; I set down 0, and carry 4 to the Place of Tens, saying 4 I carry and 1 is 5, and 1 is 6, and 1 is 7, and 1 is 8, and 1 is 9, half of which (or the 2's in 9) is 4, and 1 over, which I set down under the Place of Tens in the Row of Shillings, and carry the half, viz. 4, to the Unit's Place of Pounds, and proceed then as in Integers.

But the common Method (though not so concise nor yet so easy) is to go up the Unit's Place, and come down the Tens, as in the Row of Pence, which (in this Example) comes to 90 Shillings, or the 20's in 90 are 4 times and 10 over, thus 2,0)9,0(4l. 10s. so that it plainly appears to the Unit's Figure in the Divisor, or, what I stop at is, 0, so that the Unit's Figure in the Dividend, or what the Unit's Row of Shillings comes to above 10, will be to set down and to carry 1 for every 10, as in Integers, to the Place of Tens, and what that Sum is, divide it by that Figure which is in the Place of Tens in the Divisor, which in this Case is by 2, consequently is the Reason of my halving the Ten's Place of Shillings: Likewise when you stop at 40, 60, or &c. proceed as above; only instead of dividing by 2, you must divide by 4, 6, or &c. observing always to set down what remains under the Place of Tens.—See Case 3. Sect. 5.

(1) £.36 19 10½ (2) £.227 18 9 (3) £.472 11 3¼

	£.	s.	d.
(4)	270	16	6½
	60	0	10¼
	60	10	0¾
	96	6	10
	176	6	6½
	2	2	0
	16	17	6½
	100	0	0
£.	783	0	4½

	£.	s.	d.
(5)	260	17	0
	67	10	10¾
	170	10	0½
	100	10	6¼
	4	16	6½
	0	19	0¾
	37	11	11½
	600	10	0
	220	0	6½
£.	1463	6	6¾

	£.	s.	d.
(6)	276	17	0
	16	0	10¼
	269	11	11½
	107	19	0
	10	0	6
	0	14	11
	367	17	6¾
	0	12	4½
	20	10	6
	1000	0	0
£.	2070	4	8

WEIGHTS, MEASURES, &c.

Here I begin with the first Example, going up the Unit's Place in Grains, and find it comes up to 23, which, on a Slate, or waste Paper, I put down the 3, and carry 2 to the Place of Tens, and find it comes to 11, which I set down to the 3, and the whole of the Row of Grains make 113, which I divide by as many Grains as make 1 Penny-weight, viz. 24. and find it will go 4 Times and 17 over, thus 24)113(4 Times, and 17 remains; put down 17 Grains under the Row of Grains, and I carry 4 Penny-weights to the Row of Penny-weights, which I proceed with as in Shillings, as I stop at the same Number, viz. 20: Likewise, wherever I stop at 4, 12, or any Number under, I proceed as in Farthings and Pence, according to what Number I stop at; but when I stop at any number exceeding 12, as 16, 28, 54, &c. then I proceed as with the Grains above, so that it is impossible, without greatly perplexing the Learner, to perform these Examples before he has learned Division; and I am very sorry not to see this Method practised more than what it is, that so much of that precious Time of Youth should be lavished with perplexing Methods of the Teachers, as it is as improper to teach a Scholar to add these Examples before he knows the primary Rules in Integers, as it is to teach a Child to read before he knows his Letters.

(1) oz. 170 dwts. 5 grs. 17

(2) lb. 114 4 0 9

(3) ℥ 62 5 1 1

(4) 86 lb. 9 0 0 8

(5) Tons 731 14 0 23

(6) lb. 121 11 11

(7) Yards 581 0 1

(8) Eng. E. 569 4 0

(9) E. Fl. 575 2 1

(10) Lea. 528 2 3 10

(11) Yards 976 0 4 0

(12) A. 668 0 6

(13) Tuns 67 0 1 25 0

(14) Pun. 155 62 2 1

(15) Tier 127 27 7

(16) An. 88 9 3

(17) A. Hhds. 118 3 0

(18) B. Hhds 125 53 3

(19) A. B. 124 0 1 7 6

(20) B. Fir. 125 4 0 0

(21) Qrs. 378 4 0 0

(22) Cha. 138 35 1

(23) La. 78 0 2 1 0

(24) Mo. 110 1 6 10

(25) D. 513 18 0 0

APPLICATION.

(1) D.
June 29
July 31
Aug. 31
Sept. 30
Oct. 31
Nov. 30
Dec. 31
Jan. 27

Ans. 240 Days

(2)
Thus, 1772+60=1832 the year required.

(3)
£.
Youngest had 11000+1100+11=12111
Eldest 12111+1000=13111
Left to both 25222

(4) D.
Jan. 31
Feb. 29
Mar. 31
Apr. 30
May 31
June 30
July 31
Aug. 31
Sept. 30
Oct. 31
Nov. 30
Dec. 31

Ans. 366 Days.

(5)

	£.	s.	d.
Paid in Part	17	17	6
Rem. unpaid	82	2	6
Ans.	100	0	0

(6)

	£.	s.	d.
Private Men rec.	474	17	11½
Besides which each Officer rec.	467	0	0
Answ.	941	17	11½

(7)

	£.	s.	d.
Corn Chandler,	123	19	0
Brewer,	41	10	0
Butcher,	212	0	0
Baker,	24	0	6
Tallow Chandler,	13	8	0
Taylor,	137	9	9
Draper,	74	13	6
Coach-maker,	214	16	6
Wine merchant,	68	12	0
Confectioner,	16	2	0
Rent 8 zl. + 82 s. =	86	2	0
Servant's Wages,	46	1	0
Ex. on the Road,	50	0	0
Answ. £.	1108	14	3

(8)

	£.	s.	d.
Oats,	46	7	6
Beans,	100	0	0
Pease,	16	16	0
Barley,	73	0	8
Wheat,	56	9	10
Rye,	4	4	6
Water-carriage,	13	2	7
Charges,	1	13	0
Gains,	18	18	0
Answ. £.	330	12	1

(9)

	£.	s.	d.
A paid	13	2	6
B	2	13	0
C	0	14	4
D	1	9	8½
E	11	0	6¼
F	0	17	6
G	0	12	6
H	1	10	6
I 27 s. + 13	2	0	0
K 23 + 23 + 25 s. + 1 is. =	3	12	0
L	9	3	4
M	12	12	0
N	15	0	0
O 5 × 3 + 10 s. =	1	5	0
Answ.	76	2	10¾

(10)

	£.	s.	d.
To Mer. Wares,	418	2	6
Cheshire Cheese,	52	18	0
Broad Cloth,	317	12	10
Lead,	320	0	0
Bar Iron,	173	0	3
Copper,	1110	10	1
Accepted a Bill,	88	14	0
Do. on Honour,	50	0	0
Morocco Skins,	28	15	4
Conv. Insur. &c.	43	0	0
Warehouse Room, &c.	5	5	0
Factorage,	112	6	0
Answ. £.	2720	4	0

(11)

	oz.	dwts.
Fourteen Dishes, wt.	193	6
Thirty-six Plates,	421	11
Four Doz. Spoons,	104	6
Six Salts, - -	32	0
Knives and Forks,	83	9
Four Presenters,	113	4
Mugs, Tumblers, Beakers, &c.	264	18
Tea Kettle & Lamp,	126	9
And the rest of the Equipage,	93	2
Answ.	1432	5

(12)

	Cwt.	Qrs.	lb.
Bags. No. 1	2	2	10
— 2	2	1	16
— 3	2	0	24
— 4	0	1	16
Pocket, 58½lb. =	0	2	2½
Do.	0	2	2½
Answ. wt.	8	2	15

(13)

	M.	f.	p.
From hence to B,	39	6	0
thence to C,	46	0	24
- - D,	60	4	39
- - E,	37	6	0
From A to E is	184	1	23

(14)

	Years.	m.	d.
	28	0	0
	2	10	16
	1	11	0
	3	7	25
	16	9	27
Answ. Years	53	0	12

(15) Thus, 16+14=30 Years older.

(16) Thus, 17+29+23=69th Year, his Age required.

(17)

	Years.
When the Eldest was born the Father was	24
Then 19 Years and 19 Half Years = 19+9½ =	28½
The Youngest is now	21
Answ. Father's Age,	73½

(18) 20 left
21, 42, 84 } stolen the { 3, 2, 1 } Night.

A. 167 Sheep at first.

(19)

When Seth was born	Adam was	-	130 Years old.
Enos	Seth -	-	105
Cainan	Enos -	-	90
Mahaleel	Cainan -	-	70
Jared	Mahaleel	-	65
Enoch	Jared -	-	162
Mathuselah	Enoch -	-	65
Lamech	Mathuselah	-	187
Noah	Lamech	-	182
And when the Flood happened	Noah was		600

Answ. 1656 Years.

8. SUBTRACTION.

INTEGERS.

(1) From 476004	(2) 276000	(3) 40106
Take 120706	106019	27109
Remains 355298	169981	12997

8. MONEY.

As the upper Figures in the first Example are all larger than those which are underneath, so it is only taking the lower Line from the upper without borrowing at all; but in the second Example I begin and say, 2 Farthings from 1 I cannot, but 2 Farthings from a Penny, or 4 Farthings, there remains 2, and 1 the top Farthing makes 3 Farthings, which I set down thus, $\frac{3}{4}$; then I carry 1 to 11 is 12 Pence, which from 10 Pence I cannot, but 12 from 12 there remains 0, but 10 the upper Figure is to put down, and carry 1 to 11 is 12, from 10 I cannot, but 12 from 20 there remains 8, and the top Figure 10 is 18 Shillings to put down, and carry 1 to the Unit's Place of Pounds, which proceed with as in Integers.

Note. When the Learner is pretty ready at subtracting, he need not make use of the Words *I cannot*, as it is always seen at Sight whether he can or cannot; so that when he sees the lower Figure is greater than that which stands over it, always take the lower Figure from what you borrow, and to the Re-

mainder add the top Figure, and their sum set down; thus, in the 2d Example I say 2 from 4, 2 and 1 is $\frac{3}{4}$ to set down, and carry 1 to 11 is 12, from 12, 0, but 10 to set down, and carry 1 to 11 is 12, from 20, 8, and 10 is 18 to set down, and carry 1, &c.

	£.	s.	d.
(1)	21	13	$5\frac{1}{4}$

	£.	s.	d.
(2)	33	18	$10\frac{1}{2}$

	£.	s.	d.
(3)	87	7	$5\frac{1}{2}$

	£.	s.	d.
(4)	77	14	$5\frac{3}{4}$

	£.	s.	d.
(5)	10	9	$11\frac{1}{4}$

	£.	s.	d.
(6)	103	12	$11\frac{1}{4}$

(7)	£.	s.	d.
Paid	370	17	1
Rem. unpaid	105	2	11

(7)	£.	s.	d.
Received	1450	4	9
Rem. unpaid	26	9	9

(9)	£.	s.	d.
Borrowed	2648	19	$10\frac{1}{4}$
Paid	1843	14	3
Bal.	805	5	$7\frac{1}{4}$

WEIGHTS, MEASURES, &c.

(1) lb. 2 10 17 20 (2) oz. 3 13 7 (3) lb. 3 0 5 0 14

(4) T. 1 19 3 21 (5) lb. 2 11 15 (6) Yds. 66 3 1

(7) E. Eng. 2 4 3 (8) E. Fl. 58 0 1 (9) Lea. 39 1 4 27

(10) Yds. 50 2 11 (11) A. 67 1 16 (12) T. 1 0 0 43 1

(13) Pun. 3 75 3 0 (14) Tier. 3 40 5 (15) An. 6. 6 5

(16) A. hhds. 6 41 3 (17) B. hds. 1 48 5 (18) B. fir. 3 4 6

(19) A. Bar. 27 2 5 0 0 (20) Qrs. 4 4 3 (21) Cha. 6 25 2

(22) La. 4 0 4 4 3 (23) Mo. 2 3 4 2 3

(24) D. 156 16 42 45

APPLICATION.

(1) Thus, 1789—1735=54 Years old.

(2) First, 102—72=30 Diff. and 102 + 72=174 Sum.

(3)

	£.	s.	d.
A's Sum	74	17	0
Diff.	49	13	6
Ans. B's Sum £.	25	3	6

(4)

	£.	s.	d.
Borrowed	100	0	0
Paid	41	17	6
Ans. Rem. £.	58	2	6

(5)

	£.	s.	d.
Income - -	600	0	0
King's Tax -	140	0	0
Repairs - -	94	17	6
Pays in all -	234	17	6
Ans. neat Income	365	2	6

(6)

	£.	s.	d.
Left to both,	13111	10	6
Daughter 11,000+1100+11=12111	0	0	
Ans. the Son had £.	1000	10	6

(7)

	£.	s.	d.
Value of both, - - - -	35	10	0
Ditto Horse, - - - -	12	12	0
Ditto Furniture, - - - -	22	18	0
Answ. Difference, £.	10	6	0

(8)

Dr.	£.	s.	d.	Cr.	£.	s.	d.
To A. -	71	12	6	By Cash, -	3	13	6
B. -	34	9	9	Commodities,	23	10	0
C. -	16	8	8	Housh. Furnit.	13	8	6
D. -	44	0	0	Plate, -	7	18	5
E. -	66	7	6	Tenement,	56	15	0
F. -	11	2	3	Book Debts,	87	13	10
G. -	19	19	0				
H, 13s. 4d. × 30 =	20	0	0	Cr. £.	192	19	3
Dr.	283	19	8				
Cr.	192	19	3				
Ans. Bal. lost £.	91	0	5				

(9)

	£.	s.	d.
First Bond,	114	10	0
Interest,	19	0	0
Amount,	£.133	10	0
Paid off,	42	0	0
Second Bond,	91	10	0
Interest,	13	4	8
Amt.	£.104	14	8
Paid off,	37	14	2
Third Bond,	67	0	6
Interest,	9	11	3
Ans. Amt.	£.76	11	9

(10)

	£.	s.	d.
Altog. are worth	50	0	0
Horse and Harness	38	16	6
Chaise	£. 11	3	6
Chaise and Harness	13	13	0
Chaise	11	3	6
Harness	£. 2	9	6
Horse and Harness	38	16	6
Harness	2	9	6
Horse	£.36	7	0

(11)

	C.	qrs.	lb.
Gross	12	0	19
Tare	1	3	26
Net { first	10	0	21
Net { second	7	0	15
Answ. *C.*	3	0	6 Difference

Venture.

	C.	qrs.	lb.
Gross	8	2	2
Tare	1	1	15
Net	7	0	15

(12)

A. Dr.	£.	s.	d.
1793.			
Jan. 21, To Cash,	9	10	0
Feb. 29, Ditto	5	6	8
Mar. 17, Ditto,	12	2	6
19, Ditto,	5	5	0
	£. 32	4	2

Per Contra, Cr.

	£.	s.	d.
1793.			
Jan. 11. By Cash,	17	17	0
Feb. 24, Ditto,	42	0	0
Mar. 24, Ditto	11	18	4
Cr.	71	15	4
Dr.	32	4	2
Bal. to A.	39	11	2

B.	Dr.			Per Contra	Cr.		
1793.	£.	s.	d.	1793.	£.	s.	d.
Jan. 24, To Cash,	6	10	0	Jan. 11. By Cash,	34	11	6
March 2, Ditto,	6	6	0	Feb. 12, Ditto	16	14	0
19, Ditto,	5	5	0	March 14, Ditto,	17	8	8
				24, Ditto,	11	18	4
	£. 18	1	0				
				Cr.	80	12	6
				Dr.	18	1	0
				Bal. to B.	£. 62	11	6

C.	Dr.			Per Contra,	Cr.		
1793.	£.	s.	d.	1793.	£.	s.	d.
Jan. 30, To Cash,	19	8	4	Jan. 11, By Cash,	28.	18	10
March 19, Ditto,	5	5	0	21, Ditto,	12	5	0
				March 2, Ditto,	20	0	0
	£. 24	13	4	24, Ditto,	11	18	4
Sept. To A.	39	11	2	Cr.	74	2	2
— B.	62	11	6	Dr.	24	13	4
— C.	49	8	10				
				Bal. to Cr.	£. 49	8	10
Jointly,	£. 151	11	6				

(13) Stock, Dr.

	£.	s.	d.
To D. E	713	13	0
M. F.	352	10	8
L. P.	180	12	0
I. B.	57	12	10
Insurance,	190	0	0
£.	1494	8	6

Per Contra, Cr.

	£.	s.	d.
By Brandy,	874	10	6
Claret,	754	4	0
Corn,	675	17	3
Canary Seed,	113	0	0
Indigo,	632	12	0
Saffron,	253	5	0
W. P.	384	10	0
Wines p. F. G.	1011	10	0
Pepper p. S. Q.	1252	16	8
Bond on R. O.	300	0	0
Note on T. M.	260	14	0
India Bonds,	459	0	0
Interest,	25	14	0
Bank Stock,	2134	4	6
Banker,	1892	17	6
Cr.	11024	15	5
Dr.	1494	8	6
An. Pres. Worth £.	9530	6	11

(14)

Stock, Dr.

	£.	s.	d.
To debts, &c.	280	0	0

Per Contra. Cr.

	£.	s.	d.
By Cash, &c.	11505	10	0
By Trade he cleared	393	13	1
Cr.	11899	3	1
Dr.	280	0	0
Ans. Bal. £.	11619	3	1

(15)

Factor,	Dr. £	s.	d.	Per Contra,	Cr. £	s.	d.
To Tin, -	197	12	0	By Wines, -	226	16	6
Bees-wax, -	71	7	6	Figs, -	150	11	3
Stockings, -	47	3	6	Fruit, -	104	6	0
Tobacco,	943	15	10	Olives, -	136	10	0
Cotton, -	123	3	7	Oil, -	193	17	0
Wheat, -	116	5	6	Raisins, -	148	0	4
				Wool, -	75	13	8
Dr. £	1499	7	11	Commission,	71	18	11
Cr.	1102	13	8				
				£	1102	13	8
Employer draws for £	396	14	3				

(16)

	Y.	m.	w.	d.	h.	m.
He was to serve - - -	14	0	0	0	0	0
Of which he accomplished	Y.	m.	w.	d.	h.	m.
	11	11	11	11	11	11 — 12	1	0	4	11	11
Answer, to serve, Years	1	11	3	2	12	49

(17) W.

1784.	Dr. £	s.	d.	1784.	Per Contra, Cr. £	s.	d.
June 30, To Cash,	66	3	0	June 4, By Cash,	47	18	2
July 5, Ditto,	15	10	9	Note,	200	0	0
12, Ditto,	10	10	0	24 Ditto,	14	12	10
				July 15, Ditto,	52	0	0
£	92	3	9				
due to W.	222	7	3		314	11	0
	314	11	0				

X.	Dr.			Per Contra,	Cr.		
1784.				1784.			
June 24, To Cash,	47	10	8	June 4, By Bill,	33	14	9
July 7, Ditto,	7	3	1	Cash,	60	5	3
19, Ditto,	38	18	10	July 24, Ditto,	1	19	0
Bal. due to X.	26	6	5				
	£.119	19	0		£.119	19	0

Y.	Dr.			Per Contra,	Cr.		
1784.				1784.			
June 16, To Cash,	43	12	6	June 4, By Bal.	116	14	10
30, Ditto,	88	13	4	14, Ditto,	120	0	0
July 12, Ditto,	81	19	8	28, Ditto,	18	5	0
Bal. due to Y.	82	14	4	July 15, Ditto,	42	0	0
	£.296	19	10		£.296	19	10

Z.	Dr.			Per Contra,	Cr.		
1784.				1784.			
June 20, To Cash,	11	11	0	June 4, By Cash,	70	8	0
July 7, Ditto,	12	8	3	July 15, Ditto,	31	12	4
Bal. due to Z.	141	5	9	Assignment,	63	4	8
					23	19	3
	£.165	5	0		£.165	5	0

		£.	s.	d.
Bal. due to	W. -	222	7	3
	X. -	26	6	5
	Y. -	82	14	4
	Z. -	141	5	9
Jointly,		£.472	13	9

	Feet.
(18) Had at first, - - - -	127
Let off, 57+42 - - - -	99
Answer, remains - - - -	28

(19) Thus 42—14=28 the Difference required.
(20) Now 134—93=41 Son's Age.
And 112—41=71 Father's.
(21) First, 25—14=11 Difference of their Ages.
Then 25+11=36 C's Age.
(22) Now, 1789+13=A. D. 1802, B. born.
Then, 1802—1693=109 Years, A. older than B.

	lb.
(23) In fair Weather, - - -	33905
— foul, - - - -	30624
Answer,	3281

(24)	*Years.*	*Years.*	*Years.*
Before Christ	200	50	1784
Since	+1784	+1784	—140
Hip. and Arch. flourished.	1984	Passidonius 1834 and Ptolomy 1644 Y.	

(25)	*lb.*		*lb.*
Pekin Bell weighs	120000	Nankin Bell,	50000
From which take	94600	Erford Bell,	25400
Rem. Erford Bell,	25400	Answ. Difference	24600

(26)	*Years.*		*Years.*
Grandfather's Age,	119	Father's Age is	65
From which take	83	From which take	36
Leaves your Age	36	Answ. Difference	29

(27) *Feet.*

Goes up the first Day,	8	
Comes down at Night,	4	
	4	Feet made good the first Day.
Goes up the second Day,	8	
	12	
Comes down at Night,	4	
	8	made good the second Day.
Goes up the third Day,	8	
	16	
Comes down at Night,	4	
	12	made good the third Day.
Goes up the fourth Day,	8	
	20	the fourth Day at Night, Ans.

(28) First, 1784—1747=37.
then 37+24=61 C's
and 61+17=78 A's } Age.
also 78+13=91 B's

(29)

Conquest, - -	Anno	1066
To which add 134+10 -		144
Edifice finished, -	Anno	1210
The Peace of Utrecht,	Anno	1713
From which deduct		70
Demolition, -	Anno	1643
From which deduct -		1210
Answer, Duration, -		433 Years.

(30)

	Years.		*Years.*
Revolution,	Anno 1688	Forfeited in	Anno 1551
Forfeited before,	137	Granted -	1239
Forfeited in	Anno 1551	Answer, subsisted	40

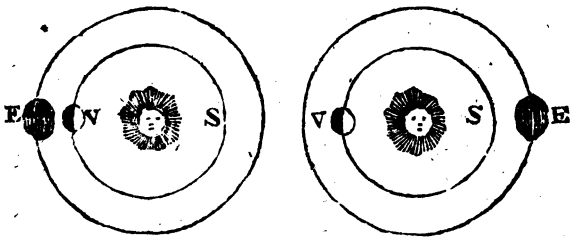
(31)

Moses born	Anno	2433	Christ born	Anno	4000
To which add		832	From which take		40
Homer born	Anno	3265	Cæsar born	Anno	3960
			From which take		312
			Alexander born	Anno	3648

Then 3960, 4000, 3648 — 3265 = 695 From Homer to Cæsar. 735 ——— Christ. 383 ——— Alexander.

Answer, 1813 Years, Sum of the Intervals.

(32) Fig. 1. Fig. 2.



Suppose S. the Sun; V. Venus; and E. the Earth; then Fig. 1 will represent them when in Perigæo, and Fig. 2, when they are in Apogæo.

81000000—59000000=22000000=S E—S V=E V Fig. 1, or the distance the Earth is from Venus, when in Perigæo.

And 81000000+59000000=140000000=V S+S E=V E Fig. 2, or the Distance the Earth is from Venus, when in Apogæo.

Therefore 140000000—22000000=118000000 Miles the Answer.

(33)

First	23+ 8=31 A.'s	Age.
And	31+19=50 B.'s	Age.
Also	50—14=36 C.'s	
Now	31+50+36=117 the Sum.	
Then	117—22=95 D.'s Age.	

(34)

When Selah was born Arphaxad was — 35 Years old.
—— Eber — Selah —— 30 ——
—— Peleg — Seber —— 34 ——
Eber lived after the Birth of Peleg — 430 Years.

From the Birth of Arphaxad to the death of Eber was —— } 529
Shem died after the birth of Arphaxad, 500

Eber was the Survivor by — 29 Years.

Then per Quest. $2+500+29+194=725$ Years, the whole Interval, and $1000-725=275$ Years the Interval wanted.

(35) First $19+27=46$ K.'s Age in 1720.
And $1738-1720=18$

Sum 64 K.'s Age in 1738.
Then $18+27=45$ L.'s —— ——

Sum, 109
$1740-1738=2$

Sum, $111-24=87$ Years the Age of M. in 1740.

(36) First $318+207=525$ B.
Then $525-104=421$ C.
And $421-84=337$ D. } Flourished.
Then $337+112=449$ E.
Also $449+47=496$ F.

(37) Sam was born before Toby 28 Years.
Toby died at - - 12 —— old.
After which Sam lived 19

Sam's Age, - - - 59 Years.
From which deduct $16+11=27$

Leaves Rachel's Age 32
To which add $7+4=11$

Gives Joshua's Age - 43 Years.

Then $12+59+32+43=146$, the Sum of their Age.

(38)

			Years.
B. born A. D.	1108	Z. born A. D.	1527
Lived before C.	48	After which Y. was born	74
C. born	1156	Y. born Anno	1601
Lived before D.	113	Before which X. was born	114
D. born Anno	1269	X. Born Anno	1487

(39) I shall be $34+17=51$ Years of age.
And you will be $70-34=36$

(40)

The Reformation, anno	1517
From which deduct	215
Invention of the Compass, Anno	1302
To which add	42
Gunpowder invented, Anno	1344
To which add	148
America discovered, Anno	1492
The Reformation,	1517
From which deduct	77
Printing invented, Anno 1440	1440

(41)

Restoration, Anno	1660
Grant made before	33
First Grant made, Anno	1627
Duration,	210
First Grant ended, Anno	1837
Reversionary Grant's Continuance,	99
Its Expiration, Anno	1936

(42) When B. is 41 A. will be $41-18=23$ Years old.
But when A. is 72, B. will be $72+18=90$ Years old.

(43)

Temple built, Anno	3000	Christ born A. D.	4000
Troy before	443	Rome built before,	744
Troy built, Anno	2557	——————A. D.	3256
London after	260	Carthage built before,	113
London built, Anno	2817	——————A. D.	3143
	——	London built,	2817
		London older than Carthage by	326

(44)

Suppose E the Earth, M the Moon, and S the Sun: then the Eclipse of the Sun will be represented by Fig. 3, and that of the Moon by Fig. 4.

Therefore 81000000—240000=80760000=S M Fig. 3, or the Distance these two Luminaries are asunder, in an Eclipse of the Sun.

Likewise 81000000+240000=81240000=S M Fig. 4, or the Distance these two Luminaries are asunder, in an Eclipse of the Moon.

Fig. 3.

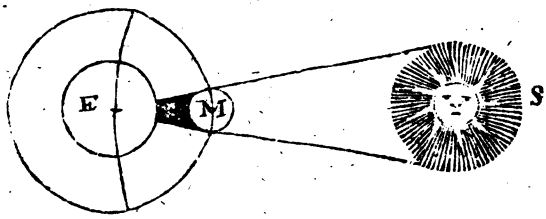
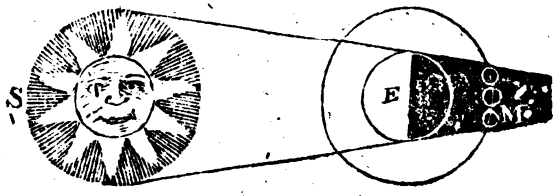


Fig. 4.



(45) From the Creation to the Flood, Years.
per Q. 10, P. 32.) 1656
Thence to the Building of Solomon's Temple, 1336
to Mahomet, 1630

Together, - - - - 4622
Mahomet after Christ, - - 622

Answer, Christ born, A. D. 4000

(46)

When Seth was born	Adam was	-	130 Years old.
Enos	Seth	-	105
Cainan	Enos	-	90
Mahalaleel	Cainan	-	70
Jared	Mahalaleel	-	65
Enoch	Jared	-	162
Methuselah	Enoch	-	65
Lamech	Methuselah	-	187
Noah	Lamech	-	182

When the Flood happened Noah was 600 Years.

Year of the Flood - - 1656
To Adam's Death 130+800= - 930

After his Death, - - - 726 Years.

(47)

	£.		£.
Miss Kitty's Fortune,	20000	And Miss Charlotte,	15000
Left by the Father,	13200	My Father,	13200
Left by the Grandmo.	6800	My Grandmother,	£.1800
	+1800		
Answer, left them,	£.8600		

(48) Reformation was in Anno - 1517
To which add - - - - - 88

Powder Plot was discovered 1605
To which add - - - 43

King Charles murdered, Anno 1648

Then 1714—54=1660 K. Charles II. returned.
And 1660—1648=12 Years, the Answer.

(49) First 180—47=133 Years since B. died.
And 161—133= 28 —— B.'s age.
Then 47+ 43= 90 —— B.'s age.
Sum, 118 the Answer.

(50) A. D. A. D.
First, 1789+28=1817 Ja. will be 50.
And, 1817—50=1767 Jacob.
Also, 1767—26=1741 Timothy.
Likewise, 1741—17=1724 Sampson.

(51) First, 1733—445=1288 A. born.
Then, 1362—1288= 74 A.'s
And, 37+ 18= 55 B.'s } Age.
Also, 256— 197= 59 C.'s
Again, 1733— 75=1658 D. died.
Then, 1658—1578= 80 D's. age.

(52) Reformation, Anno - - 1517
From which deduct - - 23

A. died Anno - - 1494
— born - - - 1441

53 A.'s age.

Then 1517+49= Anno - 1566 B. died.
And 1494—7 =— - - 1487 B. born.

79
Also, 36—7+9= - - 20 } A.
Then, 53+79+20=152 the Answer.

(53) A. D. A. D.
A. born 1438, lived 48 Years; then 1438+48=1486 died.
B. died 1502, bo. bef. 77 Yrs. ——1502—77=1425 born.
C. in 1577, was 22 Yrs. old, ——1577—22=1535 ——.
And in 1577+54=1631 died
D. died 1648, and in 1616 had lived ½ his Time; then
1648—1616=32 half his age, double of which is 64.
Then 1684, born bef. 64 Yr. then 1648—64=1584 born.
E. in 1648 was 13 Yrs. old, —— 1648—13=1635 born.
And in 1635+13+14+31+7=1700 died.
F. in 1635 born after 27 Yrs. then 1635+27=1662 born.
G. in 1662 ——— 31 ——— 1662+31=1693——

(54)

Anno		Anno		
1727—42	=	1685	A. died	Aged 47 Years.
1685—47	=	1638	— born	
1638+13	=	1651	B. born	Aged 53. ——
1712— 8	=	1704	— died	
1638—17	=	1621	C. born	Aged 91 ——
——	—	1712	— died	
1621—23	=	1598	D. born	Aged 64 ——
1598+64	=	1662	— died	
1704+11	=	1715	E. born	Aged 30 ——
1633+12	=	1745	— died	
1638—1598	=	40, Half of which is 20.		
1638—20	=	1618	F. born	Aged 113 Years.
1745—14	=	1731	— died	

The Sum of all their Ages is 398 Years.
Then 113—91=22 Years F. Survivor.

MULTIPLICATION.

INTEGERS.

(1) 14276084 × 4 = Prod. 57104336

(2) 20749509 × 9 = Prod. 186745581

(3) 1204674 × 12 = Prod. 14456088

(4) 4074746 × 16 = Prod. 65195926

(5) 147624 × 69 = Prod. 10186056

(6) 42768 × 748 = Prod. 31990464

(7) 10646 × 5278 = Prod. 56189588

(8) 14276 × 39674 = Prod. 566386024

(9) 3142708 × 467852 = Prod. 1470322223216

(10) 27680709 × 40700609 = Prod. 1126621713851781

(11) 2142760 × 4100 = Prod. 8785316000

(12) 21700 × 954000 = Prod. 20701800000

(13) Mul. 46904
By 9×3=27

Prod. 1266408

(14) Mul. 10709
By 8×7×6=336

3598224

The above Examples are performed as in Sect. 4.

CONTRACTIONS.

EXAMPLES.

(1) Thus, 476270000
47627

Prod. 476222373

(2) Thus, 2746400000
27464

9)2746372536

Prod. 305152504

(3) 46740000
4674

9)46735326

5192814
2

Prod. 10385628

(4) 476940000
47694

9)476892306

52988034
7

Prod. 370916238

(5) 747600000
7476

3)747525240

Prod. 249175080

(6) Thus, 427630000
42763

3)427587237

142529079
2

Prod. 285058158

(7) 4276
126

25656
51312=6×2=12

53[illegible]76

(8) Mul. 6946
486

41676
333408=6×8

Prod. 33[illegible]5756

COMPOUND MULTIPLICATION.

MONEY.

(1)

	£.	s.	d.
Mul.	14	17	11
By			2
Prod.	29	15	10

(2)

	£.	s.	d.
	140	10	$0\frac{1}{2}$
			6
	£.843	0	3

(3)

£.	s.	d.
17	6	4
		9
£.155	17	0

(4)

s.	d.
17	$6\frac{1}{2}$
	4
£.3	10 2

(5)

£.	s.	d.
3	0	6
		5
£.15	2	6

(6)

s.	d.
7	10
	7
£.2	14 10

(7)

s.	d.
18	$9\frac{1}{2}$
	8
£.7	10 4

(8)

s.	d.
12	8
	9
£.5	14 0

(9)

£.	s.	d.
2	6	4
		10
£.23	3	4

(10)

s.	d.
12	7
	11
£.6	18 5

(11)

£.	s.	d.
1	17	$6\frac{3}{4}$
		12
£.22	10	9

(12)

s.	d.
6	$7\frac{3}{4}$
	$7\times2=14$
£.4	13 $0\frac{1}{2}$

(13)

s.	d.
0	$10\frac{1}{4}$
	$6\times3=18$
£.0	15 $4\frac{1}{2}$

(14)

£.	s.	d.
2	9	6
		$9\times3=27$
£.66	16	6

(15)

s	d.
4	$11\frac{1}{2}$
	$6\ 5=30$
£.7	8 9

(16)

s.	d.
10	8
	$6\times6=36$
£.19	4 0

(17) d. $2\frac{1}{2}$ $9\times5=45$

£. 0 9 $4\frac{1}{2}$

(18) £. s. d. 1 7 0 $10\times5=50$

£.67 10 0

(19) s. d. 2 $7\frac{1}{2}$ $8\times7=56$

£.7 7 0

(20) £. s. d. 1 11 0 $8\times8=64$

£.99 4 0

(21) s. d. 15 9 $9\times8=72$

£.56 14 0

(22) d. $11\frac{3}{4}$ $10\times8=80$

£. 3 18 4

(23) s. 9 $12\times7=84$

£.37 16

(24) s. d. 1 $10\frac{3}{4}$ $12\times8=96$

£. 9 2 0

(25) s. d. 18 $11\frac{1}{2}$ $11\times9=99$

£.93 16 $10\frac{1}{2}$

(26) s. d. 11 10 $10\times10=108$

£.59 3 4

(27) s. d. £.1 6 $12\times10=120$

£.9 0 0

(28) s. d. 1 10 $12\times11=132$

£.12 2 0

(29) s. d. 1 $7\frac{3}{4}$ $12\times12=144$

£.11 17 0

(30) £. s. d. 1 4 $10\frac{1}{2}$ $4\times4+1=17$

£.21 2 $10\frac{1}{2}$

(31)

	s.	d.
	19	$11\frac{3}{4}$

$6\times3+1=19$

£.	s.	d.
18	19	$7\frac{1}{4}$

(32)

	s.	d.
	1	$7\frac{1}{2}$

$7\times4+1=29$

£.	s.	d.
2	7	$1\frac{1}{2}$

(33)

£.	s.	d.
2	17	6

$6\times6+2=38$

£.	s.	d.
109	5	0

(34)

	s.	d.
	5	10

$9\times5+2=47$

£	s.	d.
13	14	2

(35)

	s.	d.
	10	$4\frac{1}{2}$

$8\times7+2=58$

£.	s.	d.
30	1	9

(36)

£.	s.
5	17

$8\times8+3=67$

£.	s.
391	19

(37)

	s.	d.
	6	$4\frac{1}{2}$

$9\times8+3=75$

£.	s.	d.
23	18	$1\frac{1}{2}$

(38)

	s.	d.
	19	$1\frac{3}{4}$

$9\times9\times5=86$

£.	s.	d.
82	6	$6\frac{1}{2}$

(39)

	s.	d.
	7	6

$10\times10+6=106$

£.	s.	d.
39	15	0

(40)

£.	s.	d.
3	0	6

$7\times5+\frac{1}{2}=35\frac{1}{2}$

£.	s.	d.
[illegible]	7	9

(41)

£.	s.	d
1	12	10

$9\times8+4\frac{1}{2}+\frac{1}{4}=76\frac{3}{4}$

£.	s.	d.
125	19	$11\frac{1}{2}$

(42)

£.	s.	d.
1	16	$6\frac{1}{2}$

$4\times4+1+\frac{1}{4}=17\frac{1}{4}$

£.	s.	d.
31	10	4

(43)

£.	s.	d.
4	6	7

$8+\frac{1}{2}=8\frac{1}{2}$

£.	s.	d.
30	15	$11\frac{1}{2}$

(44)

£.	s.	d.	
26	17	6	
			10×10+½=100½
£.2700	18	9	

(45)

s.	d.	
1	10¼	
		7×4×4=112
£.10	12	4

(46)

s.	d.	
2	5	
		8×7×6=336
£.40	12	0

(47)

d.	
11¾	
10	7×5=350
£.17	2 8½

WEIGHTS, MEASURES, &c.

Proceed here in the like manner with the Product of each Denomination, as you did with the Sum of each Denomination in Addition.

(1)

lb.	*oz.*	*dwts.*	*grs.*
14	10	0	21
			4
59	4	3	12

(2)

T.	*C.*	*qrs.*	*lb.*
17	17	0	24
			2
35	14	1	20

(3)

C.	*qrs.*	*lb.*	*oz.*	*drs.*
14	0	21	0	14
				7
99	1	7	6	2

(4)

lb.	℥	ʒ	℈	*grs.*
10	6	4	1	17
				9
94	11	1	1	13

(5)

Yds.	*qrs.*	*na.*
127	0	3
		12
1526	1	0

(6)

E. E.	*qrs.*	*na.*
40	4	2
		11
449	4	2

(7)

Lea.	*m.*	*fur.*	*p.*
120	0	7	24
			5
601	1	6	0

(8)

Yds.	*f.*	*in.*	*b.c.*
147	2	11	2
			6
887	2	10	0

(9)

W. hds.	*ga.*	*pts.*
46	47	7
		3
140	17	5

(10)

Tu.	*p. hd.*	*g.*	*qts.*
6	1	1	46
			8
55	0	1	59

	Tier.	*g.*	*qts.*
(11)	27	41	2
			6
	167	39	0

	B. hds.	*g.*	*pts.*
(12)	4	47	6
			9
	43	51	6

	A. hds.	*gal.*	*q.*	*p.*
(13)	10	17	3	1
				4
	41	23	2	0

	B. b. fi.	*g.*	*p.*
(14)	12	2	7 7
			6
	76	1	2 2

	A.	*r.*	*p.*
(15)	140	2	26
			5
	703	1	10

	La. qr.	*bu.*	*p.*
(16)	74	7 4	1
			7
	523	2 5	3

	D.	*h.*	*m.*	*sec.*
(17)	365	5	48	57
				12
	4382	21	47	24

APPLICATION.

(1) Now, $54 \times 54 = 2916$.
And $46 \times 19 = 874$.
Then $2916 - 874 = 2042$, the Number required.
For $2916 - 2042 = 874$ = Proof.

(2) Thus, $142 \times 50 = 7100$*l.* the Sum required.

(3) Now, $12 \times 12 \times 6 = 864$ = Six Dozen Dozen.
And $12 \times 6 = 72$ Half a Dozen Dozen.

Answer, { 792 Difference.
936 Sum.

(4)

52 Counties.
7×6=42

364
6

2184 Parishes.
246

13104
52416=6×4=24

537265 Houses.
10

5372640 Persons.

(5)

First 28×2 =56
And 8×2+20=36

Answ. Difference 20

Again 55×2= 110
And 5×2+50= 60

Answ. Difference 50

(6)

	Years.	£.		£.
The first	4 he cleared	364	per Ann.	=364×4=1456
The next	3 ———	586		=586×3=1758
And the last	3 ———	873		=873×3=2619

His whole Gain, £.5833

Then 13000—5833=7167 his original Stock.

To find the State of his Fortune at each Year's End, thus: He began with 7167. cleared the first Year 364*l.* which, added to 7167*l.*=7531*l* his Worth at the first Year's End; then 7531+364=7895*l.* at the second Year's End: and in this manner proceed by a continual adding the preceding Year' Gain.

(7)

	£.	*s.*	*d.*		£.	*s.*	*d.*	
Each Parish paid	37	16	4	then	373	14	8	assessed.
Hamlets	28	3	10		343	16	8	paid.
Townships	19	19	0					
Sum	85	19	2	Ans.	£29	18	0	the Defic.
			×4					
£.343	16	8	paid in all.					

(8)

	C.	*qrs.*	*lb.*	
C. qrs.	137	2	10	Gr. Wt.
13 1×3=39	3	0		Tare.
Answ. *C.*97	3	10		Net Weight.

(9)

		£.	*s.*
To the Widow,	- - -	1780	0
——— Children,	- 1250× 4=	5000	0
——— Relations,	31*l.* 10*s.*×5×3 15 =	472	10
——— Charity	- - -	150	0
	£. Died worth	7402	10
Gained by Trade,	126×5×5+½ 25½ =	3213	0
	Answer, begun with £.	4189	10

(10)

		£.	*s.*	*d.*
Income,	- - - -	500	0	0
Expends,	19*s.* 11*d.* 9×8×5 365)=	363	9	7
	Answer, lays up	136	10	5

(11)

		£.	*s.*	*d.*
Expends,	1*l.* 12*s.* 6*d.*×9×8×5+5(365)=	593	2	6
Lays up,		294	12	6
	Answer, Income	887	15	0

(12) First 20+423+19=462 the Divisor.
Then 423×462+20=195446, the Dividend.

(13)

	£.	*s.*	*d.*	
Rent,	7	7	6	per Quar.
Repair	0	18	6	———
Tax	0	8	9	———
Sum	8	14	9	———
		4		
Answ. £.	34	19	0	per Ann.

(14)

		£.	s.	d.
Widow,		50	0	0
Charity,	£.	846	10	0
Nephew,	1230×3=3690	0	0	
Nieces,	1050×4=4200	0	0	
House-keepers,	5*l.* 5.×5×4 20)=	105	0	0
Executors	200 Guin.=	210	0	0

Answer, died worth £.14051 10 0

(15)

£.	s.	d.
100	0	0
1	7	0
0	10	6

In each Division was 101 17 6
×6

In each Drawer was 611 5 0
×12

Answer, £.7335 0 0

(16)

Yards.
73726 at one Circumvolution.
3

221178 in a Minute.
60

13270680 in an Hour.
10

132706800 in a Day.
365—63= 302 Days worked.

2654136
3981204

Answer, 400774536oo Yards.

(17) First 20× 60=1200 Feet, Piers stand on.
And 21×170=3570——Arches Span.

Width of the Danube, 4770
——————Thames, 1200

Answer, 3570 Feet the Difference.

(18)
Now 187 is the lesser Number.
And 187+34=221 the greater.
Then 221×187=41327 their Product.
Also, 41327 41327=1707920929 Square of Ditto.
Likewise, 187+221=408 their Sum.
And 408×408=166464 Square of Ditto.
Also, 34×34=1156 Ditto of their Difference.
And lastly, 1707920929+166464+1156=1708088549 the Sum of those Squares.

(19) First 109×73=7957 the greater Number.
And 28×17=476 Difference.

7481 lesser Number.

Then 7957+7481=15438 their Sum.
And 7957×7481=59526317 their Product.

(20)

		Acres.
A. had		757
B		2104
C.		16410
D.		12884
E.		11008
F.		9813
H.		13800
I.		8818
Wanted,		416
Sum, 76007=$\frac{1}{5}$ of the Whole.		
		5

Answer, 380035 Acres.

(21) One comes up 6 Ways.
6
Two comes up 36
6
Three comes up 216
6
Four comes up 1296 the Ans.

(22)

	£.	s.	d.		£.	s.	d.
100 G. =	105	0	0				
Ten Marks =	6	13	4				
W. had	98	6	8	-	98	6	8
	—0	16*	8	-			
R.	97	10	0	-	97	10	0
	—6	0	0				
T.	91	4	0	-	91	4	0
	+3	17	2				
S.	95	1	2	-	95	1	2

Answer, £.382 1 10

DIVISION.

INTEGERS.

(1) 4)14076893

3519223—1

(2) 12)30742165

2561847—1

3) 89)2410296)27081$\frac{87}{89}$

Quot. Rem.

(4) 576 98420649 170869—105

(5) 3029 308763705 101935—2590

(6) 46058 1622121249(352190—45479

(7) 127345)51799555(406—97485

(8) 3090807)78855994985 2551$\frac{8}{9}$—235994

(9) 37,00)1276421427(344978—2827

(10) 827,000)4074964.478 4927—335478

(11) 93,00 247698514,00(2663424$\frac{82}{93}$

(12) 14 { 2 } 67402 / { 7 } 33701

4814—6

(13) 63 { 7 } 701747 / { 9 } 100249—4 } 53

11138—7

(14) 112 = 4 × 4 × 7

4	4170642
4	1042660—2
7	260665
	37237—98

(15) 336 = 6 × 7 × 8

6	2741724
7	456954
8	65279—1
	8159—300

EXAMPLES.

(16)

1,000)4677	823
4	677
	4
4681	1504

Then 1504, the sum of the Overplusses, divided by 999, the Divisor, gives 1, and 505 remains.

So $4681\frac{505}{999}$, = the Quotient required.

17 First, 2692464

×9

Then 4599 ÷ 9 = 511, the Remainder.

2423	2176
	2423
2423	4599

So $2423\frac{511}{1111}$ is the Quotient required.

(18) First, 4769042

×9

Then 5670 ÷ 9 = 630, and 630 + 1111 = 1741, the Remainder; so the Quotient is $613\frac{1741}{7777}$.

10000)4292	1378
	4292
7)4292	5670
613—1	

(19) 109494
9

10000)98 | 5446

6)98 Then $5446 \div 9 = 605$, and $6+11+2222 = 2838$ the Remainder, $\therefore 16\frac{2838}{5556}$ is 16—2 the Quotient required.

(10) COMPOUND DIVISION.

MONEY.

Here, in the 3d Example, I divide by 8, saying the 8's in 17 is 2 Times and 1 over, the 8's in 11 is 1 and 3 over, which is 3*l.* this 3*l.* I carry to the 11, and it is 3*l.* 11*s.* or 71*s.* then I say the 8's in 71 is 8 times 8 is 64, and 7*s.* over, which I carry to the 4 Pence, and is 7*s.*. 4*d.* or 88 Pence: then I say the 8's in 88 is 11 Times, and nothing over, and so the Quotient is *l.*21 8*s.* 11*d.* In the same manner proceed with all the rest.

EXAMPLES.

(1)	(2)	(3)	
£. s. d.	£. s. d.	£. s. d.	
2)14 16 10½	7)267 0 0	8)171 11 4	
Quot. 7 8 5¼	38 2 10¼ 1/7	21 8 11	
(4)	(5)	(6)	
£. s. d.	£. s. d.	£. s. d.	
9 3)17 0 6	11)6 6 11½	3)7 11 6	
Quot. 35 4 6	0 11 6½	2 10 6	
(7)	(8)	(9)	
£. s. d.	£. s. d.	£. s. d.	
10)3 17 1	12)14 14 0	16 { 4 45 12 8	
Quot. 8½	£1 4 6	4)11 8 2	
		£2 1 0½	

(10) £. s. d.
24 { 4 3 13 0
6) 0 18 3
£. 0 3 0½

(11) £. s. d.
36 { 6 6 4 19 0
6) 10 16 6
£. 1 16 1

(12) £. s. d.
42 { 6) 190 4 6
7) 31 14 1
£. 4 10 7

(13) £. s. d.
48 { 6) 37 14 8
8) 6 5 9¼
£. 0 15 8½

(14) £. s. d.
32 { 4) 78 16 8
8) 19 14 2
£. 2 9 3¼

(15) £. s. d.
55 { 5) 4567 0 10
11) 913 8 2
£. 83 0 8¾

(16) £. s.
63 { 7) 264 12
9) 37 16
£. 4 4

(17) £. s. d.
72 { 8) 18 18 0
9) 2 7 3
£. 0 5 3

(18) £. s. d.
81 { 9 121 12 6
9) 13 10 3¼
1 10 0 113/729

(19) £. s. d.
120 { 10) 174 1 8
12) 17 8 2
1 9 0 1/6 2

(20)

£.	s.	d.	
17)214	17	$9\frac{1}{4}$	12£.
17			
44			
Rem. 10*l.*			
20			
17)217			(12*s.*
47			
Rem. 13*d.*			
12			
17)165			(9*d.*
Rem. 12*d.*			
4			
17)49			($\frac{1}{2}$qrs.
Rem. 15			
17			
Answ. £.12	12	$9\frac{1}{2}$	$\frac{15}{17}$

(21)

£.	s.	d.	
145)476	0	0	(3*l.*
435			
41			
20			
145)820			(5*s.*
725			
95			
12			
145)1140			(7*d.*
1015			
125			
4			
145)500			($\frac{3}{4}$
435			
$\frac{65}{145}$			
Answ. £.3	5	$7\frac{3}{4}$	$\frac{13}{20\,45}$

(22)

£.	s.	
112 { 4)30	2	
4) 7	10	6
7) 1	17	$7\frac{1}{2}$
£.0	5	$4\frac{1}{2}$

(23)

£.	
350 { 5)1009	
7)201	16
10 28	16 $6\frac{1}{2}$
£.2	17 $7\frac{3}{4}$

(24)

£.	s.	
224 { 4)610	8	
7)152	12	
8) 21	16	
£.2	14	6

(25)

£.	
640 { 8)1426	
8) 178	5
10) 22	5 $7\frac{1}{2}$
£.2	4 $6\frac{1}{2}$

(26) $6\frac{1}{4}$ × 4 = 25

£.	s.	d.
5	12	12 11
5)	2	10 7
	0	10 $1\frac{1}{4}$ per Qr.
		4
£.	2	0 5

(27) $10\frac{1}{2}$ × 7 = 21

£.	s.	d.
3)1061	8	$9\frac{3}{4}$
7) 353	16	$3\frac{1}{4}$
	50	10 $10\frac{3}{4}$
		2
£.101	1	$9\frac{1}{2}$

(28) $8\frac{3}{4}$ × 4 = 35

£.	s.	d.
5)450	13	$11\frac{1}{2}$
7) 90	2	$9\frac{1}{2}$
	12	17 $6\frac{1}{2}$
		4
£.51	10	2

WEIGHTS, MEASURES, &c.

(1)

lb.	*oz.*	*dwts.*	*grs.*
2	8	1 15	8
Quot. 4	0	17 16	

(2)

T.	*C.*	*qrs.*	*lb.*
3)24	14	0	14
8	4	2	$23\frac{1}{3}$

(3)

C.	*qrs.*	*lb.*	*oz.*	*drs.*
4)17	2	27	14	15
Quot. 4	1	20	15	$11\frac{3}{4}$

(4)

lb.	℥	ʒ	℈	*grs.*
5)4	11	4	2	12
0	11	7	1	$2\frac{2}{5}$

(5)

Yds.	*qrs.*	*na.*
9	2 14	3 2
Quot. 23	3	2

(6)

E.E.	*qrs.*	
8)120	4	
15	0	2

(7)

Lea.	*m.*	*fvr.*	*p.*
9	12	2 0	26
1	1	1	34

(8)

T.	*s.*	*f.*	*in.*	*b.c.*
10	14	7 2	11	2
14	2	4	2	$\frac{1}{10}$

(9)

W.hds.	*ga.*	
11)24	57	
2	16	2 $1\frac{1}{11}$

(10)

Tu.	*p.*	*hd*	*g.*	*qts.*
8)10	1	1 60		3
1	0	1 31		$0\frac{7}{8}$

	Tier. g. pts.	*Hhds. gal.*	*B. hhds. g. qts.*
(11)	6)16 20 7	(12) 5)76 27	(13) 4)12 49 2
Quot.	2 31 $3\frac{1}{6}$	15 15	3 12 $1\frac{1}{4}$

	B.b. fi. g.	*A. r. p.*	*La. qr. bu. p.*
(14)	3)61 2 6	(15) 12 140 2 26	(16) 7)60 6 7 2
Quot.	20 2 $\frac{1}{2}$	11 2 $35\frac{1}{2}$	8 6 5 $2\frac{1}{7}$

D. h. m. sec.

(17) 6)146 23 24 56

Quot. 24 11 54 $9\frac{1}{3}$

APPLICATION.

(1) Thus 22,0000÷1,0000=22*l.* each person.

(2) Thus, 336÷12=28 Miles per Day.

(3) First, 4429÷43=103, then 240—103=137 the Ans.

(4) First, 2262÷26=87, and 2262÷87=26.
Then 87—26=61, the Number required.

(5) 5190048÷72084=72, the Number required.

(6) First, 419844÷3=139948 the Remainder.
And 419844×9494=3986999936 Pro. of Div. and Quot.
Then 3985998936+139948=3986138884 the Answer.

(7) First 360—144=216 the greater Number.
And 216—144=72 their Difference.
Also 216×144=31104 Product.
Likewise 216÷144=$1\frac{1}{2}$ the larger Quot.

(8) First 3 Tons=3×20=60 Cwt. then 60÷15=4 Cwt. per Man, the Answer.

(9) First, 22525÷25=901 and 9696÷16=606.
Then 1440+901=2341, and 2341—606=1735 the Ans.

(10) First 134×71=9514, consequently 9514÷57=166$\frac{2}{7}$ the Answer.

(11) Fourscore and thirteen millions=93000000.
Then 93000000÷30079=3091, and the Remainder is 25811, from which deduct 21180, leaves 4631 Excess, the Answer.

(12)

10)1640 Guineas.
+82

	£.	s.	
£.1722	1722	0	College.
×4			
£.6888	6888	0	Wife.
—1383			
2 5505	5505	0	Daughter.
+2752 10			
£.8257 10	8257	10	Eldest Son.
2)12393 0 Sum of the Mo. and Sist.			
£.6196 10	6196	10	Youngest Son.
	988	10	Expenses on his Burial.
	29557	10	died worth.

Then from 30000 take 29557 leaves 442*l.* 10*s.*

(13)

s.	*d.*	
8)12	8	Worth of both.
1	7	Ditto of the purse.

Answer, £.0 11 1 Cash in ditto.

(14)

	£.	s.	d.	£.	s.	d.	
Brother owed at first,	74	18	2				
Paid in Part, -	41	14	8				
Remains on balance,	2)33	3	6	33	3	6	Bro.
Half of which is,	×16	11	9				
Sister owed at first,	49	15	3				
Paid in part, - -	—13	12	10				
Remains on Balance,	36	2	5	36	2	5	Sister.
To which add the Brother's,	33	3	6				
Uncle William owed,	69	5	11				
Paid in Part, - -	—24	7	3				
Rem. on B. unpaid by him,	44	18	8	44	18	8	Uncle.
All together owed, -				£114	4	7	

Then from 150*l.* take £.114 4 7 leaves £.35 15 5

(15)

	C.	qrs.	lb.		£.	s.	d.
Weight together,	9	3	16	Cost,	97	17	6
Difference,	—1	2	16	Diff.	8	13	3
	2)8	1	0 Rem.		2)89	4	3
Lesser Weight,	4	0	14	Cost,	£.44	12	1½
Difference,	+1	2	16	Diff.	8	13	3
Greater Weight,	5	3	2	Cost,	£.53	5	4½

(16)

	£.		
In Cash, -	100000		
Bills, £.54 10	6×8=436	4	
Left in all,	£.10436	4	
Debts, &c.	36+260=296	0	
	9)10140	4	0 Neat.
Daughter had,	£. 1226	13	$9\frac{11}{4}$
	7 9013	10	$2\frac{1}{2}\frac{2}{3}$
Sons had each,	£1287	12	$10\frac{11}{19}\frac{5}{8}$

(17)

First, $2072 \div 14 = 148$ Elms in each Row; then $148 - 1 = 147$ vacancies, and $147 \times 25 = 3675$ Feet, or 1225 Yards, the Length of the Grove required.

(18) $32 \left\{ \begin{array}{l} 4)384 \\ \hline 8)\ 96 \\ \hline \end{array} \right.$

(19) First $129 + 178 = 307$
Then $1000 - 307 = 693$
And $693 \div 3 = 231$ B's
Also $231 + 129 = 360$ A.'s
Likewise $231 + 178 = 409$ C.'s } Crowns.

An, in Rank 12 Men.

20 First $37 - 28 = 9$, then $250 - 9 = 241$.

$\therefore 3 : 241$
gives $80\frac{1}{3}$ B.'s
And $80 + 37 = 117\frac{1}{3}$ A.'s } £.
Also $80 - 28 = 52\frac{1}{3}$ C.'s

(21) $207 + 213 + 189 + 234 + 222 + 250 = 1315$ the Sum, which being divided by 5, the Times each being mentioned, viz. - - $5)1315$

Quotient, - - -	263
To which add Fourscore and 13 =	93
Sum, - - - - -	356
From which deduct - -	1
Leaves, Answer,	355

(22)

First $10000 - 8500 = 1500$ O.'s - -
And $8500 - 6050 = 2450$ L.'s -
Then $6050 - 420 = 5630$, and $5630 \div 2 = 2815$ M.'s } Subscription.
Consequently $2815 + 420 = 3235$ N.'s -

(23) $46)1610(35$ the lesser N°. then $46 + 35 = 81$ their Sum, and $46 - 35 = 11$ their Difference; and also $46 \div 35$ $1\frac{11}{35}$ their Quotient.

Then $46 \times 46 = 2116$ the Square of greater Number.
And $35 \times 35 = 1225$ ——————— lesser.

3341 the Sum of those Squares.

Again, $11 \times 11 \times 11 = 1331$ the Cube of their Difference.

(24)

First $7050 \div 94 = 75$ the lesser Number.
And $7050 \times 7050 = 49702500$ the Square of the greater.
Also $75 \times 75 = \ldots\ldots 5625$ Ditto of the lesser.

49696875 Difference of those Squares.

Again $7050 + 75 = 7125$ their Sum, and $7050 - 75 = 6975$ the Difference; then $7125 \times 6975 = 49696875$ the Product of their Sum and Difference.
Consequently $49696875 \times 49695875 = 2460779384765625$ the Square of the Product of their Sum and Difference.

(25) Suppose the Expence or Profit to be 2.
Then $2 \times 2 = 4$ double the Expence.
And $2 \div 2 = 1$ half the Profit.
Answer, Difference 3, or as 4 to 1.

(26) First B. is to have 72
And C. $72 + 112 = 184$ } more than A.

Sum, 256

Then $1500 - 256 = 1244$
$\therefore$ 3 | 1244

gives $414\frac{2}{3}$ A.'s
And $414\frac{2}{3} + 72 = 486\frac{2}{3}$ B.'s } Share.
Also $486\frac{2}{3} + 112 = 598\frac{2}{3}$ C.'s

(27) First $25000 + 33000 + 30000 + 28000 + 32000 = 148000$ their Sum; then each being repeated 4 times that Sum must be divided by 4, viz.

4) 148000

Which gives 37000*l.* Sum of their Fortunes, then

From	Take	Rem.	
£. 37000	25000 =	12000*l.*	the Youngest's Fortune.
	33000 =	4000	—— Eldest's.
	30000 =	7000	—— Second's.
	28000 =	9000	—— Fourth's.
	32000 =	5000	—— Miss Charlotte's Fortune.

Proof, 37000*l.*

(28)

		d.	s.	d.
First	120 at 2 for 1d. = 120÷2 = 60 or		5	0
And	120 at 3 —— = 120÷3 = 40 =		3	4
			8	4
Then 240 at 5 for 2d. = 240÷5 = 48 Twopences, or			8	0 Sold for.
		Difference	0	4 Lost.

(29)

	£.	s.	d.
A. and B. had	13	10	0
B. and C.	12	12	0
A. and C.	11	16	6
Sum, £.	37	18	6

Which being divided by the Number of Players at each Time will give the Sum won, viz.

£. s. d.

£.37 18 6÷2 = 18 19 3 what was won.

Then from £. s. d. 18 19 3

Take { 13 10 0 / 12 12 0 / 11 16 6 } Rem. { 5 9 3 C.'s / 6 7 3 A.'s / 7 2 9 B.'s } Gain.

(30)

	£.	s.
W. X. and Y. advanced	350	10
W. X. and Z.	344	10
X. Y. and Z.	400	0
W. Y. and Z.	378	4
They being mentioned each 3 times, } 3)1473	4	Sum.
£.	491	1	4 Joint Property.
Sold for 450 Guineas, or	472	10	0
Answer, lost £.	18	11	4

(31) Worth at the End of $3\frac{1}{4}$ Years, £. 3179 11 8
$4/100l.$ = 25 0 0

$3\frac{1}{4}$ Years = 13 Quarters, 13)3204 11 8
—246 10 $1\frac{1}{2}$

Worth at the end of 3 Years, £.2958 1 $6\frac{1}{2}$
+ 100 0 0

4/3058 1 $6\frac{1}{2}$
— 764 10 $4\frac{1}{2}$

Worth at the end of 2 Years, £.2293 11 2
+100 0 0

4/2393 11 2
— 598 7 $9\frac{1}{2}$

Worth at 1 Year's End, £.1795 3 $4\frac{1}{2}$
+ 100 0 0

4)1895 3 $4\frac{1}{2}$
— 473 15 10

Answer, he began with, £.1421 7 $6\frac{1}{2}$

(32) £.
First, $10 \div 5 = 2l.$ each Man's Share, supposing they had attended equally.
And $2l. \div 7 = 5s.\ 8\frac{4}{7}d.$ each Man's daily Pay.
Then $5s.\ 8\frac{4}{7}d. \times 2 = 11s.\ 5\frac{1}{7}d.$ what C. and D. must each forfeit.
$\therefore$ $11s.\ 5\frac{1}{7}d. \times 2 = £.1\ 2\ 10\frac{2}{7}d.$ C. and D.'s whole Defaults.
And $£.1\ 2\ 10\frac{2}{7}d. \div 3 = 7s.\ 7\frac{1}{7}d.$ what A. B. and E. each received of C. and D.'s Default.
Also $5s.\ 8\frac{4}{7}d. \div 4 = 1s.\ 5\frac{1}{7}d.$ what A. B. C. and D. each received of E.'s Default.

Therefore, $2£. + 7s.\ 7\frac{3}{7}d. + 1s.\ 5\frac{1}{7}d. = 2\ 9\ 0\frac{4}{7}$ A.
Ditto, - - - 2 9 $0\frac{4}{7}$ B.
$2£.\ 1s.\ 5\frac{1}{7}d.$ less $11s.\ 5\frac{1}{7}d. = 1\ 10\ 0$ C.
Ditto, - - - 1 10 0 D.
Also, $2£. + 7s.\ 7\frac{3}{7}d.$ less $5s.\ 8\frac{4}{7}d. = 2\ 1\ 10\frac{4}{7}$ E.

} Received.

(33) First 12—1=11, and 11×2=22 she had before she met the last Boy; then 22—2+10=30 she had before she met the second Boy; consequently 30—10=20 what she had before the first returned her back 10; so that 20 being multiplied by 2=40, the number of apples she had at first.

(11) REDUCTION.

MONEY.

(1)
130£.
20
2600 Shillings.
12
31200 Pence.
4
124800 Farthings.

(2)
4)24000 Qrs.
12 6000 Pence.
2,0,500 Shillings.
25£.

(3)
£.80 15 11¼
20
1615
12
19391
4
77567 Farthings.

(4)
4)16921 Qrs.
12)4230¼
2,0)35,2 6
£.17 12 6¼

(5)
£.110 0 6½
20
2200
12
26406
2
52813 Halfpence.

(6)
2)20553 Half-pence.
12)10276½
2,0 85,6—4
£.42 16 4½

(7) £.107 10 8

20

2150

6 Twopences=1*s.*

12904 Twopences.

(8) 6)5348 Twopences.

2,0 89,1—2 over=4*d.*

£.44 11 4

(9) £.6 17 0

20

137

4 Threepences=1*s.*

548 Threepences.

(10) 4)2782 Threepences.

2,0)69,5—2 or 6*d.*

£.34 15 6

(11) £.10 10 8

20

210

3

632 Fourpences.

(12) 3)3859 Fourpences.

2,0)128,6—4

£.64 6 4

(13) £.200 17 0

20

4017

2

8034 Sixpences.

(14) 2)795 Sixpences.

2,0)39,7—6

£.19 17 6

Any Number of Pounds, &c. may be redueed to Sixpences, Fourpences, Threepences, or Twopences, by multiplying the Pounds by as many of each Denomination that make one Pound; that is, to reduce Pounds, &c. to Sixpences, multiply by 40, if to Fourpences by 60, if to Threepences by 80, and if to Twopences by 120, observing to add in the odd shillings and Pence, if any; accordingly, on the contrary, any number of Sixpences, Fourpences, &c. may be reduced to Pounds, by

dividing by as many as make one Pound, observing to value the Remainder (if any) right; thus, the two last Examples.

(13) £.200 17 0
40× 34 Sixpences=17*s.*
8034 Sixpences.

(14) Sixpences. *s. d.*
4,0)795 and 35 over=17 6
£. 19 17 6

(15) 21 Guineas.
7×3=21
147
3
441 Shillings.
12
5292 Pence.
4
21168 Farthings.

(16) 4)24192 Farthings.
12)6048
21 { 3)504 / 7)168
24 Guineas.

(17) 12 Moidores.
27
324 Shillings.
12
3888 Pence.
4
15552 Farthings.

(18) 12)3240*d.*
27)270
10 Moidores.

(19) 30£.
4
120 Crowns.
2
240 Half Crowns.
30
7200 Pence.

(20) 3,0)20160 Pence.
2 6720—Half Crs.
4) 3360 Crowns.
£.840

(21) 25 Crowns.
5
125 Shillings.
3
375 Groats.
4
1500 Pence.

(22) 4)25200 Pence.
3 6300 Groats.
5)2100 Shillings.
420 Crowns.

(23) 25£.
20
5)500 Shillings.
100 Crowns.
60
6000 Pence.

(24) 80 Crowns.
5
2,0)40.0 Shillings.
20£.

(25) A Crown=60
Half do.=30
Shilling=12
One of each 102

£.213 15 6
20
4275
12
102)51306(503 of each.
306
...

(26) 120£.
8
2 960 Half Crowns.
480 Crowns.
15
3)7200 Groats.
2400 Shillings.

(27)

	s.	d.
A Crown =	5	0
Half do. =	2	6
Shilling =	1	0
Groat =	0	4
Sum, one of each =	8	10

6×6=36

2 13 —
6
£.15 18 —

(28) £.470 17,0
20
27 { 3 } 9417 Shillings.
{ 9 } 3139
348 Moidores, and 21 Shillings over.

(29)
240 Guineas.
7×3=21
1680
3
5)5040 Shillings.
4)1008 Crowns.
£.

(30)

	£.	s.
21 Guineas =	22	1
A Crown =	0	5
A Moidore =	1	7
In each Purse,	23	13

7×3=21
165 11
3
Answer, £.496 13

(31) First, 4s. 3d. = 17 Threepences.
And 1178×17=20026 Ditto.
Then, 2002,6÷8,0=£.250 6 6 the Answer.

(32) First, 17*s.* 6*d.*=35 Sixpences.
And, 1*l.*=40 Ditto.
Then, 470 Pistoles.

$7 \times 5 = 35$

3290
5

40)16450

Answ. £411 5*s.*

(33) First, 5*s.* 3*d.*=21 Three ds.
And, £.249 7*s.* 6*d.*=19950 Ditto.
Then, 19950÷21=950 Qr. Guin.

(34) First, 3*s.* 2*d.*=19 Twopences.

l.	*s.*	*d.*
387	18	4

20

7758
6

19)46550

2450 Florins.

(35) First, 5*s.* 4*d.*=32 Twopences, and 18*s.* 6*d.*=111. Ditto. Now 474×111=52614 Ditto. Therefore 52614÷32=1644 Crs. 1*s.* the Answer.

(36) *Moidores.*

1240

$9 \times 3 = 27$

11160
3

21 { 3)33480
7)11160

Answ. 1594 Gu. 6*s.*

WEIGHTS, MEASURES, &c.

(1)

14*lbs.*
12
168 Ounces.
20
3360 dwts.
6×4=24
20160
4
80640 Grains.

(2)

4)138240 *grs.*
6) 34560
2,0)576,0
12)288
24 *lb.*

(3)

lb. oz.dwt.grs.
19 10 17 22
12
238
20
4777
6×4=24
28662
4
114670 Grains.

(4)

grs.
4)74342
6 18585—2
2,0)309,7—3
12)154—17 dwts.
Answ. *lb.* 12 10 17 14

(5)

lb. oz. dwt. grs.
4 6 0 22
12
54
20
1080
6×4=24
6480
4
25942 grs. = 1 Ingot.

A. 103768 grs.

lb. oz. dwts.
6 11 14
12
83
20
1674
6×4=24
10044
4
40176)241056(6 Ingots.

(7)

	lb.	*oz.*	*dwt.*	*grs.*
One Dozen of Dishes, each Weight	2	1	15	0
Ditto —— Plates, ——	1	3	15	22
One of each - - -	3	5	10	22
				12
Answer, *lb.*	41	6	11	0

(8)

	oz.	*dwt.*	*grs.*
Bowls, each	24	4	0
Tankards, -	11	14	0
Tea-Pots, -	10	10	0
Lamps, -	20	17	21
Pl. per Dozen,	127	11	0
Spoons, ditto,	36	17	23
One of each,	231	14	20

20

4634

6×4=24

27804

4

111236)218440(1 of each.

Rem. 107204 *grs.*=223 *oz.* 6*dwts.* 20 *grs.*

Answer. 1 of each Sort and 223 *oz.* 6 *dwts.* 20 *grs.* over.

oz. dwt grs.

455 1 16

20

9101

6×4=24

54606

4

218440 grs.

(9)

lb. ℥ 3 ℈ *grs.*

4 10 4 1 12

12

58

8

468

3

1405

20

28112 Grains.

(10) *grs.*

2,0)5993,4

3)2996—14 grs.

8)998—2 ℈

12)124—6 ℥

lb. 10 4 6 2 14

(11) 6 Tons.
20
120 Cwts.
4
480 qrs.
7×4=28
3360
4
13440 *lb.*

(12) 28 { 4) 26880 *lb.*
7) 6720
4) 960
2,0) 24,0
12 Tons.

(13) *C. qrs. lb. oz. drs.*
74 2 16 0 7
4
298
7×4=28
2086
4
8360
16
133760
16
2140167 drs.

(14) *oz.*
16 { 4) 29768
4) 7442
} 8 oz.
28 { 4) 1860—2
7) 465
} 12 lb.
4) 66—3
C. 16 2 12 8

(15) *lb. oz. drs.*
67 12 15
16
1084
16
17359 drs.

(16) *C. qrs. lb.*
6 3 27
4
27
7×4=28
189
4
783 lbs. in 1 Hhd.
6
Answer, 4698

(17) *C. qrs.*
11 2
4
46
7×4=28
322
4
lb.
1288)12880(10 Answer.

(18) *C.*
A Fother=19½
×2
39

507 *C.*
2
39 1014(26 Fother.
234
...

(19) *C. qrs. lb.*
12 3 12
4
51
7×4=28
357
4
12)1440
120 Canisters the Answer.

(20) *lb.*
126½
2
253

C. qrs.
8 3
4
35
7×4=28
245
4
980 lb.
2
253)1960(7 Parcels, and
Remains 189 half lbs.=3 qrs. 10½lb over.

(21) First 6+8+12+16=42*lb.* One of each.
C. qrs. lb.
4 3 24
4
19
28
556
2
42 { 6)1112 lbs.
7)185—2
26—3 } 20lbs.
Answ. 26 of each, 20lb.

(22)
24 lb. great.
3
2)72
36 lb. common.

(23)
120 lb. common.
2
3)240
80 lb. great.

(24) *Yds. qrs.*
27 3
4
111
4
444 Nails.

(25) 4)352 Nails.
4)88
22 Yards.

(26) *E. Eng. qrs. na.*
30 4 3
5
154
4
619 Nails.

(27)
4)569 Nails.
3)142—1
E. Fl. 47 1 1

(28) 24 Yards.
4
96
4
384 N.= 1 Piece
14
Answ. 5376 Nails.

(29) 4)24768 Nails.
4)6192
12)1548
Answ. 129 Pieces.

(30) *E. Fl.*
20 each.
3
60 qrs. in 1 Piece.
12
5)720
144 *Ells* English.

(31) *E. Fl.* *Yds.*
24 227
3 4
72 { 8)908
9)113—4 } 44 Qrs.= 11 Yds.
12—5
Answ. 12 Ps. 11 Yds.

(32) 24 *E. Fl.*

3

72 Qrs. in 1 Piece.

12

864 Qr. in 12 Pieces.

4

5)3456 Qrs. in 4 Bales.

E. Eng. 691—1 Qr.

(33) 60 *Miles.*

8

480 Furlongs.

40

19200 Poles.

(34) 4,0)128,0 Poles.

8,320

Answ. 40 Miles

(35) 16 Miles.

1760 Yards in 1 Mile.

28160 Yards.

3

84480 Feet.

12

1013760 Inches.

3

3041280 Barley Corns.

(36) 3)2280060 B. Crs.

12) 760020

3) 63335—2

176,0) 2111,1 (11 Miles.

351

22,0)1751,7 Furlongs.

211

2

11)422 38 Poles.

Remains 4 Half Yards.
=2 Yards, 2 Feet.
Answer, 11 Miles, 7 Fur.
38 Poles, 2 Yards, 2 Feet.

(37) 276 Miles.

1760

1656

4692

485760 Yards.

6×6=36

2914560

6

17487360 Inches.

3

52462080 B. C.

(38) *Feet.*
First $18\frac{1}{2}=37$ Half Ft.
Miles.
197
1760
1182
3349
346720 Yards.
6
2080320 Half Ft.
which divide by $37=56224\frac{16}{37}$ Times, the Answer.

(39) *M. fur.*
69 4=1°.
8
556
11×20=200 Yds. in 1 Fur.
6116
20
122320 Yards in 1°.
6×6=36
733920
6
4403520 Inches in 1°.
3
13210560 Bar. C. in 1°.
6×60=360°.
79263360
60
Answ. 4755801600 Barley Corns.

(40) 64 Acres.
4
256 Roods.
40
10240 Poles.

(41) 4,0)2176,0 Poles.
4)544
Answ. 136 Acres.

(42) 774 Acres.
4
3096
4,0
270 { 3,0)12384,0
9,4128

Ans. 458 Shares, & 180 Poles.

(43)

	A.	*r.*
Rents	200	0
Tills	96	2
Rem.	103	2

4
414
40

Ans. 16560 Perches.

(44) 12 Tierces.
42
504 Gal.
8
4032 Pts.

(45) 8)6048 Pts.
42 { 6) 756
7) 126
18 Tierces.

(46)

Hhd.	*gal.*	*qts.*
4	42	2

63
294
4
Answ. 1178 Qts.

(47) 8)5746 Pts.
63 { 7) 718—2 Pts.
9) 102—4
11—3 } 25 Gal.
Answ. 11 hds. 25 gal. 2 pts.

(48) *Gal.*
A Pipe=126
4
12,504 Gal.
Answ. 42 Doz.

(49) A Tun=252 Gal.
A Quart=4 Half Pts. 4
A Pint =2
Half Pt.=1———1008 Qts.
4
7)4032=Half Pts.
Answ. 576 of each.

(50)

A Pipe = 126 Gal.
A Punch. = 84
A Hogsh. = 63
A Tierce = 42

Sum 315)1890(6 of each.

(51) 12 A. Bar.
32
384 Gal.
8
3072 Pints.

(52) 8)1704 Pts.
32 213(6 Bar. 21 Gal.
Rem. 21 Gal.

(53) *B. bar. fir. gal.*
6 2 7
4
26
9
Answ. 241 Gal.

(54) *A. hds. gal. pts.*
10 42 4
48
522
8
4180 Pints.

(55) 4)2017 Qrs.
48 { 6)504—1 Qt.
8) 84
Hhds. 10 24 1

(56) 12 B. Hhds.
3
2 36 Half Barrels.
Answer, 18 Barrels.

(57) [illegible] A. Bar.
2
3 36 Half Barrels.
Answer, 12 Hhds.

(58)

	T.	b.	hd.	gal.
Gal.	4	1	1	49
A Hhd. =	54	2		
A Bar. =	36	—		
A Firk. =	9	9		
	—	2		
Sum,	99	—		
	—	19		
		54		

99)1075(10 of each, and 85 Gal. over.
Rem. 85 Gal.

(59) 24 Qrs.
8
192 Bush.
4
768 Pecks.
2
1536 Gal.

(60) 2)3360 Gal.
4)1680
8) 420
Answ. 52 qrs. 4 Bush.

(61) *La. qrs. bu.*
42 4 7
10
424 Qrs.
8
3399 Bushels.

(62) 40 Chal.
36
1440 Bush.
4
Answ. 5760 Pecks.

(63) 36 { 6)4762 Bush.
6)793—4 } 10 Bush.
132—1
Cha. 132 10

(64) *Cha. bu.*
47 30
6×6=36
282
6
3)1722
Answ. 574 Sacks.

(65) 12)645 Sacks.
Answ. 537 Cha. 6 Sacks.

(66) *D. H.*
365 6
6×4=24
2190
4
8766
60
Answ. 525960 Minutes.

(67) 6,0)207360,0 Sec.
6,0)3456,0
24 { 4.576 / 6)144
Answer, 24 Days.

(68) *D. h. m. sec.*
27 7 43 5
6×4=24
162
4
655
60
39343
60
An. 2360585 Seconds.

(69) *D. h. m. sec.* m
365 5 48 57 39
6×4=24
2190
4
8765
60
525948
60
31556937
60
Answer, 1893416253 Thirds.

(70) 6,0 3155760,0 Sec.
6,0) 52596,0
24 { 4)8766 / 6)2191—2 } 6 hrs.
365—1
Ans. 365 D. 6 H.

(71)

D.	*h.*
365	6
1800 Years since.	

657000
450=1800 for the 6 Hours.

Answer, 657450 Days.

(72)

Years.
London built 1108 before the Birth of our Saviour.
1789 since

In all, 2897 Then 365 d. 6 h. = Hours. 8766 by 66 Ex.

2897

61362
78894
245448 =7×4=28.

24 { 4 } 25395102
{ 6 } 6348775—2

Answer, 1058129 Days, 6 Hrs.

12. THE RULE OF THREE DIRECT.

(1)

Yds.	*s.*	*Yds.*
If 3	—18—	17

17

3 306

2,0)10,2

Answer, £.5 2 0

(2) *lb. d. lb.* *lb. s. d. lb.*
St. thus—If 2 : 18½ : : 24 If 24 : 18 6 : : 2

4 12

74 222
6×4=24 2

4444 4)444
4 24 { 6)111

2)1776 Proof, 18½*d.* = 1*s.* 6*d.*½

4) 888

12) 222

Answ. 18*s.* 6*d.*

Or whenever the first and second Terms consist of any Numbers that are in the Multiplication Table; then the most expeditious Way will be to multiply and divide as in Sect. 9 and 10.—Thus the last Example will be performed as follows:

(2) *lb. s. d. lb.* *lb. s. d. lb.*
If 2 : 1 6½ : : 24 If 24 : 18 6 : : 2
6×4=24 2

9 3 4)1 17 0
4 24 { 6 0 9 3

2)1 17 0 Proof, 1*s.* 6½*d.*

Answ. £.0 18 6

But in Teaching I would advise the Tutor to follow the general Rule, to make the Pupil perfect therein, after which he will, with more Ease both to himself and Master, learn the Abbreviations.

lb. s. d. doz. lb.
(3) Stand thus—If 4 : 2 6 : : 6 or 72
Or, as 4 : 30*d.* : : 72 : 540 Pence.
For 72×30=2160, this÷4=540 Pence, which ÷ by 12 and 20, will give £.2 5*s.* the Answer.

Or, 2*s.* 6×9×8÷4=£.2 5*s.*

C s. C.

(4) Stated thus—If 1 : 26 : : 40

Then $26 \times 40 \div 20 = 1040 \div 20 = 52$£. the Answer.

Or thus, £.1 6*s.* $\times 8 \times 5$ 40) $= 52$£.

(5) Stated thus—If 1 oz. : $7\frac{1}{2}$*d.* : : 1 Cwt. or 112*lb.*

Or, as 1 : 15 : : 1292 : 26880 Halfpence.

Then by Reduction $7\frac{1}{2}$*d.* $= 15$ Halfpence, and 112*lb.* $= 1792$ *oz.*

Consequently 1792 $\times$ 15 $= 26880$ Halfpence, which divide by 2, 12, and 20, will give 56£. the Answer,

oz. s. d. oz.

(6) Stand thus—If 1 : 5 4 : : 36

Or, as 1 : 16 : : 36 : 576 Fourpences.

For by Reduction 5*s.* 4*d.* $= 16$ Fourpences, these $\times$ by 36 (the third Term $= 576$ Fourpences; which $\div$ by 60 will give £.9 12 Answer.

Or thus, 5*s.* 4*d.* $\times 6 + 6(36) =$ £.9 12.

Y. s. d. Ps. Yd.

(7) Stated thus—If 1 : 17 6 : : 12 each 30.

Or, as 1 : 35 : : 360 : 12600 Sixpences.

For by Reduction 17*s.* 6*d.* $= 35$ Sixpences, and 12 $\times$ 30 $=$ 360 Yards, consequently $360 \times 35 = 12600$ Sixpences, these $\div$ by 40 will give 315£. the Answer.

Or thus, 17*s.* 6*d* $\times 10 \times 6 \times 6(360) = 315$£.

lb. s. d. C. lb.

(8) Stated thus—If 1 : 7 9 : : 1 or 112.

Or, as 1 : 31 : : 112 : 3427 Threepences.

For by Reduction 7*s.* 9*d.* $= 31$ Threepences, these $\times$ by 112 (the last Term) $= 3472$ ditto, which $\div$ by 80 will give £.43 8*s.* the Answer.

Or thus, 7*s.* 9*d.* $\times 4 \times 7 \times 4$ (112) $=$ £.43 8*s.*

(9) First, Cost £.34 6*s.* gained 6£. which added together, $=$£40 6*s.*—to be sold for—then

C. qr. lb. £. s. lb.

Stated thus—If 2 1 14 : 40 6 : : 1

Or, as 266*lb.* : 806*s.* : : 1*lb.* : 3*s.* $0\frac{1}{4}d.\frac{119}{245}$

For by Reduction 2*C.* 1*qr.* 14*lb.* $=$ 266*lb*, and £40 6*s.* $=$ 806*s.* which $\div$ by 266 will give 3*s.* $0\frac{1}{4}d.\frac{118}{266}$ the Answer.

(10) First, Cost 90*l.* lost *l.*7 10*s.*; Difference *l.*82 10*s.* sold for. Then,

Yds. *£.* *s.* *El.Eng.*

Stated thus—If 200 : 82 10 : : 1

Or, as 800 Qrs. : 1650*s.* : : 5 Qrs. : 10*s.* $3\frac{1}{4}$*d.*

For by Reduction 200 Yards=800 Qrs. and £82 10*s.*= 1650*s.* which × by 5 (the last Term)=8250*s.* these ÷ by 8,00 (the first Term) will give 10*s.* $3\frac{1}{4}$*d.* the Answer.

Doz. *lb.* *£.* *s.* *lb.*

(11) Stated thus—If 9 or 108 : 2 5 : : 4

Or, as 108 : 45*s.* : : 4 : 1*s.* 8*d.*

For 45×4=180*s.* these ÷ 108 will give 1*s.* 8*d.* the Answer.

Or thus, 45 2 ÷ 9 × 12 (108)=1*s.* 8*d.*

(12) *lb.* *d.* *Hhds.* *C. qr. lb.*

Stated thus—If 1 : $6\frac{1}{2}$: : 4 each 12 2 24

Or, as 1 : 13 Halfpence, : : 5696 : 74048.

For by Reduction $6\frac{1}{2}$*d.*=13 Halfpence, and 12 *C.* 2 *qrs.* 24*lb.* =1424*lb.* in one Hogshead, which × by 4 the Number of Hhds.=5696*lb.* these × by 13 the second Term =7448 Halfpence, which ÷ 2, 12, and 20, will give £.154 5*s.* 4*d.* the Answer.

(13)

		£.	s.
Cost,	-	579	12
Freight,	-	46	0
Loading, &c.		6	0
Custom,	-	10	0
Cellar,	-	4	0
Gain,	-	360	0

Tuns. ——— *Tuns.*

Stated thus—If 46 : 1005 12 : : 26

Or, as 46 : 20112*s.* : : 26 : 11367*s.* $7\frac{3}{4}$*d.* $\frac{1}{2}\frac{7}{3}$

For by Reduction £.1005 12*s.*=20112*s.* these × by 26 (the third Term)=522912*s.* which ÷ by 46 the first Term) will give 11367*s.* $7\frac{3}{4}d.\frac{1}{4}\frac{7}{6}$, or £.568 7*s.* $7\frac{3}{4}$*d.* $\frac{1}{2}\frac{7}{3}$ the Answer.

(14) *C.* *lb.* *s.* *d.* *C. qrs. lb.*

Stated thus—If 1 or 112 : 32 6 : : 12 2 14.

Or, as 112 : 3906*d.* : : 1414 : 49233/4*d.*

For by Reduction 32*s.* 6*d.*=3906*d.* and 12 *C.* 2 *qrs.* 14*lb.* =1414*lb.* which × by 3906*d.*=551604*d.* these ÷ by 112 (the first Term) will give 49233/4*d.* or £20 10*s.* $3\frac{3}{4}$*d.* the Answer.

(15) First, Cost £.89 16*s.* 4*d.* Lost £.12 Difference; £.77 16*s.* 4*d.* sold for—then

Stated thus—If 1 or 112 : 77 16 4 :: 1

Or, as 112 : 18676*d.* :: 1 : 166¾*d.*

For by Reduction £77 16*s.* 4*d.* = 18676*d.* which ÷ by 112 will give 166¾*d.* or 13*s.* 10¾*d.* the Answer.

Or thus, £.77 16*s.* 4*d.* ×4×7×4 (112) = 13*s.* 10¾*d.*

(16) Here, as he spends as much in 4 Months as he gains in 3, consequently he will spend in 8 Months what he gains in 6———then

M.	£.	*s.*	*d.*		£.	*s.*	*d.*
Stated thus—If 8	: 185	5	6	: : 12	185	5	6
		12				×	2
	8)2223	6	0		Gains 370	11	0 per Ann.
					Spends 277	18	3
	Spends £.277	18	3				
					Answ. 92	12	9

(17) First, 4*s.* 6*d.* ×by 12×6×+8 (36 *Doz* 8 *Prs.*) will give 99£. the value of the 36 Dozen 8 Pair of Stockings; then

s. *d.* *P.* £.

Stated thus—If 1 4 : 1 :: 99.

As 4 : 1 :: 5940 : 1485.

For by Reduction 1*s.* 4*d.* = 4 Fourpences, and 99£. = 5940 ditto, which ÷ by 4 (the first Term will give 1485 Pair; these ÷ 12 will give 123 Dozen and 9 Pair, the Answer.

(18) First 18*s.* 9*d.* ÷ by 5 = 3*s.* 9*d.* sold for per Yard, and 6*s.* 6*d.* ÷ 2 = 3*s.* 3*d.* cost per Yard; then from 3*s.* 9*d.* take 3*s.* 3*d.* remains 6*d.* gained per Yard; and he gained as much as 180 Yards cost; thus, cost per Yard 3*s.* 3*d.* or 39*d.*; consequently 39×180 = 7020*d.* his whole Gain—then

d. *Yd.* *d.*

Stated thus—If 6 : 1 :: 7020

6)7020

Answer, 1170 Yards.

(19) *s.* *d.* *Ell. qrs.* £. *s.*

Stated thus—If 6 6 : 1 or 5 :: 23 8

Or as 13 : 5 qrs. :: 936 : 360 qrs.

For by Reduction 6*s.* 6*d*=13 Sixpences, and 23*£.* 8*s.*=936 Ditto, which multiplied by 5 (the second Term)=4680 *qrs.* these ÷ by 13 (the first Term) will give 360 *qrs.* bought; this ÷60, (the Quarters contained in 20 Ells F. or 1 Piece) gives 6 Pieces, the Answer.

(20)

	s.	*d.*	*oz.*	*dwts.*		*£.*	*s.*	*d.*
Stated thus—If	5	10	: 1	or 20	::	102	16	6

Or, as 70 : 20 *dwts.* :: 24678 : $7050\frac{6}{7}$ *dwts.*

For by Reduction, 5*s*, 10*d.*=70*d.* and 102*£.* 16*s.* 6*d*=24678*d.* which × by 20 (the second Term)=493560 *dwts.* these ÷ by 70 the first Term) gives $7050\frac{6}{7}$ *dwts.* the Quantity of Silver bought, which, ÷ by 994 *dwts.* (4*lb.* 1 *oz.* 14 *dwts.* the Weight of 1 Ingot, will give 7 Ingots, and 92 *dwts.* or 7 Ingots 4 *oz.* $12\frac{6}{7}$ *dwts.* the Answer.

(21)

		d.	*oz.*		*£.*	*s.*
Stated thus—If	$8\frac{3}{4}$	: 1		::	426	16

Or, as 35 : 1 :: 409728 : $11706\frac{8}{35}$*oz.*

For by Reduction $8\frac{3}{4}$*d.*=35 *qrs.* and *£*426 16*s.*=409728 *qrs.* which ÷ by 5×7 (35 gives $11706\frac{8}{35}$*oz.* these ÷ by 16, 28, and 4, will give 6 Par. and 2*qrs.* 3*lb.* $10\frac{8}{35}$*oz.* over, the Answer.

(22) Here first 100 Guineas=105*£.* lays by; then from *£.*488 5*s.* his yearly Income, take 105*£.* remains 383*l.* 5*s.* or 7665*s.* what he spends per Annum—then

	D.	*s.*		*D.*	*s.*
Stated thus—As	365	: 7665	::	1	: 21.

For 7665 ÷ 365=21*s.* per Day.

(23)

	£.	*s.*	*s.*	*d.*		*£.*	*s.*
Stated thus—If	1	or 20	: 3	9	::	564	12.

Or, as 20 : 45 :: 11292 : 25407*d.*

For by Reduction, 3*s.* 9*d.*=45*d.* and 564*l.* 12*s.*=11292*s.* which × 45*d.* (the second Term)=508140*d.* this ÷ by 20, (the first Term)=25407*d.* these ÷ by 12 and 20, will give 105*l.* 17*s.* 3*d.* Taxes which subtracted from his Income, viz. *£.*564 12*s.* 0*d.* leaves *£*458. 14*s.* 9*d.* his neat Income the Answer.

(24)

		oz.	*dwts.*		*£.*
Stated thus—If	5*s.* 9*d.*	: 1	or 20	::	200

Or, as 23 : 20 :: 1600 : $1391\frac{11}{23}$*dwts.*

For by Reduction, 5*s.* 9*d.*=23 Threepences, and 200*£.*=16 00 ditto, which × by 20 (the second Term)=320000 *dwts.* this ÷ by 23 the first Term) gives $13913\frac{1}{23}$ *dwts.* these *dwts.* ÷ by 692 (2*lb.* 11 *oz.* 12 *dwts.*) the *Pennyweights* in an Ingot, will give 20 Ingots and 73 *dwts.* *or 20 Ingots 3 oz.* $13\frac{1}{23}$ *dwts.* the Answer.

(25) First $6\times6\times10\times30=10800$ Yards in the six Packs, then

Yds. *£.* *s.* *d.* *Yds.*

Stated thus—As 3 : 2 4 3 :: 10800

Or, as 3 : 531*d.* :: 10800 : 1911600*d.*

For by Reduction 2*l.* 4*s.* 3*d.*=531 Pence, these × by 10800 =5734800*d.* which ÷ by 3 (the first Term)=1911600 Pence, or 7965*£.* the whole cost him, and *£*2. 4*s.* 3*d.*÷ 3=14*s.* 9*d.* cost per Yard——

(26) *W.* *£.* *s.* *W.*

Stated thus—If 16 : 14 16 :: 52

Or, as 16 : 296*s.* :: 52 : 962*s.*

By Reduction, *£*14 16*s.*=296*s.* which×52=15392*s.* these ÷ by 16 (viz. 4×4) gives 962*s.* or *£*48 2*s.* the Answer.

(27)

	£.	*s.*
Oxen, each -	10	0
Cows, - -	7	0
Calves, - -	1	10
Sheep, - -	0	19
One of each cost *£.*	19	9

£. *s.* of ea. *£.* *s.*

Stated thus—If 19 9 : 1 :: 116 14

20 20

389)2334(6 of each.

(28) *£.* *£.* *s.* *d.* *£.*

Stated thus—If 560 : 374 10 6½ :: 1

Or, or as 560 : 35954 6*qrs.* :: 1 642 $\frac{25}{56}$*qr.*

By Reduction *£.*374 10*s.* 6½=359546 Farthings, which÷ by 560 (viz. 8×70) gives 642 $\frac{25}{56}$*qrs.* these reduced to Shillings, &c. will be 13*s.* 4*d*½. $\frac{5}{14}$ the Answer.

(29) First, the Cost, Custom, Fees Freight, Factorage, and Profit, added into one Sum=*£.*14 0*s.* 2*d* then

C. *qrs.* *lb.* *oz.* *£.* *s.* *d.* *C.* *lb.*

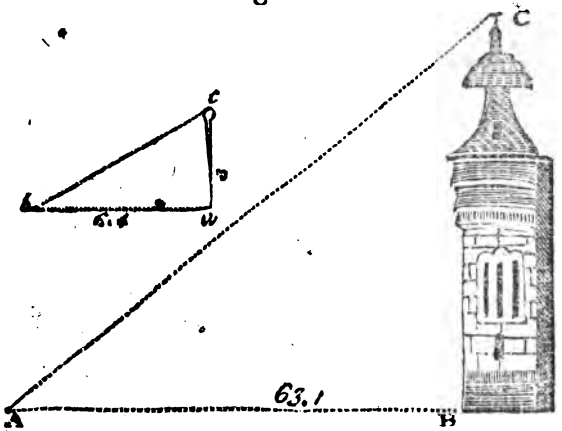
Stated thus—If 18 3 17 7 : 14 0 2 :: 1 or 112.

Or, as 33879 : 3362*d.* :: 1792 : 177 $\frac{3}{4}\frac{1}{1}\frac{1}{3}\frac{6}{8}\frac{8}{0}\frac{7}{0}$*d*

By Reduction 18 *C.* 3*qrs.* 17*lb.* 7*oz* =33879 *oz.* and 112*lb.* =1792 *oz.* Also 14*s.* 2*d.*=3362 Pence; then 3362*d.*

× 1792 *oz.*=6024704 Pence, which ÷ 33879 (the first Term) gives £77¾*d.* $\frac{9847}{33879}$, these *d.* ÷ by 12, give 14*s.* $9\frac{10847}{33879}$, the Answer.

Fig. 5.



(30) Here the above Figures *a c*=3 Feet the Length of the Staff, *a b*=6 Feet 4 Inches, or 76 Inches, Distance of its Shadow. B C = the Height of the Tower, and A B =63 Yards, 1 Foot, or 2280 Inches Distance of its Shadow—then

In. Ft. In. Ft.

Stated thus—As 76 : 3 :: 2280 : 90

For 2280 × 3=6840 Feet, which ÷ by 76=90 Feet, the Answer. For *a c* : *a b* :: B C : A B.

d. hr. m. h.

(31) Stated thus—If 6 4 : 285 :: 1

Or, as 76 : 285 :: 1 : 3 M. 6 Fur.

For 285 ÷ 76=3 Miles 6 Fur. the Answer.

Pipe G. £. s.

(32) Stated thus—If 1 or 126 : 44 2 :: 1 Pt.

Or, as 1008 : 10584*d.* :: 1 10½*d.*

For 126 Gal. =1008 Pints and 44*l.* 2*s.*=10584 Pence, which ÷ by 1008 (the first Term) gives 10½*d.* per Pint, the Answer.

(33) First $360\times69\frac{1}{2}=25020$ Miles the Circumference of the Globe—then

H. M. Miles. H. M.

Stated thus—If 23 56 : 25020 :: 1 or 60
60 60

1436)1501200(1045 Miles, 3 Fur. $9\frac{1320}{1436}$ Poles the Answer.

(34) *oz. dwts. £ s. gr.*

Stated thus—If 1 or 20 : 5 5 :: 1

Or, as 480 : 1260*d.* :: 1 : $2\frac{1}{2}d.\frac{1}{4}$.

By Reduction 20 *dwts.*=480 *grs.* and 2*l.* 5*s.*=1260 Pence, then 48,0)126,0($2\frac{1}{2}d.\frac{1}{4}$, the Answer.

(35) First $21\div3=7$ Halfpence, the worth of 12 Apples.

Ap. H. P. Ap.

Stated thus—If 12 : 7 :: 14

Then $84\times7=588$, which ÷ by 12 (the first Term) gives 49 Halfpence, or 2*s.* $0\frac{1}{2}d.$ the Answer.

(36) *s.* £. £. *s.*

Stated thus—As 4 : 1 :: 8 10

Or, as 4 : 1 :: 170 : £.42 10*s.* 0*d.*

For 8*l.* 10*s.* = 170*s.* which ÷ by 4 gives 42*l.* 10*s.* the Answer.

(37) First $\overline{184\times7\times4}+\frac{1}{2}$ $(28\frac{1}{2})=5244$ Yards of the former.

Stated thus—As 19 : 14 :: 5244 : 3864.

For $5244\times14=73416$, which ÷ $19=3864$*yds.* the Answer.

(38) First $4061\times17=47502$ Pieces : these × by 44*d.* (3*s.* 8*d.* or the value of one Piece will give 2090088 Pence, Value of all the Pieces—then

d. Reas. d.

Stated thus—If 3 : 20 :: 2090088 : 13933920

For $2090088\times20\div3=13933920$ Reas the Answer.

(39) First $274\div5$ £.54 16*s.* cost him; then

Ell E. qrs. s. d. Ell E.

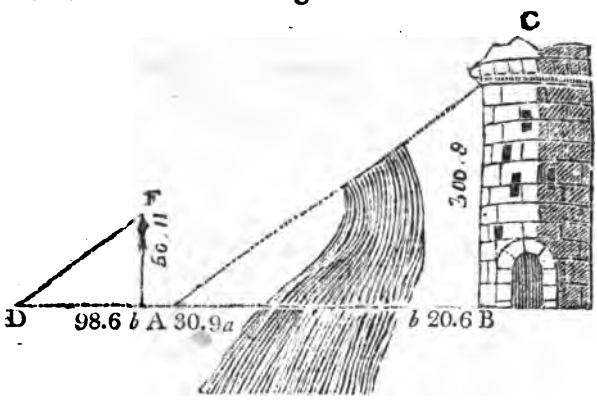
Stated thus—If 1 or 5 : 7 10 :: 274

Or, as 5 : 94*d.* :: 822 : $15453\frac{1}{2}d.\frac{2}{5}$.

By Reduction 7*s.* 10*d.*=94 Pence, and $274\times3=822$ *grs.* these × by 94 = 77268, which ÷ by 5, gives $15453\frac{1}{2}d.\frac{2}{5}$ or £64 7*s.* $9d.\frac{1}{2}d.\frac{2}{5}$ sold for.

Then from £.64 7*s.* $9\frac{1}{4}d.\frac{3}{4}$ take £.54 16*s.* and there will remain £.9 11*s.* $9\frac{1}{4}d.\frac{3}{4}$ gained, the Answer.

(40) Fig. 6.



Here F *b*=50 Feet, 11 Inches, or 611 Inches, Height of the May-Pole, and D *b*=98 Feet 6 Inches, or 1182 Inches, Length of its Shadow. Also BC=300 Feet 8 Inches, or 3608 Inches, and A. B. the Length of its Shadow—then

In. *In.* *In.* *In.*

Stated thus—As 611 : 1182 :: 3608 : 6979

Then $6979 \div 12 = 581\ 7\frac{4}{6}\frac{8}{1}\frac{7}{1} = A.\ B.$

And 30 F. 9 In. + 20 F. 6 In. = 51 3 $= Aa + bB.$

530 $4\frac{4}{6}\frac{8}{1}\frac{7}{1} = ab$ the Breadth of the Stream required.

(41) *M.* *S.* *M.*

Stated thus—If 2 : 15 :: 7 : 52*s.* 6*d.*

For $15 \times 7 = 105s$ which $\div$ by 2 = 52*s.* 6*d.* or £.2 12*s.* 6*d.* the Answer.

H. *M.* *D.* *H.*

Stated thus—If 4 : 12 :: 9 : 8

Or, as 4 : 12 :: 116 : 348 Miles.

For 9 D. 8 H. = 116 Hours, which × 12 (the second Term) 1392 Miles; these $\div$ 4 (the first Term) will give 348 Miles, the Answer.

(43) *Yds. s. d. Yds.*
Stated thus—If $5\frac{1}{2}$: 4 6 : : 1000
Or, as 11 : 54*d.* : : 2000 : 9818$\frac{2}{11}$*d.*
By Reduction $5\frac{1}{2}$ Yards = 11 Half Yards, and 1000 Yards = 2000 ditto. Also 4*s.* 6*d.* = 54 Pence. Then 54 × 2000 = 108000*d.* which ÷ by 11 (the first Term) gives 9818$\frac{2}{11}$*d.* or *l.*40 18*s.* 2$\frac{2}{11}$*d.* the Answer.

(44) *s. d. Ell. qrs. £.*
Stated thus—If 5 6 : 1 or 3 : : 352.
Or, as 11 : 3 *qrs.* : : 14080 : 3840 *qrs.*
For 5*s.* 6*d.* = 11 Sixpences, and 352*£.* = 14080 ditto, which × by 3 = 42240 *qrs.* these ÷ 11 (the first Term) gives 3840 *qrs.* or 960 Yards.
Again, 3840 *qrs.* ÷ 5 = 768 Ells English, which ÷ by 64 = 12 Ells English in each Piece.

(45) *£. £. s. £. s.*
Stated thus—If 100 : 4 15 : : 40 10
Or, as 2000 : 95*s.* : : 810 : 38*s.* 5$\frac{1}{2}$*d.*$\frac{4}{7}$.
For 100*l.* = 2000*s.* and 4*l.* 15*s.* = 95*s.* Also *l.*40 10*s.* = 810*s.* which × by 95 = 76950*s.* these ÷ by 2,000 will give 38*s.* 5$\frac{1}{2}$*d.* $\frac{6}{10}$, or *l.*1 18*s.* 5$\frac{1}{2}$*d.*$\frac{4}{7}$ the Answer.

(46)
First 24 × 20 = 480 Yards, contained in the 24 Pieces.
Stated thus—As 12 : 8 : : 480 : 320
Here 480 × 8 = 3840 Yards, which ÷ by 12, gives 320 *yds.* the Answer.

(47) *A. £. s. d. A.*
Stated thus—If 1 : : 1 19 6 : : 240
Or, as 1 : 79 : : 240 : 18960 Sixpences.
By Reduction *l.*1 19*s.* 6*d.* = 79 Sixpences, which × by 4 × 60 (240 the Product is 18960 Sixpences; these ÷ by 40 will give 474*l.* per Annum; this ÷ by 4 = *l.*118 10*s.* per Quarter, the Answer.

(48) *£. s. d. £.*
Stated thus—If 1 : 10 6 : : 1000
Or, as 1 : 21 : : 1000 : 21000 Sixpences.
By Reduction 10*s.* 6*d.* = 21 Sixpences, which × by 1000, the Product is 21000 Sixpences, these ÷ 40 = 525*l.* the Answer.

M. fur. p. ft. Min. Mile. Yds.

(49) Stated thus—If 37 2 37 $5\frac{1}{2}$: 4 :: 1 or 1760
Or, as 394592 : 4 :: 10560 : 6″ $25\frac{1460800'''}{1971960}$

By Reduction 37*m.* 2*fur.* 37*p.* $5\frac{1}{2}$*ft.* = 394592 Half-feet, and 1760 Yards = 10560 ditto, which × by 4 Minutes (the second Term) the Product will be 42240 Minutes; this × by 60, to bring the Minutes into Seconds, gives 2534400 Seconds; these ÷ by 394592 (the first Term) the Quotient is 6″ $25\frac{1460800'''}{1971960}$ past 12, the Answer.

(50) *£ s. d. E. E. £. s. d.*
Stated thus—If 1 7 10 : 4 :: 118 17 $7\frac{1}{2}$
Or, as 668 : 80 *nails* :: 57063 : 6033$\frac{960}{668}$ *nails.*

By Reduction *l.*1 7*s.* 10*d.* = 668 Halfpence. And *l.*118 17*s.* $7\frac{1}{2}d.$ = 57063 ditto. Also 4 Ells English = 80 Nails, then 57063 × 80 = 4565040 Nails, which ÷ by 663 = 6833$\frac{560}{668}$ Nails bought; then to find how many Pieces, each 33 Ells Flemish, 1 *qr.* 2 *na.* or 402 Nails, are contained in the Quantity bought; thus, 402) 833(16 Pieces, and 33 *Ells*, 1 *qr.* 1 *na.*$\frac{560}{668}$ over, the Answer.

(52) First, from 100*l.* take 60*l.* Remains 40*l.* Serge cost.

Yds. Yds. Yds. Yds.
Stated thus—As 2 : 3 :: 236 : 354.

Here 236 × 3 = 708; this ÷ by 2 = 254 Yards of Shalloon, which cost 60*l.* consequently 60*l.* ÷ by 354, will give 3*s.* $4\frac{1}{2}d.\frac{52}{354}$, what the Shalloon cost per Yard; and 40*l.* ÷ by 236, will give 3*s.* $4\frac{1}{2}d.\frac{472}{590}$, what the Serge cost per Yard.

(52) *s. lb. £. s. d.*
Stated thus—If 14 : 8 :: 4 19 $1\frac{1}{4}$
Or, as 672 : 8 :: 4757 : 56*lb.* 10*oz.* $1\frac{352}{590}dr.$

By Reduction 14*s.* = 672 Farthings, and *l.*4. 19*s.* $1\frac{1}{4}d.$ = 4757 Ditto, which × by 8 (the second Term) the Product will be 38056; this ÷ by 672 (the first Term) will give 56*lb.* 10*oz.* $1\frac{1}{8}dr.$ the Answer.

(53) *C. lb. s. C. qrs. lb.*
Stated thus—If 1 or 112 : 7 :: 20 2 16.
Or, as 112 : 7 :: 2312 : 144*s.* 6*d.*

For 20 *C.* 2 *qrs.* 16 *lb.* = 2312 *lb.* these × 7*s.* = 16184*s.* which ÷ by 112 will give 144*s.* 6*d.* or *l.*7 4*s.* 6*d.* the Answer.

(54) A Pipe=126 Gallons, from which take 12 Gallons. Remains 114 Gallons, or 456 Quarts at 18*d.* per Qt. Then

Qt. d. Qts. d.

Stated thus—As 1 : 18 :: 456 : 8208.

Here $456 \times 18 = 8208$ Pence, which ÷ by 12 and 20, gives *l.*34 4*s.* sold for, from which take *l.*25 4*s.* Remains *l.*9 gained, the Answer.

(55) *C. qrs. lb. £. s. d. lb.*

Stated thus—If 20 2 16 : 7 4 9 :: 1

Or, as 2312 : 6948*qrs.* : : 1 : $\frac{3}{4}$*qrs.*

Here by Reduction 20 *C.* 2 *qrs.* 16 *lb* =2312 *lb.* and *l.*7 4*s.* 9*d* =6948 Farthings, which ÷ by 2312 (the first Term) gives $\frac{3}{4}$ $\frac{2}{2\,3\,1\,2}$, the Answer.

(56) First $9 \times 4 = 36$ square Inches in one Brick, and $20 \times 20 = 400$ Feet square, in the Floor, which × by $144 = 57600$ square Inches, the Content of Ditto.—Then

Inch. B. Inch.

Stated thus—As 36 : 1 : : 57600 : 1600

For $57600 \div 6 \times 6(36) = 1600$ Bricks the Answer.

(57) First $22 \times 4 = 88$ Miles I have gone before you set out, and $32 - 22 = 10$——you gain upon me per Day; then

Stated thus—As 10 : 1 : : 88 : $8\frac{4}{5}$

For $88 \div 10 = 8\frac{4}{5}$ Days, before you overtake me.

Then $8\frac{4}{5} \times 4 \times 8\,32) = 281\frac{3}{5}$ Miles you travelled before I was overtaken, consequently $350 - 281\frac{3}{5} = 68\frac{2}{5}$ Miles on this Side Edinburg, the Answer.

(58) First, from 20 take 11. Remains 9 Days to be sunk.

Days. Yea. Days.

Then stated thus—As 3 : 400 : : 9 : 1200.

Here $400 \times 9 = 3600$; this ÷ by $3 = 1200$ Years after Anno 1700; then 1200 added to 1700=2900, the year of Christ required.

(59) *d. lb. d.*

First, Beef cost per Day, $2\frac{1}{4} \times 5 = 11\frac{1}{4}$, and per Week, $11\frac{1}{4}d. \times 7 = 6s.\ 6\frac{3}{4}d. = 315$ *qrs.*

Biscuit cost per Day, $1\frac{1}{2}d. \times 3lb. = 4\frac{1}{2}d.$ and per Week, $4\frac{1}{2}d. \times 7 = 2s.\ 7\frac{1}{2}d.$ 126 *qrs.*

Meat for the Ship's Company cost per Day *l.* 12 12*s.* and per Week *l.* 12 12*s.* × 7 = 88*l.* 4*s.* or 84672 Farthings.

Stated thus—If 315 : 126 :: 84672 : 33868$\frac{3}{7}\frac{1}{7}\frac{2}{7}$ *qrs.*

For 84672 × 126 = 10668672, which ÷ by 315 = 33868$\frac{3}{7}\frac{1}{7}\frac{2}{7}$*qrs.* or *l.* 35 5*s.* 7—*d.*$\frac{3}{7}\frac{1}{7}\frac{2}{7}$.

(60) *H. M. H.*

Stated thus—As 8 : 135 :: 1 : 16 *m.* 7 *fur.*

Here 135 ÷ 8 = 16 Miles 7 Furlongs, what both rode per Hour.

	M.	*fur.*	*p.*		*M.*	*f.*	*M. fur.*	*p.*
Then,	16	7	0	less 2	4 =	14	3	0
And,	14	3	0 ÷ by	2	=	7	1	20 B. rode per Hour.
Also,	7	1	20 +	2	4 =	9	5	20 A ditto.

$$\text{For } \left\{\begin{matrix} 7 & 1 & 20 \\ 9 & 5 & 20 \end{matrix}\right\} \times \text{ by } 8 = \left\{\begin{matrix} 57 & 4 & \text{B.} \\ 77 & 4 & \text{A.} \end{matrix}\right\} \text{ travelled.}$$

Proof, 135 Miles.

(61) Suppose 130 to be the whole Work; then A. will perform $\frac{1}{13}$th Part, and B. $\frac{1}{10}$th, which will be 13 + 10 = 23*d.* Part of the Work performed by them both together in one Day.

W. Day. W. D. H. M.

Then stated thus—As 23 : 1 :: 130 : 5 7 49$\frac{13}{23}$.

For 130 ÷ 23 = 5 Days, 7 Hours, 49$\frac{13}{23}$ Minutes, the Answer.

(62) Here, suppose 198 to be the whole Work, of which B. and C. will perform $\frac{1}{18}$, viz. 11, and with the Help of A. they will perform $\frac{1}{11}$, viz. the 18th Part in one Day, then 18 — 11 = 7th Part performed by A. alone.

Then stated thus—If 7 : 1 :: 198.

7) 198 (28 Days, 3 H. 25$\frac{5}{7}$ M. the Ans.

To perform Questions of this Nature, you may suppose any Number to be the Work, so that it will divide by the different Times the different Persons perform the Work in; thus, in the first Question I supposed 130, by Reason it will divide by 13 and by 10, and nothing remains. Likewise in the last Question I supposed 198, as it will divide by 18 and 11, and leave nothing remaining.

(63) Stated thus—A. 17 : 3 :: 140 : 24 14 1$\frac{5}{7}\frac{1}{7}\frac{1}{7}$

£. £. £. £. s. d.

l. *l.* *s.* *d.*

Here $140 \times 3 = 420$; this ÷ by 17 = 24 14 $1\frac{4}{17}\frac{1}{17}$ taken off. Then from *l.*140 take *l.*24 14*s.* $1\frac{1}{4}d.\frac{1}{17}$. Remains *l.*115 5*s.* $10\frac{1}{4}d.\frac{6}{17}$, the Assessment, which ÷ by 5, gives *l.*23 1*s.* 2*d.*—$\frac{8}{17}$ the Taxes; this, taken from *l.*140, leaves *l.*116 80*s.* $9\frac{3}{4}d.\frac{9}{17}$, the Answer.

(64) *l.* *l.*

First $1000 \div$ by $\begin{cases} 5 = 200 \text{ gained by Land Trade.} \\ 8 = 125 \text{ ——— Sea ditto.} \end{cases}$

Sum, 325 Gain per Year.

Then $1000 \div 2\frac{1}{2} = 400l.$ lost in 1 Year.

And $400l. - 325 = 75l.$ run out per Year.

l. *Y.* *l.*

Stated thus—As 75 : 1 :: 1000 : $13\frac{1}{3}$

For $1000 \div 75 = 13\frac{1}{3}$ Years, the Answer.

(65) Here, from 10 in the Morning till 6 in the Evening is 8 Hours.—I set out before you, then $2 \times 8 = 16$ Miles I travelled before you set out, and 170—16=154 Miles. Also, $2 + 3 = 5$ Miles both travelled per Hour.

M. *H.* *M.*

Stated thus—As 5 : 1 :: 154 : $30\frac{4}{5}$

For $154 \div 5 = 30\frac{4}{5}$ Hours they will meet.

Then $30\frac{4}{5} \times 2 = 61\frac{3}{5}$, which added to $16 = 77\frac{3}{5}$ Miles, or 77*m.* 4*fur.* 32*p.* from Exeter; and $30\frac{4}{5} \times 3 = 92\frac{2}{5}$M. or 92*m.* 3*fur.* 8*p.* from London.

Answer, $\begin{cases} 77 & 4 & 32 \\ 92 & 3 & 8 \end{cases}$ Distance from $\begin{cases} \text{Exeter.} \\ \text{London.} \end{cases}$

Proof, 170 Miles.

(66) First, from 13° take 1°, remains 12°, Moon gains of the Sun per Day.

And $30° \times 3 = 90°$ from the first of Aries to the first of Cancer.

Also, $90° + 3 = 93°$ Sun before the Moon.

Then stated thus—As 12° : 1 :: 93° : $7\frac{3}{4}$.

For $93 \div 12 = 7\frac{3}{4}$ Days, in which time the Sun will be overtaken by the Moon.

$\therefore 7\frac{3}{4} + 3 = 10\frac{3}{4}$ Degrees of Cancer, the Answer.

(67) First, from 21 take 15. Remains 6 Rods, the Dogs gained in Running 21 Rods, and fourscore=80+16=96 Rods the Hare started before the Dogs.—Then

R. R. R.

Stated thus—As 6 : 21 :: 96 : 336

Here 96 21=2016; this÷by 6, gives 336 Rods the Dogs ran, and 336—96=240 Rods the Hare ran.

(68) First, $5\frac{2}{3}$ Miles=29920 Feet.

Feet. Sec. Feet.

Stated thus—As 1150 : 1 :: 29920 : $26\,1\frac{1}{2}\frac{1}{3}$

For 29922÷1150=26 Sec. $1\frac{1}{2}\frac{1}{3}$ Third, the Answer.

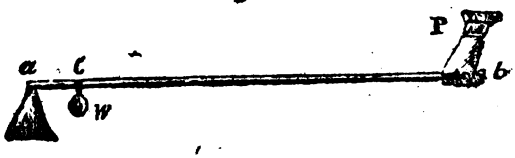
(69) First, 1 Min. 3 Seconds=63 Seconds.

" *Feet.* "

Stated thus—As 1 : 1150 :: 63 : 72450

For 1150×63=72450 Feet, or 13 Miles, 5 Furlongs, 30 Poles, 5 Yards, the Answer.

Fig. 7.



(70) Here *a b*=100 Inches and *a c*=$7\frac{1}{2}$, P the Power, or 168*lb.* and *w.* the Weight to be moved.

Then from 100 take $7\frac{1}{2}$, remains $92\frac{1}{2}$=*a b*—*a c*=*c b*.

In. lb. In.

Stated thus—If $7\frac{1}{2}$: 168 :: $92\frac{1}{2}$: 2072.

Or, as 15 : 168 :: 185 : 2072

Here $7\frac{1}{2}$ Inches=15 Half Inches, and $92\frac{1}{2}$=185 Half Inches, which×168=31080*lb.* these÷by 15=2072*lb* =*w.* the Answer.

For A's *a c* : P. :: *c. b* : *w.*

(71) First, 9×2=18 Inches the Diameter of the Crank.

Ft. In. Ft.

Stated thus—As 6 : 18 :: 9 : 27

For $\overline{18\times9}\div6=162\div6=27$ Inches the Answer.

(72) First $62\frac{1}{2} \times 3 = 187\frac{1}{2}$ *lb.* Wt. of 3 solid Feet of Water.

Feet lb. Feet.

Therefore, Stated thus—As 6 : $187\frac{1}{2}$ :: 9 : $281\frac{1}{4}$

For $187 \times 9 \div 6 = 1687\frac{1}{2} \div 6 = 281\frac{1}{4}$ *lb.* the Answer.

(73) Stated thus—As 25 : 1000 :: 1 : 40, so the less is moved with a Force 40 Times greater than the other.

(74) Stated thus—As 60 : 8 :: 100 : $13\frac{1}{3}$

Answer, as $13\frac{1}{3}$ to 1.

(75) Stated thus—As 8 : 48 :: 1 : 6

Answer, Lesser to the Greater as 1 to 6.

(76) First 2 Hours $\times 60 = 120$ Minutes.

Then—Stated thus—As 40 : 120 :: 1 : 3

Answer, the Swifter to the Slower, as 1 to 3.

(77) First $30 \times 12 = 360$, and 60 Inches $= 5$ Feet.

Then as 1 : 5 :: 360 : 1800

Consequently, $1800 - 5 = 1795$ Feet, the Answer.

(78) First $50 \div 5 = 10$.

Then—Stated thus—As 5 : 10 :: 1 : 2

So that the first Body hath been in Motion double the Time of the latter.

(13) THE RULE OF THREE INVERSE.

D. M. D.

(1) Stated thus—If 12 : 48 :: 72

48

72 { 9)576 / 8) 64

Answer, 8 Men.

D. M. D

If 72 : 8 :: 12.

72

12)576

Proof 48 Men.

(2) *In. B. In. L. In. B.*

Stated thus—As 12 : 12 :: 4 : 36

Here $12 \times 12 = 144$; this $\div 4 = 36$ Inches long the Answer.

(3) *£. M. £.*

Stated thus—As 500 : 6 :: 220 : 13 *m.* $19\frac{1}{11}$

Here $500 \times 6 = 3000$, this $\div$ by 220, gives 13 Months, $19\frac{1}{11}$ Days the Answer.

(4) *M. oz. M.*

Stated thus—As 3 : 14 :: 8 : 5 *oz.* 5 *dwts.*

Here $3 \times 14 = 42$, this $\div$ by $8 = 5$ *oz.* 5 *dwts.* the Answer.

(5) *s. d. oz. s.*

Stated thus—If 4 6 : 12 :: 3

Or, as 9 : 12 :: 6 : 18 *oz.*

Here 4*s.* 6*d.* = 9 Sixpences, and 3*s.* = 6 do. then $12 \times 9 = 108$; this $\div$ by 6, the last Term, gives 18 *oz.* the Answer.

(6) *Qrs. Yds. Qrs.*

Stated thus—If 5 : 275 :: 3

Here $275 \times 5 = 1375$, this $\div$ by 3, gives 458 *yds.* 1 *qr.* $1\frac{1}{3}$ *na.* the Answer.

(7) *£. Yrs. £.*

Stated thus—As 80 : 15 :: 600 : 2

Here $15 \times 80 = 12{,}00$, this $\div$ by 6,00 gives 2 Years, the Answer.

(8) *M. W. D. M.*

Stated thus—If 10 : 43 5 :: 6

Or, as 10 : 306 :: 6 : 510

Here 43*w.* 5*d.* = 306 Days, which $\times$ by 10 = 3060, this $\div$ by 6, gives 510 Days, or 18 Months 6 Days, the Answer.

(9) First 3 *qrs.* $= 2\frac{1}{4}$ *Feet.* Then,

Ft. Ft. F.

Stated thus—If 24 : 30 :: $2\frac{1}{4}$

Or, as 96 : 30 :: 9 : 320

Here 24*ft.* = 96*qrs.* and $2\frac{1}{4} = 9$ *qrs.* then 96×30 (the second Term = 2880 Feet; these $\div$ by 9 the last Term) gives 320 Feet = $106\frac{2}{3}$ Yards, the Answer.

Note, Paper is 27 Inches, or $2\frac{1}{4}$ Feet wide.

(10) *Mo. Men. Mo.*

Stated thus—As 3 : 1500 :: 5 : 900.

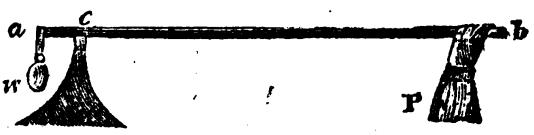
Here $1500 \times 3 = 4500$, this $\div$ by 5, gives 900 Men to continue in; then from 1500 take 900, remains 600 Men to depart.

(11) *H. Cock. M.*

Stated thus—As 6 : 1 :: 15 : 24.

Here 6 Hours 60=360 Minutes, which ÷ by 15, (the last Term) gives 24 Cocks, the Answer.

(12) Fig. 8.



In the above Figure $a\,b$= 100 Inches $a\,c$=$\frac{1}{4}$, P. the Power, or $1\frac{1}{4}$ *Cwt.* and *w*. the Weight.

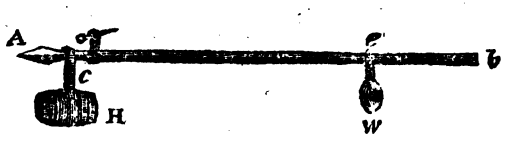
Then 100—$7\frac{1}{2}$=$92\frac{1}{2}$ Inches=$a\,b$—$a\,e$=$c\,b$.

Stated thus—As $92\frac{1}{2}$: $1\frac{1}{4}$ *C.* or 168 *lb.* : : $7\frac{1}{2}$: $18\frac{1}{4}$ *C.*

Here $92\frac{1}{2}$=185 Half Inches, and $7\frac{1}{2}$=15 do : then 185 × 168=31080; this ÷ by 15 (the last Term) gives 2072 *lb.* or $18\frac{1}{4}$ *C.* the Answer.

For as $b\,c$: w :: $a\,c$: P.

(13) Fig. 9.



In the above Fig. $c\,e$=70 Inches, $A\,c$=2 do. H. the Hogshead, and *w* the Weight.

In. *C.* *In.*

Stated thus—As 2 : $9\frac{1}{4}$: : 70

Or, as 2 : 1064 *lb.* : : 70 : 30 *lb.* 6 *oz.* $6\frac{1}{2}$ *drs.*

Here $9\frac{1}{4}$ *C.*=1064 *lb.* which × by 2=2128 *lb.* these ÷ by 70, (the last Term) gives 30 *lb.* 6 *oz.* $6\frac{1}{2}$ *drs.* the Weight required.

For as $A\,c$: w :: $o\,c$: H.

(14) Stated thus—As 100000 : 20 :: 30 : 66666 Feet 8 Inches per Second, the Answer.

(15) *lb.* *Ft.* *lb.*

Stated thus—As 200 : 100 : : 8 : 2500 Feet per Second.

(14) CONTRACTIONS IN THE RULE OF THREE.

Yds. £. *s.* *d.* *Yds.*

(1) Stated thus—If 4 : 3 17 6 :: 28 Here the first and
× 7 7 Second Term ÷ by 4.

Answer, £.27 2 6

(2) First × 3 = 9*C.* in the three Chests.

Stated thus—If 9*C.* : 11£. 13*s.* :: 72 Here the first and 3d ÷ by 9.
1 8 8

Answer, £.93 4 0

(3) *C.* £. *C.*

Stated thus—If 26 : 78 :: 156 Here 1st and 2d ÷ by 26.
1 3 3

£.468 the Answer.

(4) *lb.* *s.* *lb.*

Stated thus—If 3 : 18 :: 112 Here the 1st and 2d ÷ by 3.
1 6 6

Answer, £.33 12*s.* 0*d.*

(5) First 6 × 8 = 48 Yards in the 6 Gowns.

Yds. £. *Yds.*

Then—Stated thus—If 48 : 6 :: 6 Here the 1st and 2d ÷ by 6
8 64 8£. the Answer.

(6) First, 1½*C.* = 168*lb.*

lb. *s.* *lb.* [by 12

Then—Stated thus—If 24 : 12 :: 168 Here 1st and 2d ÷
2)168(84*s.* or 4£. 4*s.* the Answ.

(7) First 24 × 3 = 72 Yards in the 3 Pieces.

Yds. £. *s.* *Yds.*

Then—If 72 : 64 14 :: 8 Here 1st and 3d ÷ by 8.
9)64 14(7£. 3*s.* 9¼*d.*⅓ Answer.

(8) First $6 \times 3 = 18$*lb.* in the 6 Parcels.

lb. *£.* *s.* *lb.*

Then—If 18 : 12 19 :: 6 Here 1st and 3d Terms ÷ by 6.

3)12 19(4*£.* 6*s.* 4*d.* the Answer.

In. *In.* *Ft.* [by 3.

(9) Stated thus—If 12 : 12 :: 3 Here the 1st and 3d ÷

4 4 1

Answer, 48 Inches, or 4 Feet.

(10) *M.* *C.* *M.*

Stated thus—If 512 : 225 :: 64 Here 1st and 3d ÷ by 64.

8 8 1

Answer, 1800 Cwt.

(15) THE RULE OF FIVE:
Or, THE DOUBLE OF THREE.

P. *D.* *A.*

(1) Placed thus, 6 : 12 : 72

— 4 120

Now, per Rule 5th, $72 \times 4 = 288$ Divisor.

And, $12 \times 6 \times 120 = 8640$ Dividend.

Then $8640 \div 288 = 30$ Men, Answer.

By two Statings, First, As 12 : 6 :: 4 :: 18

Second, As 72 : 18 :: 120 : 30 Men, as before.

(2) *P.* *D.* *B.*

Placed thus—6 : 12 : 2, Then $6 \times 12 = 72$, the Divisor.

36 : 4 :—, And $36 \times 4 \times 2 = 288$ the Dividend.

So $288 \div$ by $72 = 4$ Bushels, Answer.

By two Statings, First, As 6 : 2 :: 36 : 12.

Second, As 12 : 12 :: 4 : 4 Bu.

(3) *P.* *W.* *£.* Now $21 \times 6 = 126$ the Divisor.

6 : 21 : 120 And $46 \times 14 \times 120 = 77280$*£.* Divid.

14 46 —— Then $77280 \div 126 =$ *£.*613 6*s.* 8*d.* the Answer.

By two Statings, First, As 6 : 120 :: 14 : 280.

Second, As 21 : 280 :: 46 : *£.*613 6*s.* 8*d.*

(4) Here £9. 5*s.* = 8880 Farthings, and £70 10*s.* $3\frac{1}{4}$*d.* = 67695 ditto,

C. C. qrs. Then 8880 × 125 = 1110000 the Divisor,
40 : 100 : 8880 and 67695 × 40 × 100 = 270780000 Div.
— : 125,67695 So 270780,0000 ÷ 111,0000 = 243 *C.* 3 *qrs.* 21 *lb.* 15 *oz.* 2$\frac{6}{77}$ *drs.*

By two Statings, First, A's 100 : 40 :: 125 : 32.
Second, As 8880 : 32 :: 67695 : 243 *C.* 3 *qrs.* 21 *lb.* 15 *oz.* 2$\frac{6}{77}$ *drs.*

(5) First, from £.125 8*s.* take £.120 Remains £.5 8*s.* or 108*s.* Interest.
£. *M. s.* Then 120 × 9 = 1080 the Divisor.
120 : 9 : 108 And 108 × 12 × 100 = 129600*s.* Dividend.
1000 12 : —. Therefore 12960,0 ÷ 108,0 = 120*s.* or 6£. the Answ.

By two Statings, First, 9 : 108 :: 12 : 144*s.*
Second, As 120 : 144 :: 100 : 120 = 6£.

(6) First, 100£. = 24000 Pence, and £259 13*s.* 5*d.* = 62321 *d.* then
D. W. £. Here 52 × 24000 = 1248000 the Divisor.
24000 : 52 : 5 And 62321 × 5 × 20 = 6232100 Dividend.
62321 : 20 : — Therefore 6232100 ÷ 12480,00 = £.4 19*s.* 10$\frac{1}{4}$*d.* $\frac{12}{13}$ the Answer.

W. *W.* £. *s.* *d.*
By two Statings, First, As 52 : 5 :: 20 : 1 18 5$\frac{1}{13}$ $\frac{2}{13}$.
Second, As 24000*d.* 38*s.* 5$\frac{1}{13}$*d.* $\frac{2}{13}$. :: 62132[illegible]. £.4 19*s.* 10$\frac{1}{4}$*d.* $\frac{12}{13}$, as before.

(7) First 20 Weeks = 5 Months. Then
1400 : 5 : 14. Here 14 × 8 = 112 Divisor.
—— : 8 : 8. And 1400 × 5 × 8 = 56000 Divid.
Therefore 56000 ÷ 112 = 500 Men.

By two Statings, First, As 5 : 1400 :: 8 : 875.
Second, As 14 : 875 :: 8 : 500 Men.

(8 First, 10 Guineas = 210 Shillings, and 5*l.* = 100*s* do. then
M. D. s. Here 100 × 12 = 1200 the Divisor.
8 : 5 :: 100 And 210 × 8 × 5 = 8400 the Dividend.
— : 12 : 210 Therefore 84,00 ÷ 12,00 = 7 Men, the Answ.

By two Statings, First, As 5 : 8 :: 12 : 1$\frac{1}{2}$.
Second, As 100 : 3$\frac{1}{2}$ :: 210 : 7 Men.

(9 First, 24 Weeks, 4 Days = 172 Days; then
M. D. £. Here 140 × 12 = 1680 the Divisor.
5 : 172 : 140 And 172 × 5 × 200 = 172000 the Dividend.
12 : —— : 200 Therefore 17200,0 ÷ 168,0 = 102$\frac{8}{21}$ Days, or 3 *Mo.* 2 *W.* 4$\frac{8}{21}$ *D.* the Answer.

By two Statings, First, As 5 : 172 : : 12 : $71\frac{2}{3}$.
Second, As, 140 : $71\frac{2}{3}$: : 200 : $102\frac{6}{7}$ *D.* or 3 *Mo.* 2 *W.* $4\frac{6}{7}$ *D.*

(10) First, *l.*3 10*s.*=70*s.* and *l.*38 10*s.*=770*s.* Also 1 Year=4 *qrs.* and $1\frac{1}{4}$=5 *qrs.*

£.	*qrs.*	*s.*
100	4	70
——	5	770

Here 70×5=350 the Divisor.
And 770×4×100=308000 the Dividend.
Therefore 30800,0÷35,0=880*l.* Answer.

By two Statings, First, As *l.*4 : 100 *l.*5 : 87*s.* 6*d.* or 1050*d.*
Second, As 70 : 1050*d.* : : 770 : 211200*d.*=880*l.*

(11) First, 36*s.*÷by 12 (the Sacks in a Chaldron) = 3*s.* what the Coals cost per Week, and 41*l.*=820*s.* then

P.	*w.*	*s.*
7	1	3
—	: 6	820

Here 3×6=18 the Divisor.
And 820×7=5740 the Dividend.
Therefore 5740÷18=318$\frac{8}{9}$ Poor, the Answer.

By two Statings, As, 1 : 7 : : 6 : $1\frac{1}{6}$
Second, As, 3 : $1\frac{1}{6}$: : 820 : 318$\frac{8}{9}$ Poor.

(12) First, As, 2 : 11 : : 3 : $16\frac{1}{2}$. A. goes, while B. goes 17. Then 17—$16\frac{1}{2}$=$\frac{1}{2}$ Toise B. gains of A. in going 17 Toises.
—Again.

As $\frac{1}{2}$: 17 : : $\frac{1}{2}$: 17 Times round gone by A. and $16\frac{1}{2}$ by B. the Answer.

(13) First 18×2=36 Inches, Length of the Lever; then

In.	*Ft.*	*In.*	*Ft.*

Stated thus—As 40 : 104 : : 36 : $93\frac{3}{5}$

Feet.	*Hou.*	*Feet.*

Again,—As $93\frac{3}{5}$: 1 : : 73 : 10 Hours, 8 Min. 20 Sec. the Answer.

In.	*lb.*	*In.*	*lb.*

(14) Stated thus—As 1 : $1\frac{1}{2}$: : 12 : 18wt. 12 In. } from the
Again, As 1 : $1\frac{1}{2}$: : 28 : 42wt. 28 — } Shoulders.
Consequently 42—18=24*lb.* the Answer.

(15) First, 30 Inches=$2\frac{1}{2}$ Feet, 24=2 ditto, and 4*C.* 70*lb.* =518*lb.* then 11—$2\frac{1}{2}$=$8\frac{1}{2}$ Feet; also 11—2=9.

Feet.	*lb.*	*Feet.*	*lb.*

[former Case.
Stated thus—As 11 : 518 : : $8\frac{1}{2}$: $400\frac{3}{11}$ Force in the
And Contra, As $8\frac{1}{2}$: $400\frac{3}{11}$: : $2\frac{1}{2}$: $117\frac{8}{11}$ Pressure.
Again, As 11 : 518 : : 9 : $423\frac{9}{11}$ Force in lat.
Also, As 9 : $425\frac{9}{11}$: : 2 : $94\frac{1}{11}$ Pressure. [Ca.

H.	*D.*	*H.*	*D.*

(16) Stated thus—As 12 : 7 : : 16 : $5\frac{1}{4}$ of 16 Hours each.

M.	*D.*	*M.*	*D.*	*H.*	*M.*

Then, As 152 : $5\frac{1}{4}$: :576 : 19 14 $18\frac{18}{19}$, the Answer.

M. Gall. M. Gall.

(17) Stated thus—As 9 : 14 :: 31 : $48\frac{2}{9}$ fills in 31 Minutes.
Then $48\frac{2}{9}=40=8\frac{2}{9}$ Gallons in the Tub at the End of 31 Minutes.
And from 2 to 5=3 Hours, or 180 Minutes.

M. Gall. M. Gall.

Again, As 31 : $8\frac{2}{9}$ :: 180 : $47\frac{2}{11}$ fills in 3 Hours.
And $147-47\frac{3}{11}=99\frac{8}{11}$ Gallons wants of being full.

Gall. M. Gall. M. Sec.

Also. As 14 : 9 :: $99\frac{8}{11}$: 63 $48\frac{180}{484}$, the Tub will be full.
Which added to 5 o'Clock, will give 3 Min. $48\frac{180}{484}$ Sec. after 6, the Tub will be full.

(18) First $1\frac{1}{2}d.=3$ Halfpence, and $6s.$ $7\frac{1}{2}d.=159$ Ditto.

l. *l.*

Stated thus—As 3 : 1 :: 159 : $53=\frac{4}{7}$ of the Rent.
Then $53l.-4=l.13$ $5s.=\frac{1}{7}$ of the Rent, the Answer.
For $53l.$ added to $13l.$ $5s.$ gives $66l.$ $5s.$ the whole Rent.

l. *s.* *l.* *s.* *s.* *l.* *s.*

Again, As 1 : 4 :: 66 5, 265 or 13 5 the Answer.
Or, instead of the last Stating, thus; from *l.*66 $5s.$ the whole Rent, take $\frac{4}{7}$, viz. $53l.$ Remains $13l.$ $5s.$ the King's Tax, as before.

(19) First, 45+13=58 Persons, and 17 Guineas=375 Shillings.

P. *l.* *P.*

Stated thus—As 45 : 20 :: 58 : $25\frac{7}{9}l.$ in the same Time.—And in twice that Time the 58 Persons will spend $25\frac{7}{9}\times 2=51\frac{5}{9}$*l.*'s Worth at 17 Guineas per Head.
Then—As $51\frac{5}{9}$: 17, or 357 : 63 : 436 $2\frac{1}{4}$, $\frac{2}{1}\frac{6}{9}\frac{16}{84}=l.11$ 16*s.* $2\frac{3}{4}d.\frac{6}{7}\frac{9}{4}\frac{0}{7}$ per Hhd. which $\times$ by 2—*l.*23 12*s.* $5\frac{3}{4}d.\frac{3}{7}\frac{2}{7}\frac{7}{3}$ per Pipe, the Answer.

(20) First 8—5=3 Miles B.
And 10—5=5—— C } gained of A. in one Day.

M. D. M. D.

Then Stated thus—As 3 : 1 :: 73 : $24\frac{1}{3}$ when A. and B. meet.
Again, As 5 : 1 :: 73 : $14\frac{3}{5}$ ——A and C ——
So that B nor C can never meet with A. but at the End of these Periods, when A and C will have travelled 219 Miles.

D. D. D. M.

∴ Then, As $14\frac{1}{7}$: 219 :: $24\frac{1}{3}$: 365.

$\left.\begin{matrix}219\\465\end{matrix}\right\}$ × by $\left\{\begin{matrix}24\frac{1}{3}\\14\frac{1}{7}\end{matrix}\right.$ = 5329 Days, the 73d Time of their general Meeting ∴ 5329 ÷ 73 = 73 Days, their first general Meeting.

M. D. M.

For, As 73 : $\left|\begin{matrix}24\frac{1}{3}\\14\frac{1}{7}\end{matrix}\right|$ :: $\left|\begin{matrix}219\\365\end{matrix}\right|$: 73 Days.

(21) First, for every Day he worked he received 12 Pence.
And for every Day he played he paid 8 ———

Sum, 20

Likewise, as his idle Days came to the same Money as those he worked, therefore the Proportion will be as follows:

D D. D. D.

As 20 : 390 8 : 156 Days he worked.
And, As 20 : 390 12 : 234 ——— played.

For 156, at 12*d.* per Day, comes to the same Money as 234 at 8*d.* per Day, viz. *l.*7 16*s.* the Proof.

(22) First, fourscore and eleven, or 91 Guineas = 5733 Fourpences.
And from December 11 to May 10th = 150 Days.
Also, 100 Marks = 4000 Fourpences.
And from September 3d to Christmas = 113 Days.
Then stated thus—As 5733 : 150 :: 4000 : $214\frac{7995}{8000}$, or rather 215 Days, from which take 113 = 102 Days, and *l.*40 = 2400 Fourpences.
Again, Recep. As 4000 : 102 :: 2400 : 170 Days, the Answer nearly.

(23) First *l.*2 1*s.* 8*d.* = 500 Pence, and 500 ÷ 20 = 25 Days Wages; then 40—25 = 15 Days more.

For every Day he worked he had — 20 Pence.
And for every Day he played, — 10

Sum, 30

Then—As 30 : 15 :: $\begin{cases} d. & D. \\ 10 : 5 & \text{worked.} \\ 20 : 10 & \text{idle.} \end{cases}$

∴ He was idle 10 Days and worked (5+25)=30 Days.

(18) PRACTICE.

(1)

¼	¼	2107 at ¼.
12		526¼
2,0		4,3 10
		£.2 3 10¼

(2)

½	½	1470 at ½d.
12		735
2,0		6,1 3
		£.3 1 3

(3)

		d.
¼	½	1276 at ¼d.
¼	½	638
		319
12		957
2,0		7,9 9
		£.3 19 9

Or thus,

¼	½	1276 at ¼d.
1½	⅛	638
2,0		7,9 9
		£. 3 19 9

(4)

12)	1762 at 1d.
2,0)	14,6 10
	£.7 6 10

(5)

¼	¼	1400 at 1¼d.
		350
12		1750
2,0		14,5 10
		£.7 5 10

(6) $1\frac{1}{3}$ | $\frac{1}{3}$ | 2462 at $1\frac{1}{2}d.$

2,0 | 30,7 9

£.15 7 9

(7) $1\frac{1}{2}$ | $\frac{1}{2}$ | 1041 at $1\frac{1}{4}d.$

$\frac{1}{4}$ | $\frac{1}{10}$ | 130 1

21 $8\frac{1}{4}$

2,0 | 15,1 $9\frac{3}{4}$

£.7 11 $9\frac{3}{4}$

(8) 2 | $\frac{1}{10}$ | 2490 at 2*d.*

2,0 | 41,5

£.20 15

(9) 2 | $\frac{1}{10}$ | 2408 at $2\frac{1}{4}d.$

$\frac{1}{4}$ | $\frac{1}{8}$ | 401 4

50 2

2,0 | 45,1 6

£.22 11 6

(10) 2 | $\frac{1}{10}$ | 640 at $2\frac{1}{2}d.$

$\frac{1}{2}$ | $\frac{1}{4}$ | 106 8

26 8

2,0 | 13,3 4

£.6 13 4

(11) 2 | $\frac{1}{10}$ | 1740 at $2\frac{1}{4}d.$

$\frac{1}{2}$ | $\frac{1}{4}$ | 290

$\frac{1}{4}$ | $\frac{1}{2}$ | 72 6

36 3

2,0 | 39,8 9

£.19 18 9

(12) 3 | $\frac{1}{4}$ | 746 at 3*d.*

2,0 | 18,6 6

£.9 6 6

(13) 3 | $\frac{1}{4}$ | 1417 at $3\frac{1}{4}d.$

$\frac{1}{4}$ | $\frac{1}{2}$ | 354 3

29 $6\frac{1}{4}$

2,0 | 38,3 $9\frac{1}{4}$

£.19 3 $9\frac{1}{4}$

(14) 3 | $\frac{1}{4}$ | 3091 at $3\frac{1}{2}d.$

$\frac{1}{2}$ | $\frac{1}{10}$ | 772 9

128 $9\frac{1}{2}$

2,0 | 90,1 $6\frac{1}{2}$

£.45 1 $6\frac{1}{2}$

(15) *d.*

3		214 at $3\frac{1}{4}d.$
$\frac{3}{4}$	$\frac{1}{4}$	53 6
		13 $4\frac{2}{4}$
2,0		6,6 $10\frac{3}{4}$
		£.3 6 $10\frac{3}{4}$

(16) *d.*

4	$\frac{1}{5}$	2000 at 4*d.*
2,0		66,6 8
		£.33 6 8

(17)

3	$\frac{1}{4}$	569 at $4\frac{1}{2}d.$
1	$\frac{1}{3}$	142 3
$\frac{1}{4}$	$\frac{1}{4}$	47 5
		11 $10\frac{2}{4}$
2,0		20,1 $6\frac{1}{4}$
		£.10 1 $6\frac{1}{4}$

(18)

3	$\frac{1}{4}$	1246 at $4\frac{1}{2}d.$
$1\frac{1}{2}$	$\frac{1}{2}$	311 6
		155 9
2,0		46,7 3
		£.23 7 3

(19)

3	$\frac{1}{4}$	1426 at $4\frac{2}{4}d.$
1	$\frac{1}{3}$	356 6
$\frac{3}{4}$	$\frac{1}{4}$	188 10
		89 $1\frac{1}{2}$
2,0		56,4 $5\frac{1}{2}$
		£.28,4 $5\frac{1}{2}$

(20)

4	$\frac{1}{5}$	2740 at 5*d.*
1	$\frac{1}{4}$	913 4
		228 4
2,0		114,1 8
		£.57 1 8

(21)

4	$\frac{1}{3}$	2147 at $5\frac{1}{4}$
1	$\frac{1}{4}$	715 8
$\frac{1}{4}$	$\frac{1}{4}$	178 11
		44 $8\frac{3}{4}$
2,0		93,9 $3\frac{3}{4}$
		£.46 19 $3\frac{3}{4}$

(22)

4	$\frac{1}{3}$	674 at $5\frac{1}{2}d.$
$1\frac{1}{2}$	$\frac{1}{8}$	224 8
		84 3
2,0		30,8 11
		£.15 8 11

(23)

d.			d.
3	¼	1746 at 5¼d.	
2	⅒	436 6	
¾	¼	291 0	
		109 1½=¼ of 3d.	
2,0		83,6 7½	
		£.41 16 7½	

(24)

			d.
6	½	1741 at 6d.	
2,0		87,0 6	
		£.43 10 6	

(25)

4	⅓	2142 at 6¼d.	
2	½	714	
¼	⅙	357	
		44 7½	
2,0		111,5 7½	
		£55 15 7½	

(26)

6	½	1040 at 6½d.	
½	1/12	520	
		43 4	
2,0		56,3 4	
		£.28 3 4	

(27)

6	½	1746 at 6¾d.	
¾	⅛	873	
		109 1½	
2,0		98,2 1½	
		£.49 2 1½	

(28)

4	⅓	1000 at 7d.	
		333 4	
3	¼	250 0	
2,0		58,3 4	
		£.29 3 4	

(29)

6	½	1656 at 7¼d.	
1	⅒	828	
¼	¼	138	
		34 6	
2,0		100,0 6	
		£.50 0 6	

(30)

6	½	1420 at 7½d.	
1½	¼	710	
		177 6	
2,0		88,7 6	
		£.44 7 6	

(31)

	d.		
	4	$\frac{5}{7}$	674 at $7\frac{1}{4}d.$
	3	$\frac{1}{4}$	2248
			1686
	$\frac{3}{4}$	$\frac{1}{4}$	42 1 $\frac{1}{2}$ = $\frac{3}{4}$ of 3
		2,0	42,5 3$\frac{1}{2}$
			l. 21 15 3$\frac{1}{2}$

(32)

	d.		
	6	$\frac{1}{2}$	2170 at $8d.$
	2	$\frac{1}{3}$	1085
			361 8
		2,0	144,6 8
			l. 72 6 8

(33)

	6	$\frac{1}{2}$	1700 at $8\frac{1}{4}d.$
	2	$\frac{1}{3}$	850
	$\frac{3}{4}$	$\frac{1}{8}$	283 4
			35 5
		2,0	116,8 9
			l. 58 8 9

(34)

	6	$\frac{1}{2}$	1765 at $8\frac{1}{2}d.$
	2	$\frac{1}{3}$	882 6
	$\frac{1}{2}$	$\frac{1}{4}$	294 2
			73 6$\frac{1}{2}$
		2,0	125,0 2$\frac{1}{2}$
			l. 62 10 2$\frac{1}{2}$

(35)

	6	$\frac{1}{2}$	740 at $8\frac{3}{4}d.$
	2	$\frac{1}{3}$	374 6
			124 10
	$\frac{3}{4}$	$\frac{1}{8}$	46 9$\frac{3}{4}$ = $\frac{1}{8}$ of $6d.$
		2,0	54,6 1$\frac{3}{4}$
			l. 27 6 1$\frac{3}{4}$

(36)

	6	$\frac{1}{2}$	1417 at $9d.$
	3	$\frac{1}{2}$	708 6
			354 3
		2,0	106,2 9
			l. 53 2 9

(37)

	6	$\frac{1}{2}$	2373 at $9\frac{1}{4}d.$
	3	$\frac{1}{2}$	1186 6
	$\frac{1}{4}$	$\frac{1}{12}$	593 3
			49 5$\frac{1}{4}$
		2,0	182,9 2$\frac{1}{4}$
			l. 91 9 2$\frac{1}{4}$

(38)

	6	$\frac{1}{3}$	1476 at $9\frac{1}{2}d.$
	2	$\frac{1}{3}$	738
	$1\frac{1}{2}$	$\frac{1}{4}$	246
			1846 = $\frac{1}{4}$ of $6d.$
		2,0	116,8 6
			l. 58 8 6

(39)

d.		
6	½	1760 at 9¾*d.*
3		880
¾		440
		110
2,0		143,0
		*l.*71 10

(40)

d.		
6	½	6000 at 10*d.*
4	⅓	3000
		2000
2,0		500,0
		*l.*250

(41)

6	½	4652 at 10¼*d.*
3	½ of ½	2326
1	½ of ⅓	1163
¼	¼	387 8
		96 11
2,0		397,3 7
		*l.*198 13 7

(42)

6	½	2476 at 10½*d.*
3	½	1238
1½	¼	619
		309 6
2,0		216,6 6
		*l.*108 6 6

(43)

6	½	2176 at 10¾*d.*
4	⅓	1088
		725 4=⅓ of 1*s.*
¾	⅛	135 0=⅛ of 6*d*
2,0		194,9 4
		*l.*97 9 4

(44)

6	½	1276 at 11*d.*
3	½	638
2	⅓	319
		212 8=⅓ of 6*d*
2,0		116,9 8
		*l.*58 9 8

(45)

6	½	2142 at 11¼d.
4	⅓	1071
		714
1	¼	178 6
¼	¼	44 7½
		200,8 1½
		*l.*100 8 1½

(46)

6	½	4760 at 11½*d.*
		2380
4	⅓	1586 8
1½	¼	595 0=¼ of 6*d*
2,0		456,1 8
		*l.*228 1 8

(47)

	d.	
6	½	640 at 11¼d.
3	½	320
		160
2	⅓	106 8 = ⅓ of 6d.
¾	¼	40 0 = ¼ of 3.
2,0		62,6 8
		£.31 68

(48)

6	½	1074 at 1s. 10½d.
4	⅓	557
		358
¾	¼	67 1½
2,0		203,6 1½
		£.101 16 1½

When the Price is, 2, 3, 4, 6, or 8*d.* you may make use of this Method—Thus, for 2*d.* divide the given Quantity by 120; for 3*d.* by 80; for 4*d.* by 60; for 6*d.* by 40; and for 8*d.* by 30, which will give the Answer in Pounds, &c. See the Note under those Prices in the Guide.

(49)

	d.	
¼	¼	2140 at 1s. ¼
12		535
		44 _ 7 added to [2140,
2,0		218,4 7
		£109 4 7

(50)

	d.	
6	½	1749 at 1s. 11½d.
1½	½	874 6
4	½	218 7½
		583 0
2,0		342,5 1½
		£.171 5 1½

(51)

d.		
4	1/3	2140 at 1s. 5d.
1	1/4	713 4
		178 4
2,0		303,1 8
		l.151 11 8

(52)

d.		
6	1/2	1453 at 1s. 7½d.
1½	1/4	726 6
		181 7½
2,0		236,1 1½
		l.118 1 1½

(53)

6	1/2	1614 at 1s. 10d.
4	1/3	807
		538
2,0		295,9
		l. 147 19

(54)

6	1/2	2647 at 1s. 11¼d.
3	1/2	1323 6
		661 9
2	1/3	441 2 = 1/3 of 6d.
¼	1/4	165 5¼
2,0		523,8 10¼
		l.261 18 10¼

The 53d Example may be performed thus—1s. 8d. is 1/12 of a Pound, 2d. is 1/60 of 1s. 8d.

(55) 2476 at 2s.
1
l.247 12

(56) 1476 at 4s.
2
l.295 4

(57) 276 at 6s.
3
l.82 16

(58) 2100 at 8s.
4
l.840 0

(59) 274 at 10s.
5
l.1370

(60) 674 at 12s.
6
l.404 8s.

(61) 2680 at 14s.
7
l.1876 0

(62) 267 at 16s.
8
l.213 12

(63) 1267 at 18s.
9
l.1140 6

The 55, 56, and 59th Examples may be performed thus: —2*s.* is $\frac{1}{10}$, 4*s.* is $\frac{1}{5}$, and 10*s.* is $\frac{1}{2}$ of a Pound, therefore the given Quantities being divided by 10, 5, and 2, will give the Answer, as above.

(64) 2174 at 7*s.*

7

2,0)1521,8

*l.*760 18

(65) .1427 at 9*s.*

9

2,0)1284,3

*l.*642 3

(66) 647 at 11*s.*

11

2,0)711,7

*l.*355 17

(67) 267 at 13*s.*

13

2,0)357,1

*l.*173 11

(68) 274 at 17*s.*

17

2,0)465,8

*l.*232 18

(69) 1260 at 19*s.*

19

2,0)2394,0

*l.*1197 0

(70) 4*s.*$\frac{1}{2}$)2420 at 4*s.*

*l.*484

(71) 5*s.*$\frac{1}{2}$)1764 at 5*s.*

*l.*441

(72) 1*s.* 8*d.*$\frac{1}{12}$ 4762 at 1*s.* 8*d.*

*l.*396 16 8

(73) 2*s.* 6*d.*$\frac{1}{2}$)476 at 2*s.* 6*d.*

*l.*58 7 6

(74) 3*s.* 4*d.*$\frac{1}{6}$)1760 at 3*s.* 4*d.*

l 293 6 8

(75) 6*s.* 8*d.*$\frac{1}{3}$)167 at 6*s.* 8*d.*

*l.*58 13 4

(76)

d.		
3	$\frac{1}{4}$	1420 at 3*s.* 3*d.*
		3
		4260
		355
2,		4615
		*l.*80 15

(77)

d.		
6	$\frac{1}{2}$	427 at 5*s.* 9*d.*
		5
		2135
3	$\frac{1}{2}$	213 6
		106 9
2,0		245,5 3
		*l.*122 15 3

(78)

		d.	
6	½	402 at 10*s.* 8¾*d.*	
		10	
		4020	
2	⅓	201=402÷2	
		67	
¾	⅛	25 1½=⅛ of 201.	
2,0		431,3 1½	
		£.215 13 1½	

(79)

		d.	
6	½	174 at 17*s.* 9½*d*	
		17	
		2958	
3	½	87=174÷2	
½	1/12	43 6	
		7 3	
2,0		30,5 9	
		£.154 15 9	

(80)

3	¼	273 at 19*s.* 4¾*d.*	
		19	
		5187	
1	⅓	68 3=273÷4	
		22 9	
¾	¼	17 0¾=¼ of 3*d.*	
2,0		529,5 0¾	
		*l.*264 15 0¾	

(81)

6	½	260 at 14*s.* 11¾*d.*	
		14	
		3640=14*s.*	
3	½	130= 6*d.*	
		65= 3	
2	⅓	43 4=2	
¾	¼	16 3=¼ of 3*d.*	
2,0		389,4 7	
		*l.*194 14 7	

Example 76 may be performed thus—2*s.* is $\frac{1}{10}$, 1*s.* is ½ of 2*s.* and 3*d.* is ¼ of 1*s.*—Example 77, thus; 5*s.* is ¼ 6*d.* is $\frac{1}{20}$ of 5*s.* and 3*d.* is ½ of 6*d.*—Likewise Example 78, thus; 10*s.* is ½ of a Pound, 6*d.* is $\frac{1}{20}$ of 10*s.* 2*d.* is ⅓ of 6*d.* and ⅛ is ⅓ of 6*d.* which Parts added together will give the Answers as before.

(82) 1[illegible]*l.*

£.480

(83) 96 at 17*l.*

17

£.1632

(84) 100 at 3*l.*
3
*l.*300

(85) 142 at 42*l.*
6×7=42*l.*
*l.*5964

(86) 694 at 2*l.* 6*s.*
2
1298
194 14
*l.*1492 14

(87) 526 at 7*l.* 16*s.*
7
3682
420 16
*l.*4102 16

(88) 142 at 1*l.* 17*s.*
17
2,0,24,1,4
120 14 added to 142*l.*
*l.*262 14

(89) 164 at 24*l.* 19*s.*
5×5=25
4100
—8 4=164*s.* subt.
*l.*4091 16

(90) 271 at 5*l.* 7*s.*
5
1355*l.*
20,)189,7=271×7
94 17 added to 1355*l.*
*l.*1449 17

(91) 5 | ¼ | 604 at 20*l.* 9*s.*
20
12080
151=604÷4
4 | ⅙ | 120 16
£.12351 16

(92) 10*s.* | ½ | 914 at 10*l.* 15*s.*
11
10054,0
5 | ½ | 228,10 Sub.
£.9825 10

(93) [illegible] at 1*l.* 14*s.*
515 18 added to 737*l.*
£.125[illegible]8

(94)

6*s.* 8*d.* | ⅓ | 274 at 7*l.* 6*s.* 8*d.*
7
1918
91 6 8
*l.*2009 6 8

(95)

3*s.* 4*d.* | ⅙ | 120 at 12*l.* 3*s.* 4*d.*
12
1440
20
*l.*1460

(96)

1*s.* 8*d.* | 1½ | 97 at 9*l.* 1*s.* 8*d.*
9
837
8 1 8
£.881 1 8

(97)

5*s.* | ¼ | 512 at 42*l.* 5*s.*
7×6=42
21504
128
£.21632

(98)

1472 at *l.*4 6*s.* 7½*d.*
4
6 5/12*d.* 5888
441 12 See Case 4.
1½ 36 16
9 4
£.6375 12

(99)

10*s.* | ½ | 279 at *l.*6 11*s.* 9¼*d.*
6
1674 0
1 139 10
6*d.* 1/10 13 19
3 1/20 6 19 6
¾ 1/80 3 9 9
¼ 0 17 5¼
£.1838 15 8¼

(100)

10*s.* | ½ | 1420 at *l.*19 14*s.* 11¾*d.*
19

26980
710
4 | ⅕ | 284
6*d.* | ⅛ | 55 10 = ⅛ of 4*s.*
3 | ½ | 17 15 = ½ of 6
2 | ⅓ | 11 16 8
¾ | ¼ | 4 8 9

*£.*28043 10 5

(101)

4*d.* | ⅕ | 2074 at *l.*1 17 5½*d.*
37 20

76738 37
691 4
1½ | ⅛ | 259 3
2,0 | 7768,8 7

*£.*3884 8 7

(102)

27 at 4 11 8¼ — 9×3=27

£. *s.* *d.*
41 5 6¼
3

*l.*123 16 8¼

(103)

64 at 12 13 7¼ — 8×8=64

£. *s.* *d.*
101 8 10
8

*l.*811 10 8

(104)

7 *C.* 3 *qrs.*18 *lb.* at

£. *s.* *d.*
2 *qrs.* | ½ | 17 6
7

6 2 6
1½ | 8 9
4 4½
16*lb.* | 2 6
2 | 3¾

*£.*6 18 5¼

(105)

420 *oz.* 15 *dwts.* 16 *grs.* at

£. *s.* *d.*
3 16 10½
10*d.* | ½ | 6

23 1 3
7

161 8 9
10

1614 7 6
1 18 5¼
4 | ⅕ | 15 4½
1 | ¼ | 3 10
gr. 8 | ⅓ | 1 3¼
8 | ⅓ | 1 3¼

*£.*1617 7 8¼

(106)

72*C.* 3 *qrs.* 19 *lb.* at

qr.	*lb.*	£.	*s.*	*d.*
2	$\frac{1}{2}$	3	17	$4\frac{1}{2}$
				9×8=72
		34	16	$4\frac{1}{2}$
				8
		278	11	0
	$\frac{1}{2}$	1	18	$8\frac{1}{4}$
16	$\frac{1}{7}$		19	4
2	$\frac{1}{8}$		11	$0\frac{1}{2}$
1	$\frac{1}{2}$		1	$4\frac{1}{2}$
				$8\frac{1}{4}$
		£.282	2	$1\frac{1}{2}$

(107)

12*C.* 1 *qr.* 17*lb.* at

lb.	£.	*s.*	*d.*
1 $\frac{1}{4}$	4	4	4
			12
	50	12	0
14 $\frac{1}{2}$	1	1	1
2 $\frac{1}{7}$		10	$6\frac{1}{2}$
1 $\frac{1}{2}$		1	6
			9
	£.52	5	$10\frac{1}{2}$

(108)

23 *C.* 18*lb.* at

	£.	*s.*	*d.*
	4	14	$1\frac{1}{2}$
			11×2+1=23
	51	15	$4\frac{1}{2}$
			2
lb.			
16 $\frac{1}{7}$	103	10	9
	4	14	$1\frac{1}{2}$
2 $\frac{1}{8}$		13	$5\frac{1}{4}$ $\frac{2}{7}$
		1	8 $\frac{4}{7}$
	£.109	0	0

(109)

s.	*d.*			*£. s. d.*
5	¼	476	*A.* 3 *r.* 28 *p.* at	3 7 11
		3		
		1428		
		119		
2	6 ¼	59	10	
	5 ⅛	9	18 4	
2	*r* ½	1	13 11½ = ½ of	3 7 11
1	*r* ½		16 11¾	
	20 ½		8 5¾	
	8 ⅓		3 4¼	
		£.	1619 11	1¾

(110)

16 *C.* 2 *qrs.* at

		£.	*s.*	*d.*
2	½	2	6	11
				4
		9	7	8
				4×4=16
		37	10	8
		1	3	5½
		l.	38 14	1½

(111)

48 *C.* 2 *qrs.* 7*lb.* at

		£.	*s.*	*d.*
2 *qrs.*	½	74	16	6
				8×6=48
		598	12	0
			6	
		5591	12	0
7*lb.*	⅛	37	8	3
		4	13	6¼
		l.	3633 13	9¼

(112) 24 *lb.* at

lb.		*£.*	*s.*
16	⅛	4	17 per *C.*
8	½	0	13 10¼
		0	6 11
		l.	1 0 9¼

(113 17 *lb.* at

lb.		*£.*	*s.*	*d.*
14	⅛	3	5	4
2	⅐	0	8	2
1	½	0	1	2
		0	0	7
		l.	0 9	11

When the Pupil is perfect in all the preceding Cases, he may then learn the Contractions, some of which are as follow:

CONTRACTIONS TO CASE I. AND II.

(1)

d.

3 | $\frac{1}{8\,0}$ | 1276 at $\frac{3}{4}$

$\frac{1}{4}$ | $\frac{1}{4}$ | 15 19

£.3 19 9

(2)

d.

3 | $\frac{1}{8\,0}$ | 1740 at $2\frac{3}{4}d.$

$\frac{1}{4}$ | $\frac{1}{1\,2}$ | 21 15 0 } subtract.
1 16 3 }

£.19 18 9

(3) See Example 3.

s. d.

2 6 | $\frac{1}{3}$ | 1420 at $7\frac{1}{2}$

$7\frac{1}{2}$ | $\frac{1}{4}$ | 177 10

£.44 7 6

See Example 30.

CONTRACTIONS TO CASE III. VII. AND IX.

(4)

s. d.

1 8 | $\frac{1}{1\,2}$ | 3162 at 1*s.* 7*d.*

1 | $\frac{1}{2\,0}$ | 163 10 0 } subtract.
13 3 6 }

£.250 6 6

(5)

3*d.* | $\frac{1}{8\,0}$ | 427 at 5*s.* 9*d.*

3

128 2 } subtract.
5 6 9 }

£.122 15 3

(6)

2*d.* | $\frac{1}{1\,4\,0}$ | 246 at 17*s.* 10*d.*

9

221 8 } subtract.
2 1 }

£.219 7

(7)

1*s.* $\frac{1}{10}$ | 241 at *l.*6 19*s.*
7

1687
12 1 0 } subtract.

*£.*1674 19 0

(8)

5*s.* $\frac{1}{4}$ | 641 at *l.*1 15*s.*
2

1282 0
160 5 } subtract.

*£.*1121 15

17. TARE AND TRETT.

(1)

	C.	*qr.*	*lb.*		*C.*	*qr.*	*lb.*
Gross,	6	2	16	×6×4(24*hhds.*)=	159	2	16
Tare,	-	-	-	-	17	3	27
Answer, Net Wt.					141	2	17

(2)

	C.	*qr.*	*lb.*
Gross,	23	2	19
Tare,	1	0	12
Answer, Net Wt.	22	2	7

(3)

	C.	*qr.*			*C.*	*qr.*	*lb.*
Gross,	2	3	× 8 Frails.	=	22	0	0
Tare,	22*lb.*			=	1	2	8
Answer, Net Wt.					20	1	20

(4)

	C.	*qr.*	*lb.*		*C.*	*qr.*	*lb.*
Gross Wt.	3	0	27	×5×8 (40) =	129	2	16
Tare	0	0	18	=	6	1	20
Answer, Net Wt.					123	0	24

(5)

	C.	*qr.*	*lb.*		*C.*	*qr.*	*lb.*	*oz.*
Gross Wt.	2	1	17	×2×11 (22 Barrels) =	52	3	10	0
Tare 14*lb.* per. *Cwt.* is $\frac{1}{8}$	-	-	-	=	6	2	11	12
Answer, Net Wt.					46	0	26	4

(6) Gross Wt. 6*C.* 2 *qr.* 12 *lb.* × 9 = 59 *C.* 1 *qr* 24 *lb.*

14*lb.*	$\frac{1}{8}$	59	1	24	Gross.
	$\frac{1}{7}$	7	1	20	8
	$\frac{1}{2}$	1	0	6	$14\frac{6}{7}$
	1	0	2	3	$7\frac{1}{7}$
		9	0	2	$14\frac{2}{7}$ Tare.
		Answer, 50	1	21	$1\frac{5}{7}$ Net Wt.

(7)

		C.	*qr.*	*lb.*	
		16	3	0	Gross.
		1	1	12	Tare.
4*lb.*	$\frac{1}{28}$	15	1	16	Suttle.
		1	2	$10\frac{14}{13}$	Trett.
		Answer, 14	3	$5\frac{9}{13}$	Neat Weight.

(8)

		C.	*qr.*	*lb.*		*C.*	*qr.*	*lb.*
		Gross, 2, 3, 27 × 3 × 9 (27 Bags) =				80	3	8
16*lb.*	$\frac{1}{7}$	80	3	1	Gross.			
8	$\frac{1}{2}$	11	2	4	2 Tare, supposing it had been 16*lb.*			
4	$\frac{1}{2}$	5	3	2	1			
1	$\frac{1}{4}$	2	3	15	0 8			
		0	2	24	12 2			
		9	1	13	13 10 Tare.			
4*l.*	$\frac{1}{28}$	71	1	15	2 6 Suttle.			
		2	2	27	8 $1\frac{6}{13}$ Trett.			
		Answer, 68	2	15	10 $4\frac{7}{13}$ Net.			

(9)

2*lb.*	$\frac{2}{108}$	5647	3	13	Gross.
		33	2	$13\frac{41}{100}$	Cloff.
		Answer, 5614	0	$14\frac{119}{100}$	Net.

(10)

		C.	qr.	lb.	
	3)	42	3	0	Gross.
2lb. $\frac{1}{28}$	7)	14	1	0	
	8)	2	0	4	
		0	1	0	8 Cloff.
Answer,		42	1	27	8 Net.

(11)

		C.	qr.	lb.	oz.	
14 lb.	$\frac{1}{8}$	15	3	20	0	
7	$\frac{1}{4}$	1	3	27		Tare, supposing it had been 14lb.
		0	3	27	8	Tare.
4	$\frac{8}{28}$	14	3	20	8	Suttle.
		0	2	8	5	Trett.
2	$\frac{1}{28}$	14	1	12	2	=Second Suttle.
		0	0	9	8	Cloff.
Answer,		14	1	2	10	Net.

(12)

		C.	qr.	lb.		
16 lb.	$\frac{1}{7}$	[illegible]				Gross
8	$\frac{1}{2}$	2	3	25	[illegible]	Tare, supposing 16lb. per Cr.
		1	1	26	$14\frac{6}{7}$	Tare.
4	$\frac{8}{28}$	19	1	14	$1\frac{1}{7}$	Suttle
		0	2	27	7	Trett.
2	$\frac{1}{28}$	18	2	14	$10\frac{1}{7}$	=Second Suttle.
		0	0	12	$6\frac{2}{7}$	Cloff.
Answer,		18	2	2	$3\frac{6}{7}$	Net.

(13)

	£.	s.	d.	C.	qrs.	lb.	
Neat proceeds,	4	14	$6\frac{1}{10}$	9	3	10	Gross.
Custom, &c.	2	8	6	0	3	$26\frac{1}{7}$	Tare.
Freight,	1	2	8				
Factorage,	0	4	9	C. 8	3	$11\frac{4}{7}$	Net=4959 Fifths.
				And 1 C.=560 Ditto.			
	£.8	10	5=2045 Pence.				
			d.				

Stated thus—If 4959 : 2045 :: 560
560

4959)1145200(230$\frac{3}{4}\frac{6}{9}\frac{4}{5}\frac{3}{5}$*d.* or *l.*0 19*s.* $2\frac{3}{4}\frac{6}{9}\frac{4}{5}\frac{3}{5}$*d.* the Answer.

(14)

First, 1180 × 80=94400 Inches, which, ÷ by 231, (the Number of Inches in a Gallon) gives 408$\frac{1}{4}\frac{3}{3}\frac{2}{1}$ Gallons; these × $7\frac{1}{2}$*lbs.* (the Number of *lbs.* in a Gallon) gives 3064$\frac{2}{4}\frac{1}{3}\frac{5}{1}$*lbs.* Gross, which ÷ by 10=306$\frac{1}{2}\frac{1}{1}\frac{4}{1}$*lb.* Tare; this, taken from the Gross=2758$\frac{1}{2}\frac{0}{1}\frac{2}{1}$*lb.* Neat.

Then, stated thus—As 112*lb.* 4*s.* 6*d.* or 54*d.* :: 2758$\frac{1}{1}\frac{0}{4}\frac{2}{4}$*lb.* 1329$\frac{3}{4}\frac{2}{2}\frac{2}{3}\frac{0}{8}\frac{3}{7}\frac{2}{2}$*d.* or *l.*5 10*s* 9$\frac{3}{4}\frac{2}{2}\frac{2}{3}\frac{0}{8}\frac{1}{7}\frac{3}{2}$*d.* the Answer.

18. SIMPLE INTEREST.

(1) First, *l.*824 18*s.* 2*d.* Principal, × by 4, Rate per Cent. =*l.*3299 12*s.* 8*d.* which ÷ by 100, gives *l.*32 19*s.* 11*d.* $\frac{3}{25}$, the Answer.

(2) First, 500*l.*÷2,0=25. Interest for 1 Year; this × by 4, (the Number of Years)=100*l.* the Answer.

(3) First, *l.*264 0*s.* 4*d.* ÷ 20=*l.*13 4 $4\frac{4}{5}$ Interest for 1 Year, this × by 12 (the Time)=*l.*158 8*s.* $2\frac{4}{5}$*d.* $\frac{4}{5}$ Interest; which, added to the Principal, gives *l.*422 8*s.* $6\frac{1}{4}$*d.* $\frac{1}{5}$ the Amount required.

(4) First, *l.*246 18*s.* × $4\frac{1}{4}$=*l.*1049 6 6 which ÷ 100= *l.*10 9 $10\frac{1}{4}$ $\frac{3}{25}$ the Interest required.

(5) First, *l.*140 10*s.* × $4\frac{1}{2}$ = *l.*632 5*s.* 0*d.* ÷ 100 = *l.*6 6*s.* $5\frac{1}{4}$*d.* $\frac{1}{2}$ Interest for a Year, which × by 7, = *l.*44 5*s.* $1\frac{3}{4}$*d.* $\frac{1}{2}$ Interest, for 7 Years, the Answer.

(6) First, *l.*470 × $3\frac{3}{4}$=*l.*1762 10*s.* ÷ 100=*l.*17 12*s.* 6*d.* Interest for a Year which × 5=*l.*83 2*s.* 6*d.* Interest for

5 Years; this added to the Principal, viz. *l.*470=*l.*558 2*s.* 6*d.* the Amount required.

(7) First, *l.*474 14*s.* 6*d.*×3¼=1542 17*s.* 1½*d.* this ÷ by 100, gives *l.*15 8*s.* 6¾*d.*, $\frac{2}{5}\frac{1}{0}$ the Commission required.

(8) First, 2½*d.*=$\frac{1}{40}$, then *l.*74,2 12*s.* 6*d.*÷4,0=*l.*18 11*s.* 3¾*d.* the Answer.

BROKERAGE.

(9) First, *l.*420 12*s.* 6*d.*÷100=*l.*4 4*s.* 1½*d.*

		£.	s.	d.
4*s.*	$\frac{1}{5}$	4	4	1½
		0	16	9¾
2	$\frac{1}{2}$	0	8	4¾
4*d.*	$\frac{1}{6}$	0	1	4¾

Answer, *l.*1 6 7¼ Brokerage.

(10) First, 1000*l.*÷100=10*l.* Then

s.		d.	£.
4 0	$\frac{1}{5}$	10 at 4*s.* 6*d.*	
6	$\frac{1}{4}$	2	0
		0	5

Answer, £.2 5

(11) First, *l.*540 10*s.*÷100 =*l.*5 8*s.* 1*d.* then

s.	d.		£.	s.	d.	
10	0	½	3	8	1	at 13 10
3	4	⅓	2	14	0½	
			0	18	0	
	6	$\frac{1}{10}$	0	2	6¼	

Answer, *l.*3 14 8¾

(12) First, *l.*2474 15*s.*÷100 =*l.*24 14*s.* 11¼*d.* then

s.	d.		£.	s.	d.	
10	0	½	24	14	11¼	at 19 9½
			12	7	5¼	
6	8	⅓	8	4	11¾	
2	6	¼	3	1	10¼	
	7½	¼	0	15	5½	

Ans. *l.*24 9 9

INSURANCE.

(13) First, 14,60*l.*÷100=*l.*14 12*s.*

Then, *l.*14 12*s.*÷8 (2*s.* 6*d.*=⅛)=*l.*1 16*s.* 6*d.* Insurance required.

(14) First, *l.*2460 × 10¾ = *l.*26445, which ÷ 100 = *l.*264 9*s.* the Insurance required.

(15) First, *l.*2500 × 6⅞ = *l.*17187 10*s.* which ÷ by 100 = *l.*171 17*s.* 6*d.* the Insurance required.

(16) First, *l.*7406 17*s.* 6*d.* × 15¾ = *l.*116658 5*s.* 7½*d.* which ÷ 100 = *l.*1166 11*s.* 7¼*d.*½ the Insurance required.

PURCHASING OF STOCKS.

(17) First, £.400 × 20¼ ÷ 100 = 81*l.*
Then, 400 × 81 = 481*l.* the Purchase required.

(18) First, *l.*470 × 87¼ = *l.*41242 10*s.* which ÷ by 100 = *l.*412 8*s.* 6*d.* the Purchase required.

(19) First, *l.*2470 17*s.* 10*d.* × 3½ (the Excess) = *l.*8648 2*s.* 5*d.* which ÷ 100 = *l.*86 9*s.* 7¼*d.*⅖; this, added to *l.*2470 17*s.* 10*d.* the Stock, gives *l.*2557 7*s.* 5¼*d.*⅔ the the Answer.

(20) First, *l.*876 × 14⅝ (the Excess) = £12811 10*s.* which ÷ by 100 = *l.*128 2*s.* 3½*d.*⅖; this, added to 876*l.* the Stock, gives *l.*1004 2*s.* 3½*d.*⅖ the Purchase required.

(21) First, *l.*427 10*s.* × 4 ÷ 100 = *l.*17 2*s.*
Then, *l.*17 2*s.* ÷ 3 4 Mo. = ⅓ = 5*l.* 14*s.* the Interest required.

(22)

£.		£.	*s.*	*d.*	
5	⅕	246	12	6	
1 Year		= 12	6	7½	Interest.
¼	¼	3	1	7¾	½
Answer,		£.15	8	3¼	½

(23)

		£.	s.		
5	$\frac{1}{10}$	209	18		
		10	9	$10\frac{3}{4}$	$\frac{1}{5}$
$\frac{2}{4}$	$\frac{1}{10}$		10	$5\frac{1}{4}$	$\frac{3}{5}$
$\frac{1}{2}$	$\frac{1}{2}$	9	19	$5\frac{3}{4}$	$\frac{3}{5}$
				4	
		39	17	$8\frac{1}{5}$	$\frac{2}{5}$ = 4 Years Interest.
		4	19	$8\frac{3}{4}$	$\frac{3}{5}$ = $\frac{1}{2}$ — —
		209	18	0	= Principal.

Subtract.

Answer, £.254 15 4 $\frac{4}{2}$

(24) £

		£.	s.		
5	$\frac{1}{10}$	604	17	6	
$\frac{1}{2}$	$\frac{1}{10}$	30	4	$10\frac{5}{2}$	
		3	0	$5\frac{3}{4}$	$\frac{3}{5}$
6 Mo.	$\frac{1}{2}$	33	5	$4\frac{1}{4}$	$\frac{3}{5}$
				3	
		99	16	1	$\frac{4}{5}$ = 3 Years Interest.
3	$\frac{1}{2}$	16	12	8	$\frac{4}{5}$ = 6 Months.
		8	6	4	$\frac{2}{5}$ = 3 Ditto.
		604	17	6	Principal.

Answer, £729 12 $7\frac{1}{2}$ the Amount.

(25 First, *l.*400 15*s.*×4÷100=*l.*16 0 $7\frac{1}{5}$ Interest for 1 Year. Then

w.	£.	*s.*	*d.*	*w.*	£.	*s.*	*d.*
As 52	: 16	0	7	: : 4	1	4	$7\frac{3}{4}$ the Interest required.

(26) First, 600×$3\frac{3}{4}$=22,50*l.* this÷by 1,00=*l.*22 10*s.* the Interest for one Year.

w.	£.	*s.*	
26 $\frac{1}{2}$	22	10	
	11	5	Interest for 26 Weeks.
	600	0	Principal.

Answer, £611 5

(27) First, 740*l.* ÷ 20 = 37*l.* Interest for a Year; then
If 52 *W.* : 37*l.* : : 42 *W.*
Here 37 × 42 = 1554*l.* these ÷ by 52 = *l.*29 17*s.* $8\frac{1}{4}d.\frac{3}{13}$ the Interest for 42 Weeks, which added to 37*l.* × 4, viz. 148*l.* the Interest for 4 Years, gives *l.*177 17*s.* $8\frac{1}{4}d.\frac{3}{13}$, the Interest for 4 Years, 42 Weeks, the Answer.

(28) First, 200*l.* × $4\frac{1}{2}$ = 900*l.* ÷ 100 = 9*l.* Interest for a Year, which × 5 = 45*l.* Interest for 5 Years. Then
If 52 *W.* : 9*l.* : : 50 *W.*
Here 9 × 50 = 450*l.* ÷ 52 = *l.*8 13*s.* $0\frac{3}{4}d.\frac{9}{13}$, Interest for 50 Weeks; which added to 45*l.* the Interest for 5 Years, gives *l.*53 13*s.* $0\frac{3}{4}d.\frac{9}{13}$, the whole Interest; this added to 200*l.* = *l.*253 13*s.* $0\frac{3}{4}d.\frac{9}{13}$, the Amount required.

(29) First 200 × $3\frac{1}{2}$ ÷ 100 = 7*l.* Interest for 1 Year.
Now, 73 Days = $\frac{1}{5}$ of 1 Year.
Therefore 7*l.* ÷ 5 gives *l.*1 8*s.* 0*d.* the Interest required.

(30) First, *l.*340 10*s.* ÷ 20 = *l.*17 0*s.* 6*d.* the Interest for a Year.
And from January the 1st, to July the 18th, = 199 Days by the Table; or see Q. 1. P. 27; then,

D.	£.	*s.*	*d.*	*D.*
If 365	:	17	0 6	: : 100

Here *l.*17 0*s.* 6*d.* = 4086 Pence; these × 199 = 813114*d.* which ÷ 365 = 2227$\frac{1}{2}d.\frac{1}{3}\frac{0}{6}\frac{0}{5}$, or *l.*9 5*s.* 7$\frac{1}{2}d.\frac{1}{3}\frac{0}{6}\frac{6}{5}$ Answer.

(31) First, 500*l.* × 4 = 2000*l.* ÷ 100 = 20*l.* Interest for a Year.
And from December the 4th, 1791, to March the 10th, 1793, = 1 Year, 96 Days; then

D.	£.	*D.*
If 365	: 20	: : 96.

Here 96 × 20 = 1920*l.* this ÷ 365 = 5*l.* 5*s.* $2\frac{1}{4}d.\frac{6}{7}\frac{3}{3}\frac{1}{6}\frac{5}{5}$, Interest for 96 Days; which, added to 20*l.* (Interest for a Year) = *l.*25 5*s.* $2\frac{1}{4}d.\frac{3}{7}\frac{1}{3}\frac{5}{6}\frac{5}{5}$, the Interest required.

(32) First, *l.*4 10*s.* Rate × 9$\frac{1}{2}$ Year's Time = *l.* 42 15*s.* Interest of 100*l.* for 9$\frac{1}{2}$ Years; which Interest added to 100*l.* = *l.*142 15*s.* the Amount; then,

£.	*s.*	£.	£.	*s.*
If 146	15 :	100	: : 856	10

Here *l.*142 15*s.* = 2855*s.* and *l.*856 10*s.* = 171307*s.* which × 100 = 1713000*l.* these ÷ 2855 = 600*l.* the Answer.

(33) First, 3*l.* Rate, × 7¾. Time=*l.*23 5*s.* Interest; which, added to 100, Principal,=*l.*123 5*s.* the Amount; then,

£.	s.	£.	£.	s.	d.
If 123	5	: 100	: : 614	3	11.

Here *l.*123 5*s.*=29580*d.* and *l.*614 3*s.* 11*d.*=147407*d.* × 100=14740700*l.* ÷ 29580=*l.*498 6*s.* 8*d.* the Answer.

(34) First, *l.*600×4½=2700*l.* ÷ 100=27*l.* Interest of 600*l.* for a Year; and from *l.*856 10*s.* take 600*l.* remains 256*l.* 10*s.* Interest of 600*l.* for the whole Time; then,

£.	Y.	£.	s.
If 27	1	: : 256	10
20		20	
54,0		)513,0(9½ Years, Answer.	

(35) First, *l.*498 6*s.* 8*d.* × 3 =*l.*1495 this ÷ 100=*l.*14 19*s.* Interest of the given Principal for a Year; then from *l.*614 3*s.* 11*d.* take *l.*498 6*s.* 8*d.* leaves *l.*115 17*s.* 3*d.* Interest of the Principal for the whole Time; then,

£.	s.	Y.	£.	s.	d.
If 14	19	: 1	: : 115	17	3

Here *l.*14 19*s.*=3588*d.* and *l.*115 17*s.* 3*d.*=27807*d.* which ÷ 3588=7¾ Years, the Answer.

(36) First, from *l.*856 10*s.* take 600*l.* Remains *l.*256 10*s.* Interest of the 600*l.* for 9½ Years.

£.	£.	s.	s.	£.
If 600	: 256	10	or 5130	: : 100
			100	
		6,00	)5130,00(855*s.*	

Interest of 100*l.* for 9½ Years, which ÷ 9½ viz. 19 Half-Years, gives 45*s.* or *l.*2 5*s.* Interest of 100*l.* for Half a Year, which ×2=*l.*4 10*s.* per Cent. the Answer.

(37) First, from *l.*614 3*s.* 11*d.* take *l.*498 6*s.* 8*d.* Remains *l.*115 17*s.* 3*d.* Interest; then,

£.	s.	d.	£.	s.	d.	£
If 498	6	8	: 115	17	3	: : 100
Reduced, 119600*d.*			: 27807			: : 24000*d.*
			24000			
1196,00			)6673680,00(5580*d.* or 465*s.*			

And $7\frac{1}{4}$=31 Qrs. then 31 465 15*s.* × 4=3*l.* per Cent. the Answer.

(38) First, 5000*l.* × $4\frac{1}{2}$=225,00*l.* ÷ 1,00=225*l.* Interest for a Year: which ÷ 4=*l.*56 5*s.* Interest due to Lady-Day, which is a Quarter, and in this Manner proceed with each new Principal for the Interest.

1793.	£.		
Christmas, lent - -	5000	at $4\frac{1}{2}$ per Cent.	
1794. Inter. due to Lady D.	56	5	
Amount - - -	5056	5	
Drew out - - -	194	5=185 Guineas.	
Remains - - -	4862	0 New Principal.	
Interest of which to Mids.	54	13 $11\frac{1}{4}$	
Amount - - -	4916	13 $11\frac{1}{4}$	
Paid in 500 Moidores -	675	0 0	
Sum - - - -	5591	13 $11\frac{1}{4}$	
Drew out - - -	700	0 0	
Remains - - -	4891	13 $11\frac{1}{4}$ New Principal	
Interest to Michaelmas -	55	0 $7\frac{1}{2}$	
Amount - - -	4946	14 $6\frac{3}{4}$	
Paid in Part - - -	569	17 0	
Answer,	£5516	11 $6\frac{3}{4}$	

(39)

1793.			Days	Products.
May 1.	Lent per Bill at one Day's Date	500	13	6500
13.	Received in Part - -	50		
	Balance - - -	450	22	9900
June 4.	Received in Part - -	56		
	Balance - - -	394	40	15760
July 14.	Received in Part - -	44		
	Balance - - -	350	9	3150
	Carried over, Sum			35310

		£.	Days	Products.
	Brought over	350		35310
28.	Received in Part	50		
	Balance	300	26	7800
Aug. 18.	Received in Part	87		
	Balance	213	12	2556
30.	Received in Part	13		
	Balance	200	22	4400
Sept. 21.	Received in Part	30		
	Balance	170	27	4590
Oct. 18.	Received in Part	30		
	Balance	140	11	1540
29.	Received in Part	40		
	Balance	100	13	1300
Nov. 11.	Received in Part	50		
	Balance	50	47	2350
Dec. 28.	Received in full of Principal	50		

Sum of the Products 59846

Then, 73,00)598,46,8 £. 3s. 11d. $\frac{1}{2}$. $\frac{58}{365}$ Interest due on this Account.

(40)

1793.		£.	s.	d.	Days	Products. £.	s.	d.
Jan. 18.								
	To a Bill at one Day's Date	878	19	10	40	35159	13	4
Feb. 27.	Received in Part	57	15	7				
	Balance	821	4	3	19	15603	0	9
Mar. 18.	Received in Part	37	14	0				
	Balance	783	10	3	42	32907	10	6
Apr. 29.	Received in Part	34	11	0				
	Carried over	748	19	3	13	9736	10	3
		93406	14	10				

		£.	s.	d.	Days	Products. £.	s.	d.
	Balance - -	748	19	3		93406	14	10
May 12.	Received in Part -	136	15	7				
	Balance - -	612	3	8	58	23262	19	4
June 19.	Received in Part -	67	13	4				
	Balance - -	544	10	4	26	14157	8	8
July 15.	Received in Part -	15	15	6				
	Balance - -	528	14	10	10	5287	8	4
5.	Received in Part -	111	11	11				
	Balance - -	417	2	11	70	29200	4	2
Oct. 3.	Received in Part -	78	7	4				
	Balance - -	338	15	7	47	15922	12	5
Nov. 19.	Received in Part -	100	0	0				
	Balance - -	238	15	7	4	955	2	4
23.	Received in Part -	100	0	0				
Dec. 30	Received in full of the Principal	138	15	7	37	5134	16	7
		138	15	7				

Sum of the Products £.187327 6 8

Then 73,00)187347*£*. 6*s*. 8*d*. (£.25 13*s*. $2\frac{1}{2}d.\frac{60}{73}$ the Interest required.

(41) First, from 109 Moidores take 21*s*. 6*d*. Rem. 147 0 6
Amount of the Bond,
And 109 Guineas - - - =114 9 0

Difference or Interest £.32 11 0

Also, £.114 9*s*. Principal, × 4 Rate, =£.4,57 16*s*. this ÷ 1,00=£.4 11*s*. $6\frac{1}{2}d.$ Interest for a Year.

£. *s*. *d*. *Yea*. £. *s*. *d*.
Then If 4 11 $6\frac{1}{2}$: 1 :: 32 11 6

Reduced, 2197 Halfpence : 1 :: 15636 Halfpence; therefore 2197)15636(7 Years, 42 Days, the whole Time.

July hath 31 Days
Till August 18th. } Sum 49—42=7th of July, the Ans.

(42) First, £39. 19*s*. 8*d*.=9596 Pence; then per Sect. 15.

	Prin.	*Yrs.*	*Interest.*
Placed thus,	100	12	9596
	1	0	240

Then 240×12×100=288000 Dividend, which ÷ by 9596 =30 Years, $4\frac{5}{7}+\frac{15}{98}$ Days, the Answer.

(43) First, from Aug. 7, 1786, to May 11, 1789=2 Years, 277 Days.

	£.	*s*.	*d*.
The first Bond was made for -	1111	10	0 at 6 per Cen.
Interest of which for 2 Yrs. 277 D. is	184	9	$9\frac{1}{7}$
Sum,	1298	19	$9\frac{2}{7}$ Amount.
May 11, Paid off, - -	140	0	0
2d Bond, - -	1158	19	$9\frac{2}{7}$
Due Sept. 19, - -	21	16	8
Sum,	1180	16	$5\frac{2}{7}$ Amount.
Sept. 5, Paid off, - -	87	11	9
3d Bond, - - - -	1093	4	$8\frac{2}{7}$ } Subt.
Sept. 11. Received in full, -	1409	16	8 }
Interest	316	12	$0\frac{2}{5}$

Then, from September 19, 1789, to September 19, 1799,=7 Years, 357 Days, or 2912 Days, 3d Bond or Principal, viz. £1093 4*s*. 8*d*.=262376 Pence, £.316 12*s*. 0*d*.= 303956 Farthings, and a Year=365 Days; then, by Case 6, or rather by Sect. 15; thus—

Prin.	*Time.*		*Interest.*
1049504	29·2	::	3056155648
24000	365	::	

Then, 262376×2912=764038912 the Divisor;
And 299136×365×24000=2620431360000 the Dividend.
Therefore 764038912) 2620431360000 gives $3429\frac{541010752}{764038912}$ Farthings, or *l*.3 11*s*. $5\frac{1}{4}$*d*. the Answer required.
The Time of the 2d Bond's Continuance is found thus—

P.	*Time.*	*Int.*					*qrs.*	*D.*	*d.*
100	365	5	5	0	Red.		24000	365	1260
1158 19 9	——	21	16	8			278,57	——	5240

Then $278157 \times 1260 = 350477820$ the Divisor.

$5240 \times 365 \times 24000 = 45902400000$ the Dividend.

350477820)45902400000(131 Days nearly since May 11, which Answers to Sept. 19, as before mentioned.

(44)

If he puts in $\left\{\begin{matrix}100\\200\\300\end{matrix}\right\}$ £. to have $\left\{\begin{matrix}40\\55\\70\end{matrix}\right\}$ £. per Year.

Then from $\left\{\begin{matrix}55\\70\end{matrix}\right\}$ £. take $\left\{\begin{matrix}40\\55\end{matrix}\right\}$ £. $= 15$ per C. for his Mon.

And $\left\{\begin{matrix}40\\55\\70\end{matrix}\right\} - \left\{\begin{matrix}15\\30\\45\end{matrix}\right\} = 25$ £. per Year for his Attendance.

(45)

	£.	*s.*	*d.*
First *l.*900 at 111⅛ per Cent, per Case 2. -	= 1002	7	6
Brokerage of Ditto, at 2*s.* 6*d.* per Cent. -	= 1	2	6
	1003	10	0
Midsummer Dividend, at 2 per Cent. -	18	0	0
	985	10	0
Interest of *l.*1003 10*s.* for 49 Days, at 5 per Cent.	6	14	8
Brokerage of £.400 at 2*s.* 6*d.* per Cent.	0	10	0
	£.992	14	8

	£.	*s.*	*d.*
	992	14	8
Sold 400*l.* at 92½ per Cent. - - -	370	0	0
	622	14	8
Interest for ½ Year, due Feb. 10, 1746,	15	11	4¼
	638	6	0¼
Dividend received at that Time, - -	10	0	0
Carry over	628	6	0¼

	£.	s.	d.
Brought over,	628	6	0¼
Interest due to August 10,	15	4	1¾
	644	0	2
Dividend received at that Time,	10	0	0
	634	0	2
Interest due to Feb. 1747,	15	17	0
	649	17	2
Dividend received then,	10	0	0
	639	17	2
Interest to the 10th of August,	15	19	11
	655	17	1
Midsummer Dividend, received August 10,	10	0	0
	645	17	1
Sold off 500£. at 102⅝ per Cent.	512	2	6
	133	14	7
Brokerage,	0	12	6
To my Damage in the whole,	£133	2	1

(19) COMPOUND INTEREST.

(1)

£.	£.	s.	d.	
5 1/20	600	0	0	=1st Year's Principal.
	30	0	0	=————Interest.
5 1/20	630	0	0	=second Year's Principal.
	31	10	0	=————Interest.
5 1/20	661	10	0	=third Year's Principal.
	33	1	6	=————Interest.
	694	11	6	=Amount.
	600	0	0	=Principal.

Answer £94 11 6

(2)

First, *l.*150 × 4 = *l.*6,00 ÷ 100=*l.*6 Interest, which added to 150=*l.*156 second Year's Principal; then *l.*156 × 4 = *l.*624÷100=*l.*6 4*s.* 9½*d.* Interest, + *l.*156=*l* 162 4*s.* 9 *d.* third Year's Principal, which × 4, = *l.*648 19*s.* 2*d.*÷1,00=*l.*6 9*s.* 9¼*d.* Interest, which added to *l.*162 4*s.* 9½*d.*=*l.*168 14*s.* 7*d.*=fourth Year's Principal, which × 4, = *l.*674 18*s.* 4*d.*÷100 =*l.*6 14*s.* 11*d.* fourth Year's Interest, which added to *l.*168 14*s.* 7*d.* = *l.*175 9*s.* 6¾*d.*=fifth Year's Principal, which × 4, =*l.*701 18*s.* 3*d.* this ÷ 100= *l.*7 0*s.* 4½*d.* fifth Year's Interest, which added to *l.*175 9*s.* 6¾*d.*=*l.*182 9*s.* 11¼*d.* the Amount required.

(3)

		£.	s.	d.	
5	1/20	440	16	0	=1st Year's Principal.
		22	0	9½	=————Interest.
5	1/20	462	16	9½	=second Year's Principal.
		23	2	10	=————— Interest.
5	1/20	485	19	7¼	=third Year's Principal.
		24	5	11¾	=————— Interest.
5	1/20	510	5	7¼	=fourth Year's Principal.
		25	10	3¼	=————— Interest
					[of 4 Years.
5£.	1/3	535	15	10½	=fifth Year's Principal, or amount
6*M.*	1/2	26	15	9½	=————Interest.
1	1/6	13	7	10¾	
15*D.*	1/2	2	4	7¼	
		1	2	3¾	
		16	14	10¼	=Interest for 7 Mo. 15 Days.
		535	15	10½	=4th Year's Amount.
		552	10	8¾	=Amount for 4 Y. 7 M. 15 D.
		440	16	0	=Principal.

Answer, £111 14 8¼ Compound Interest.

20. REBATE OR DISCOUNT.

	£.
6 *Mo.* $\frac{1}{2}$	5
16 *D.* $\frac{1}{5}$	2 10
	0 10
	3 0 0 Interest.
	100 0 0 Principal.
	£.103 0 0 Amount of 100*l.* for the given Time.

Then, as £. 103 : £. 3 :: £. 420 : £12 4 $7\frac{3}{4}$ $\frac{7}{103}$ the Rebate required.

(2) First, 100+6=106*l.* Amount of 100*l.* for a Year.
Then—If 106*l.* : 100*l.* :: 100*l.*
× 100

106)10000(*l.*94 6*s.* $9\frac{1}{2}d.\frac{2}{53}$ Answer.

(3)

Mo.		£.
6	$\frac{1}{2}$	4
		3
4	$\frac{1}{3}$	1 6 8
		Amount, 103 6 8

£. *s. d.* £. £. *s. d.*
then—If 103 6 8 : 100 :: 600 10 6

Or, 24800*d.* : 100 :: 144126*d.*

248,00)144126,00(*l.*581 3*s.* $\frac{3}{4}$ $\frac{3}{17}d.$
Inte. for 10 M. 3 6 8
Principal, 100 0 0

present Worth, which taken from the Principal, leaves *l.*19 7*s.* 5*d.* Answer,

(4) First, from December 12, to July 27,=227 Days.
Then—If 365 *D.* : 5*l.* :: 227 Days.
227

365,1135(*l.*3 2*s.* $2\frac{1}{4}d.$ Rebate of 100*l.* for 227 Days.

	£.	s.	d.	£.	s.	d.	£.	s.
Again—If	103	2	2¼	: 3	2	2¼	:: 890	16

Or, 98985 *qrs.* : 2985 *qrs.* :: 855168 *qrs.*

855168

98985)2552676480(25788 *qrs.* or 26*l.* 17*s.* 3*d.* the Answer.

(5) First, one half of 430*l.*=215*l.*

Mo.	£.			£.	s.	d.	£.	s.	d.	£.
4	⅓	5	If	101	13	4	: 1	13	4	:: 215

Interest,	1	13	4	24400*d.*	: 400*d.* :: 51600*d.*
Prin.	100	0	0		51600*d.*
Amt.	101	13	4	244,00)	206400,00(845¼*d.* or *l.*3 10*s.*

5¾*d.* Disc. of one half for 4 Months.

Again, *l.*1 13*s.* 4*d.* × *l.*2=3*l.* 6*s.* 8*d.* Interest for 100*l.* for 8 Months.

	£.	s.	d.	£.	s.	d.	£.
Then—If	103	6	8	: 3	6	8	:: 215

Or, 24800*d.* : 800*d.* :: 51600*d.*

51600

248,00)412800,00(1664½*d.* or 6*l.* 18*s.* 8½*d.* Discount of Half for 8 Months, which, added to that for 4 Months, gives 10*l.* 9*s.* 2¼*d.* Answer.

(6) First, From May 21, to Christmas,=218 Days; then—

If 365 *D.* : 5*l.* :: 218 Days.

218

365)1090 *l.* 19*s.* 8½*d.* Interest of 100*l.* for 218 Days.

	£.	s.	d.	£.	£.
Again,—If	102	19	8½	: 100	:: 550

Or, 98866*qrs.* : 100 :: 528000*qrs.*

528000

98866 52800000 534*l.* 1*s.* 1¼*d.* Answer.

Again, 360—120=240*l.* to continue 10 Months.

Mo.	£.			£.	*s.*	£.	£.
6	½	3	then—If	102	10	: 100 ::	240
		1	10	Or, 2050*s.*		: 100 ::	4800*s.*
4	⅓	1	0	205,0)480000			234*l.* 2*s.* 11*d.* present
Interest,		2	10	Worth of 240*l.* for 10 Months, which, added to that for 5 Months, will make			
Princip.		100	0	352*l.* 13*s.* 3¼*d.* the present Worth required.			
Amt.		102	10				

(10) 5*l.* $\frac{1}{20}$ 500*l.* Principal.

25 Interest of 500*l.* for one Year; then 25*l.*

× 12=300*l.* Interest for 12 Years.

Again, 5×12=60*l.* Interest of 100*l.* for 12 Years.

Then. As, 160*l.* : 60*l.* :: 500*l.* : 187*l.* 10*s.*

∴ 300*l.*—187*l.* 10*s.*=112*l.* 10*s.* Advantage to Interest.

(11)

Mo.	£.	*s.*	*d.*	then—If 112*l.* 6*s.* 8¼*d.* : 100*l.*
6	½	4	7	6=4 ⅜*l.* :: 13377*l.* 13*s.* 4*d.*
				2
				107841)12842560(11908*l.* 15*s.* 9¾*d.*
		8	15	0 $\frac{\text{o}\,\text{s}\,\text{o}\,\text{r}}{\text{r}\,\text{o}\,\text{7}\,\text{8}\,\text{4}\,\text{1}}$ the ready Money required.
3	½	2	3	9
16*D.*	6	1	1	10½
		0	3	7¾
10	⅓	0	2	5
Interest,		12	6	8¼
Prin.		100	0	0
Amt.		112	6	8¼

21. EQUATION OF PAYMENTS.

(1)

£.	*Mo.*	*Prod.*
200	× 3 =	600
150	× 4 =	600
250	× 6 =	1500
6,00)		27,00

(2)

£.	*Mo.*	*Prod.*
200	× 7 =	1400
260	× 5 =	1300
46,0)		270,0'5 *M.* $26\frac{4}{46}$ *D.*

Answer 5 *M.* $26\frac{2}{23}$ *d.*

Answer, 4 Months 15 Days.

(3) Here suppose 120*l.* to be the Sum owed.

then	of £.	£.	*Mo.*	*Prod.*
$\frac{1}{2}$	120 =	60	× 3 =	180
$\frac{1}{3}$		40	× 4 =	160
$\frac{1}{6}$		20	× 9 =	180
		12,0		)520 4 Mo. 10 D. Answer.

(4) Here, as the Debt is to be had at 4 equal Payments, and $\frac{1}{4}$ being paid down, there remains $\frac{3}{4}$ to be paid, at three equal Payments; consequently, the Sum of the different Times that each Payment is to be made, being divided by 3, will give the Answer, thus—

4+5+6=15, this ÷ 3 = 5 Months the Answer.

(5)

	£.	*Mo.*	*Prod.*
Owed,	240	× 5 =	1200.
Paid down,	40		
Remains,	20,0)		12,00(6 Months, the Answer.

22. SINGLE FELLOWSHIP.

(1) First, 80+60=140*l.* their Stock.—Then,

£.	£.	£.	£.
As 140	: 28	:: { 80	: 16 A.'s } Gain.
		{ 60	: 12 B.'s }

(2) First, 320+340=660*l.* G. and D. put in; then, from 824*l.* take 660*l.* remains 164*l.* E's Stock, and 824+70=894*l.* their whole Gain; therefore,

	£.	£.	s.	d.	Rem.	
As 824 : 894 ::	320 : 347	3	8¼	72	C.'s	Gain.
	340 : 368	17	8	128	D.'s	
	164 : 177	18	7½	624	E.'s	

(3) Here, suppose 600*l.* to be their Stock; then

	£.		£.	£.
½	300	A.'s	Stated thus,	300
⅓	200	B.'s	As 770 : 120 ::	200
¼	150	C.'s		150
⅕	120	D.'s		120

of 600*l.* = Stock.

770 Sum. Answer as below.

By Fractions, thus—½, ⅓, ¼, and ⅕, = $\frac{60}{120}$, $\frac{40}{120}$, $\frac{30}{120}$, and $\frac{24}{120}$.—Then 60+40+30+24=154.

	£.	£.				
∴—As 154 : 120 ::	60 : 46	15	0¾	9	A.'s	Loss.
	40 : 31	3	4¼	6	B.'s	
	30 : 23	7	6¼	43	C.'s	
	24 : 18	14	0¼	19	D.'s	

(4) First, 30 + 48 + 42 = 120*l.* their whole Gain.—Then

	£.	£.		
As 120 : 400 ::	30 : 100	D.'s	Stock.	
	48 : 160	E.'s		
	42 : 140	F.'s		

(5)

	£.	s.		
First,	1000	=20000	A.	Debt.
	640	=12800	B.	
	900	=18000	C.	
	842 16	=16856	D.	

Sum, 3382 16=67656, the whole Debt.
And 2420*l.* 17*s.* 6*d.*=581010 Pence, his whole Worth.

		Then, s.	d.	£.	s.	d.
As 67656 : 581010 ::	20000 : 171754			=715	12	10
	12800 : 109922½			=458	0	2½
	18000 : 154578¼			=644	1	6¾
	16856 : 144754¼			=603	2	10¼

Rem.		
45504	A.	Received.
45360	B.	
360	C.	
44088	D.	

(6) Here, suppose 420, as it will divide by 3, 4, 5, 6, and by 7, and have no Remainder, then $\frac{1}{3}$d of 420=140 A, $\frac{1}{4}$ =105B, $\frac{1}{5}$=84 C, $\frac{1}{6}$=70 D, and $\frac{1}{7}$=60 E, therefore, added together,=$\frac{459}{420}$, the neglecting the Denominator, the Statings will stand thus:

	£.	£.	£.	s.	d.		
	140	152	10		$1\frac{1}{4}$	105	A.'s
	105	114	7		$6\frac{3}{4}$	423	B.'s
As 459 : 500 ::	84	91	10		$0\frac{3}{4}$	63	C.'s } Share.
	70	76	5		$0\frac{1}{2}$	282	D's
	60	65	7		$2\frac{1}{2}$	45	E.'s

Or by Fractions, thus, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}=\frac{8\,2\,0}{1\,1\,5\,5}$, $\frac{6\,3\,0}{2\,3\,1\,0}$, $\frac{5\,2\,4}{2\,3\,1\,0}$, $\frac{4\,2\,0}{2\,3\,1\,0}$, and $\frac{3\,9\,0}{2\,3\,1\,0}$. Here neglecting the Denominator, and the Sum of the Numerators will be the first Term, and each Numerator the third; then proceed as above, will give the Answer.

(7) First, $\frac{3}{8}+\frac{1}{5}=\frac{2\,1}{5\,5}+\frac{2\,4}{5\,5}=\frac{4\,5}{5\,5}$ A. + B.'s=Part. Then, $\frac{5\,5}{5\,5}-\frac{1\,5}{5\,5}=\frac{1\,1}{5\,5}$ C.'s Part; then by neglecting the Denominator.

	£.	£.	£.	s.	d.		
As 11 : 140 ::	21	267	5		$5\frac{17}{17}$	A }	paid.
	24	305	9		$1\frac{4}{17}$	B. }	

(8 First, 10+7=17; then—

As 17*l.* : 52*l.* 10*s.* :: 10*l.* : 30*l.* 17*s.* $7\frac{3}{4}d.\ \frac{1}{17}$ A.'s Gain.

	£.	£.	£.	s.	d.
And—If	45	: 100 ::	30	17	$7\frac{3}{4}\ \frac{1}{17}$

Or, 734400 : 100 :: 504000 Seventeenths.

7344,00)504000,00 68*l.* 12*s* $6\frac{1}{2}d.\ \frac{2\,5\,0\,2}{7\,3\,4\,4}$ A.'s Adventure.

Again,—As 17*l.* : 52*l.* 10*s.* :: 7*l.* : 21*l.* 12*s.* 4*d.* $\frac{16}{17}$ B.'s Gain.

	£.	£.	£.	s.	d.
Also—If	45	: 100 ::	21	12	$4\frac{16}{17}$

Or, 734400 : 100 :: 352800

7344,00 352800,00(48*l.* 0*s.* 9*d.* $\frac{4\,5\,5\,2}{7\,3\,4\,4}$ B.'s Adventure.

(9) First, 50 Guineas=52*l.* 10*s.* B.'s Gain, which take from 74*l.* 11*s.* A.'s Gain, leaves 22*l.* 1*s.* Difference, 52*l.* 10*s.*—74*l.* 11*s.*=12*l.* 1*s.* the whole Gain.

	£.	s.	£.	s.	d.	£.	s.
Then—If	22	1	: 19	19	8 ::	127	1

Or, 441*s.* : 4796*d.* :: 2541*s.*
2541

441 12186636(27634*d.* or 115*l.* 2*s.* 10*d.* $\frac{42}{441}$ Cost, which added to the Gain, viz. 127*l.* 1*s.*=2421*l.* 3*s.* 10*d.* or 58126 Pence, which ÷ by 55*s.* viz. 660 Pence, will give 88 Anchors, and 3*s.* 10*d.* over; then 127*l.* 1*s.* ÷ 88 =1*l.* 8*s.* 10½*d.* gained per Anchor.

(10) First, from 14*s.* 6*d.* take 8*s.* 6*d.* remains 6*s.* A. gains more than B.; then,

	s.	d.					
As	6	35	::	14	6 : 4	4	7 A.'s
				8	6 : 2	9	7 B.'s

} Stock.

(11) First, 210+312=522*l.* Sum of A. and B.'s Stock, and 140—37*l.* 10*s.*=102*l.* 10*s.* their Gain, then,

	£.	£.	s.		£.	£.	s.	d.	
As	522	102	10	::	210 :	41	4	8¼	108 A.'s
					312 :	61	5	3¼	414 B.'s

} Gain.

As 102*l.* 10*s.* : 522 :: 37*l.* 10*s.* : 190*l.* 19*s.* 6*d.* $\frac{6}{41}$ C.'s Stock.

(12) First, 8+5=13*l.* their Gain per Cent.

	£.	£.		£.	s.	d.	
Then—As	13	154	::	8 :	94	15	4$\frac{6}{7}$ A's
				5 :	59	4	7$\frac{7}{13}$ B.'s

} Gain.

Ans. A. had for his Trouble, 35 10 9 $\frac{12}{13}$

AB, AC, BC, AB, AC, BC.

(13) First, $\frac{2}{7}$, $\frac{3}{5}$, and $\frac{4}{9}$, = $\frac{160}{560}$, $\frac{336}{560}$, $\frac{248}{560}$, by Case 5, Sect. 38: th rejecting the Denominator, we shall have,

First, 210—160=50 C.'s share more than B.'s.
And 210 168=42 A.'s
Also, 160—42=118 by A. and B. equally.
∴ 118 2= 59 B.'s Share
And 59 42 =101 A.'s
Also, 59+50=109 C's

} =269 their Sum.

		£.	s.	d.	Rem.	
As 269*l.* : 30*s.* ::	59 :	6	6¼	225	B.'s	
	101 :	11	3	180	A's	Share of the 30*s.*
	109 :	12	1¾	$\frac{1}{2}\frac{3}{6}$	C.'s	

Meal. Malt. Meal. Malt.

(14) Firſt,—As 3 : 5 :: 8 : $13\frac{1}{3}=\frac{40}{3}$
Then, $8+7+13\frac{1}{3}=28\frac{1}{3}=\frac{85}{3}$, their Sum.

Qrs. Buſh.

As $\frac{85}{3}$: $\begin{cases} \frac{40}{3} :: \frac{100}{1} : \frac{800}{17}=47\frac{1}{17}=376\frac{8}{17}=6400 & \text{Malt.} \\ \frac{8}{1} :: \frac{100}{1} : \frac{480}{17}=28\frac{4}{17}=225\frac{16}{17}=3840 & \text{Meal.} \\ \frac{7}{1} :: \frac{100}{1} : \frac{420}{17}=24\frac{12}{17}=197\frac{11}{17}=3300 & \text{Oatmeal.} \end{cases}$

$\frac{800}{17}=\frac{2}{3}=\frac{1200}{17}$, $\frac{480}{17}$, $\frac{420}{17}\times\frac{1}{2}=\frac{210}{17}$ } Price of the { Malt. Meal. Oatmeal.

Here the Denominator (17) may be omitted, and the Numerators divide by 30, the Quotient will ſtill retain the ſame Proportion.

3,0)12,00+48,0+21,0(=40+16+7=63 Sum.

	£.	s.	d.		
As 63 : 142 :: 40 : $\frac{5680}{63}=90$	3	$2\frac{2}{21}$	Malt.	} Coſt.
16 : $\frac{2272}{63}=36$	1	$3\frac{5}{21}$	Meal.	
7 : $\frac{994}{63}=15$	15	$6\frac{14}{21}$	Oatmeal.	

	£.	d.		
Then, $\frac{5680}{63}\div\frac{6400}{17}=\frac{9656}{1260}$£.	=4	$9\frac{10}{63}$	Malt.	} per Buſh.
And, $\frac{2272}{63}\div\frac{3840}{17}=\frac{38624}{24192}$	=3	$2\frac{20}{63}$	Meal.	
Alſo, $\frac{994}{63}\div\frac{3300}{17}=\frac{16898}{20790}$	=1	$7\frac{10}{63}$	Oatmeal.	

* (15) Firſt, $\frac{12}{11}$ of $\frac{3}{8}=\frac{9}{22}$, B's Part.

And, $\frac{4}{11}$, $\frac{9}{22}$, $\frac{1}{10}=\frac{528}{1760}$, $\frac{792}{1760}$, $\frac{286}{1760}$, or $\frac{264}{880}$, $\frac{361}{880}$, $\frac{143}{880}$,
Then, $\frac{351}{880}-\frac{264}{880}=\frac{87}{880}$=Difference betwixt A.'s and B.'s.
And, $\frac{143}{880}+\frac{87}{880}=\frac{230}{880}$ C.'s Part.
Alſo, $\frac{264}{880}+\frac{351}{880}+\frac{230}{880}=\frac{845}{880}$=A.'s+B.'s+C.'s.
$\frac{880}{880}-\frac{845}{880}=\frac{13}{880}$ D.'s Part.

	£.	£.	s.	d.		
As 858 :: 400 :: 264 :	223	1.	$6\frac{1}{4}$	726	A.'s	} Share.
351 :	163	12	$8\frac{1}{2}$	780	B.'s	
230 :	107	4	$6\frac{1}{4}$	54	C.'s	
31 :	6	1	$2\frac{1}{2}$	156	D.'s	

23. DOUBLE FELLOWSHIP; or FELLOWSHIP with TIME.

	£.	£.	
(1) Firſt,	240×4=	960	A.'s } Stock and Time.
And,	120×6=	720	B.'s
Alſo,	200×8=	1600	C.'s

3280 Sum; then

* See Queſtion the 9th for Exerciſe in Fractions.

	£.	£.	s.	d.	
As 3280 : 260 ::	960 :	76	1	11¼	216 A.'s
	720 :	57	1	5¼	80 B.'s } Gain.
	1600 :	126	16	7	32 C.'s

(2) First, 4×4×50= 800 Officer's Pay and Time.
And, 8×4×40=1280 Midshipman's Ditto.
Also, 120×3×28=10080 Sailor's Ditto

12160 Sum—Then

	£.	s.	d.	
As 12160 : 4000 ::	800 :	263	3	1¾ 704 Officers.
	1280 :	421	1	0½ 640 Midship.
	10080 :	3315	15	9¼ 1088 Sailors.

£.	s.	d.		£.	s.	d.	
263	3	1¾ ÷	4 =	65	15	9¼	Officers. } each, Fractions rejected.
421	1	0½ ÷	8 =	52	12	7½	Midship.
3315	15	9¼ ÷ 120 =		27	12	7¼	Sailors.

(3) 60×4=240 A's Stock and Time.
40×5=200 B.'s ditto.
30×3= 90 C.'s ditto.

530 Sum of their Stocks and Times.

	£.	s.	d.		Rem.
As 530 : 40 ::	240 :	18	2	3	36 A.'s
	200 :	15	1	10½	30 B.'s } Part.
	90 :	6	15	10	40 C.'s

(4) £. Mo.
400×6=2400
200×6=1200 } =3600 A.'s Stock and Time.
360×7=2520
460×2= 920
340×3=1020 } =4460 B.'s ditto.
£. Mo
190×8=1520
300×2= 600
200×2= 400 } =2520 C.'s ditto.
then 3600+4460+2520=10580 their Sum.

			£.	s.	d.	Rem.	
As 10580 : 460 ::	£.	£.	3600 : 156	10	5	920 A.'s	Share.
			4460 : 193	18	3	552 B.'s	
			2520 : 109	11	$3\frac{1}{2}$	$\frac{644}{1015}$ C.'s	

(5) Reciprocally, As 19 : 84 12 6 :: 7 : 229 13 11 $\frac{5}{7}$

$6\times3+1=19$

7)1607 17 6

Ans. B.'s Adventure, £.229 13 11 $\frac{5}{7}$

(6) First, 20+26*l.* 5*s.*+32=78*l.* 5*s.* Sum of their Gain.

		£.	s.	£.	s.	d.	Rem.	
£. s.	£.	20	0 : 163	11	$6\frac{3}{4}$	225 A.'s	Stock	
As 78 5 :	640 ::	26	5 : 214	13	11	980 B.'s	and	
		32	0 : 261	14	6	$\frac{360}{1765}$ C.'s	Time.	

Therefore

£.	s.	d.	£.	s.	d.		
163	11	$6\frac{3}{4}\div9$	=18	3	6	A.'s	Stock, Fract. rejected.
214	13	11 ÷7	=30	13	5	B.'s	
201	14	6 ÷5	=52	6	$10\frac{3}{4}$	C.'s	

(7) First, 72*l.* 10*s.*=1450*s.* C.'s Stock; then by Sect. 15.

S.	*T.*	*G.*	
1450 :	9 :	23	Here 23×6=138 Divisor.
—— :	6 :	13	And 1450×9×13=169650 Dividend.

£. s. d.

138)169650(1229*s.* $4d.\frac{4}{23}$=61 9 4 A.'s Stock.

Again,—

S.	*T.*	*G.*	
1450 :	9 :	23	Here 23×5=115 Divisor.
—— :	5 :	18	And, 145×9×18=234900 Dividend.

£. s. d.

115)234900(2042*s.* $7\frac{1}{4}d.\frac{3}{115}$ 102 2 $7\frac{1}{4}$ B.'s Stock.

72 10 0 C.'s ——

61 9 4 A.'s ——

Answer, the whole Stock, £.236 1 $11\frac{1}{4}$

(8) Here suppose X's Gain to be 2
Then by the Question Y.'s will be 3 } =9 their Sum.
And Z.'s — — 4

	£.	£.		£.	s.	d.	*Pence.*	
Now as	9	420	2	93	6	8	=22400	X.'s
			3	140	0	0	=33600	Y.'s } Gain.
			4	186	13	4	=44800	Z.'s

Therefore, 22400×4= 89600 X.'s
33600×6=201600 Y.'s } Gain and Time.
44800×9=403200 Z.'s

694400

Now rejecting the 0's we shall have the following Proportion.

	£.	£.		£.	s.	d.		
As	6944	4262	896	549	18	8½	448	X.'s
			2016	1237	7	1	4480	Y.'s } Stock.
			4032	2474	14	2¼	2016	Z.'s

(9) First, $\frac{1}{5}+\frac{2}{3}=\frac{3}{15}+\frac{10}{15}=\frac{13}{15}$, then $\frac{15}{15}-\frac{13}{15}=\frac{2}{15}$ B.'s Gain; and rejecting the Denominator, we shall have A.'s Gain =3, B.'s 2, and C.'s 10.; then

S. T. G.
400 : 7 : 10 Here, 10×5= 50 Divisor.
—— : 5 : 3 And, 400×7×3=8400 Dividend.
5,0)840,0(168*l.* A's Stock.

Again,—

S. T. G.
400 : 7 : 10 Here, 10×8=80 Divisor.
—— : 8 : 2 And, 7×2×400=5600 Dividend.
8,0)560,0(70*l.* B's Stock.

(10)

			Days.	*Days.*	
From Feb.	10, to June	10, —	= 122		
Jan.	17, to April	30, —	= 61	=208	A.'s Time.
July	14, to 14 after	St. James's	= 125		
Aug.	2, to Nov.	13, —	= 104		
May	1, to June	24, —	= 55	=179	B's. Time.
Sept.	30, to Oct.	19, —	= 20		

387 Horse.
—— in Use.

	D.	£.	s.	d.	*Rem.*	
As	387	7	10	208	4	0	7¼	280/387 A.'s } Share.
				179	3	9	4½	90/387 B.'s

24. BARTER.

(1) First, 4 *Cwt.* = 112 × 4 = 448 *lb.* at 12*s.*
Then per Case 4, Sect. 16. 448 × 6 = 268*l.* 16*s.*—
Value of the Tea.——Now,
As 1*l.* 10*s.* ; 1 *Cwt.* :: 268*l.* 16*s.* Or.
As 30*s.* : 1 :: 5376*s.* : 179 *Cwt.* 22$\frac{4}{7}$*lb.* of Sugar, the Quantity required.

(2) First, 45 Yds. at 1*s.* 4*d.* = 3*l.* or 60*s.*
Then, 60 ÷ 18 = 3$\frac{1}{3}$ Yds.

(3) First, 30 *Cwt.* = 3360*lb.* at 7$\frac{1}{2}$*d.* per *lb.* Now.

d.			
6	1	336,0	then—As 6 : 7$\frac{1}{2}$ :: 36 : 45
1$\frac{1}{2}$	$\frac{1}{4}$		

84 Again,—As 45*s.* : 1 *Cwt.* :: 2100*s.*
21 46 *Cwt.* 2 *qrs.* 18$\frac{2}{3}$*lb.* the Answer.

105 = 2100*s.*

s.	*d.*	*s.*	*s.*	*d.*	
(4) If 8	6	: 10	:: 1	6	Or,

As 17 *six-d.* : 10*s.* :: 3 *six-d.* : 1*s.* 9$\frac{3}{17}$*d.* per *lb.* the Answer.

d.	*d.*	*l.*	
(5) Then—If 6	: 4	:: 20	Or,

As 1 *six-d.* : 4*d.* : 800 *six-d.* : 3200*d.* = 13*l.* 6*s.* 8*d.* real Value of A.'s Currants.
Now 20*l.* ÷ 2 = 10*l.* or 200*s.* and 6*s.* 8*d.* = 80*d.* Value of B.'s Candles.
∴ As 7*s.* : 80 :: 200*s.* : 2285$\frac{1}{2}$*d.* $\frac{6}{7}$ =

l.	*s.*	*d.*	
9	10	5$\frac{1}{2}$	$\frac{6}{7}$ real Value of B's Candles.
10	0	0	gave in ready Money.

A received	19	10	5$\frac{1}{2}$	$\frac{6}{7}$
For	13	6	8	

Answer, £6 3 9$\frac{1}{2}$ $\frac{6}{7}$ A. got of B.

(6) *Cwt.* £. *s.* £. *s.*——£. *s.*
First, 18 at 1 11 per *C.*=1 11×3×6=27 18 real Value.
And do. at 2 2——=3 2×3×6=37 16 adv. Value.

∴ A advanced his Sugar, 9 18

Also, 3)37*l.* 16*s.*(=12*l.* 12*s.* A received in Cash, and 37*l.* 16*s.*—12*l.* 12*s.*=25*l.* 4*s.* in Paper.
Again, 15*s.* 6*d.*—14*s.*=1*s.* 6*d.* B. advanced his Paper.
Therefore,

s.	*d.*	*d.*	£.	*s.*
If 15	6 :	18 ::	25	4
×2			×40	Sixpences in a £.
31			1008	
			× 18	

31)18144(585*d.* or 2*l.* 8*s.* $9\frac{1}{4}d.\frac{5}{31}$ B.'s advance on his Paper.
Then from 9*l.* 18*s.* take 2*l.* 8*s.* $9\frac{1}{4}d.\frac{5}{31}$. Rem. 7*l.* 9*s.* $2d.\frac{26}{31}$ in A.'s Favour the Answer.

£. *s.* £. *s.* £. *s.*
(7) First 5 6÷4 5=1 1 Gain per Piece.
And 5 6÷2 =2 13 required down.

Also, 4 5—2 13=1 12 Value of the Half remaining.
+1 1

Made £.2 13 of the Half remaining.

£. *s.* £. *s.* *d.*
Then,—If 1 12 : 2 13 :: 3 Or,
As 128 *three-d.* : 53*s.* :: 1 *three-d.* : $4\frac{3}{4}d.$ $\frac{7}{8}$ per *lb.* the Answer.

(8) First, $\left.\begin{matrix}13 & 0\\ 11 & 10\end{matrix}\right\}$ ×50= $\left\{\begin{matrix}650 \text{ advanced Val. of the Cloth.}\\ 575 \text{ real Value.}\end{matrix}\right.$

£.75

s. d. *s. d.*
Again, 2 6×2=5 0 adv. Value of the Wool per Tod.
—4 2 real Value.

0 10 Gain per Tod. or 5*d.* per Stone.

s. d. *St.* *£.*
If 2 6 : 1 :: 650 Or,

six-d. *St.* *six-d.* *St.*
As 5 : 1 :: 26000 : 5200 at 5*d.* per Stone. Now,

4*d.*	$\frac{1}{20}$	5200 Stone.
1	$\frac{1}{4}$	86 13 4
		21 13 4

*£.*108 6 8 gained by the Wool.

A Sack=26 Stone; therefore, 5200÷26=200 Sacks, which will pay for the Cloth.
∴ 108*l.* 6*s.* 8*d.*—75*l.*=33*l.* 6*s.* 8*d.* B.'s Gain by this Affair.

d. *d.* *s.* *d.*
(9) If 10 : 16 :: 20 or 240
240

10 3840

12 384

32*s.* advanced Value of the Malt.
20 real Value.

12*s.* B.'s Gain per Quarter.

Here 30 Guineas×21=630 Shillings, which÷by 12=52½ Quarters, or 420 Bushels, the Answer.

10) 720 Ells, at 5*s.* per Ell, or $\frac{1}{4}$=180*l.* Real } Val. of the
Ditto ——— 6*s.* 8*d.* or $\frac{1}{3}$=240 Adv. } Holland.
240*l.* at 10*l.* per Ct. or $\frac{1}{10}$=24 Discount.
Then 240—24=216*l.*
And 216÷ 2=108 paid in ready Money.

Then,—If £. £. s.
216 : 180 :: 36
20 180

4320)6480(1*l.* 10*s.* real Value per *lb.*

Now 108*l.*=2160*s.* which÷30=72 *lb.* the Quantity delivered.

s. d. s. d.
(11) If 8 or 96 : 10 :: 6 : 7½ Bar. Price of the Pamphlets.
100 Reams at 8*s.*=40*l.* real } Value of the
Ditto, at 10*s.*=50*l.* advanced } Paper.
¼=50*l.*÷4=12*l.* 10*s.* B. to have in Cash,
40*l.* Value of B.'s Pamphlets.
×40 Sixpences in a £.

1600 Pamphlets to be delivered.

From 40*l.* take 12*l.* 10*s.* Rem. 27*l.* 10*s.* what they then stood him in, so that the Advantage to B. is 27*l.* 10*s.*

(12) First, 140 *lb.* 11 *oz.*=1691 *oz.*

5*s.*	⅓	1691 at 6*s.* 4*d.*				s.	d.				
1	⅕	422	15			6	8	⅓	1691 at 7*s.* 2*d.*		
4*d.*	⅓	84	11								
		28	3	8		6	1/40		563	13	4
									42	5	6
		£.535	9	8	real Value of the Plate.				£.605	18	10

£. s. d. £. s. d.
From 605 18 10 take 535 9 8 Remains 70*l.* 9*s.* 2*d.* whole Advantage of A.'s Plate.

⅐ £.605 18 10—
86 11 3⅐ received in ready Specie.

£.519 7 6⅙

7 *C.* 2 *qrs.* 18 *lb.*=858 *lb.*=at 11*s.* 2*d.*

10*s.*	½	858 at 11*s.* 2*d.*		5*s.*	¼	858 at 9*s.* 6*d.*	
1	1/10	429				214	10
2*d.*	⅛	42	18	4	1/20	171	12
		7	3	6*d.*	1/80	21	9
		479	1 adv. Val. of B.'s Tea.			407	11 real Value of

£.	*s.*	*d.*		*£.*	*s.*	*d.*
519	7	$6\frac{6}{7}$		519	7	$6\frac{6}{7}$
—479	1	0		407	11	9
*£.*40	6	$6\frac{6}{7}$	Disc. allowed by A.	111	16	$6\frac{6}{7}$

A.'s Advantage by the Rise of his Plate, - 70 9 2

B.'s whole Advantage, - - - *£.*41 7 $6\frac{6}{7}$

(13) 14 *C.* 2 *qrs.* 25 *lb.* at 3*l.* 3*s.* per *Cwt.*=46*l.* 7*s.* $6\frac{1}{4}d.$ Advance Value of A.'s Hops.

s. *s.* *s.*
If 59 : 63 :: 6
6

59)378(6*s.* $4\frac{3}{4}d.\frac{1}{59}$ advanced Price of B.'s Wine per Gallon. Now $1\frac{1}{2}$ *hhd.*=$94\frac{1}{2}$ *gal.* at 6*s.* $4\frac{3}{4}d.\frac{1}{59}$
$10\times9+4\frac{1}{2}=94\frac{1}{2}$

B.'s Wine comes to	30	5	$5\frac{1}{4}d.$	$\frac{9}{59}$
A.'s Hops	46	7	$6\frac{3}{4}$	

Answer, *£.*16 2 $1\frac{3}{4}d.\frac{52}{59}$

25. LOSS AND GAIN.

(1) 6*d.* | $\frac{1}{40}$ | 240 at 14*s.* 6*d.* | 240 at 18*s.*
7 | 9
168 | 216,0 Sold for:
6 | 174 Cost.
Cost *£.*174 Ans. | *£.*42 Gained.

(2) First, 9*C.* 2*qrs.* 18*lb.*=1082*lb.* and 46*l.*+12*l.* 12*s.*=
58*l.* 12*s.* or 1172*s.* sold for; therefore,
1082)1172(1*s.* $0\frac{3}{4}$*d.* $\frac{5}{7}\frac{3}{4}\frac{7}{7}$ per *lb.* the Answer.

(3) First, 10*s.* 6*d.*—8*s.* 6*d.*=2*s.* Gain by 8*s.* 6*d.*—then
s. *d.* *s.* *£.*
If 8 6 : 2 :: 100 or,
As 17 *six-d.* : 2*s.* :: 4000 *six-d* : 470*s.* $7d.\frac{5}{17}$
Or, 23*l.* 10*s.* $7d.\frac{5}{17}$ the Answer.

(4) First, 100*l.*+8=108*l.* Amount; then
s. *£.* *s.* *d.*
If 5 : 108 :: 6 3 then 135—100=35*l.* Answer.
20 25 25 Threepences.
Mr. WEBSTER's Answer is 10*l.*
2,0)270,0 135*l.*

(5) First, 100+7*l.* 10*s.*=107*l.* 10*s.* Amount.
s. *£.* *s.* *s.* *d.*
If 5 : 107 10 :: 5 9 Or,
As 20 *three-d.* : 2150*s.* :: 23 *three-d* : 2472*s.*=123*l.*
12*s* 6*d.* Amount per Cent.
Then 123*l.* 12*s.* 6*d.*—100=23*l.* 12*s.* 6*d.* the Answer.
Mr. STONEHOUSE's Answer is 8*l.* 12*s.* 6*d.*

(6) First, 100+15=115*l.* Amount; then,
If 11*s.* 6*d.* : 115 :: 12*s.* Or,
As 23 *six-d.* : 115*l.* :: 24 *six-d.* : 120*l.* the Amount
per Cent.
Therefore 120—100=20*l.* per Cent. Answer.
Mr. HILL makes the Answer 15*l.* 13*s.* $0\frac{1}{2}$*d.* $\frac{2}{23}$.

(7) 3*d.* $\frac{1}{4}$ | 500 at 1*s.* 3*d.* Now 100—9=91*l.* or 1820*s.* then
152
£. *s.* *s.* *£.* *s.*
If 1820 : 100 :: 625
625*s.*=31 5
182,0)6250,0(34*l.* 6*s.* $9\frac{3}{4}$*d.* $\frac{3}{5}\frac{3}{7}$ Amount.
Therefore, 34*l.* 9*s.* $6\frac{3}{4}$*d.* $\frac{3}{5}\frac{3}{7}$—21*l.* 5*s.*=3*l.* 1*s.* $9\frac{3}{4}$*d.* $\frac{3}{5}\frac{3}{7}$ the
Answer.
Mr DILWORTH's Answer is 2*l.* 16*s.* 3*d.*

(8) First, 100+25=125*l.* Amount; then,
As 6*l.* 15*s.* : 125 :: 8*l.* : 148*l.* 2*s.* 11½*d.* $\frac{2}{9}$, Amount per Cent.
Then 148*l.* 2*s.* 11½*d.* $\frac{2}{9}$—100=48*l.* 2*s.* 11$\frac{3}{7}$*d.* $\frac{5}{7}$ the Answer.
Mr. WALKINGHAM's Answer is 29*l.* 12*s.* 7*d.*$\frac{1}{7}$

(9) First, 108+30=130*l.* Amount; then,
As 3*l.* 10*s.* : 130*l.* :: 4*l.* 5*s.* : 157*l.* 17*s.* 1½*d.* $\frac{6}{7}$ Amount per Cent.
Then 157*l.* 17*s.* 1½*d.* $\frac{6}{7}$—100=57*l.* 17*s.* 1½*d.* $\frac{6}{7}$ Answer.

(10) First, 100—17=83*l.* and 100+20=120*l.* then,

£.	£.	£.	*s.*	£.	*s.*
If 83	: 100	:: 52	10	: 63	5$\frac{5}{83}$

£	£.	£.	*s.*	£.	*s.*	*d.*
Again, As 100	: 120	:: 83	5$\frac{5}{83}$	: 75	18,	0$\frac{3}{4}$ $\frac{49}{83}$ Worth.

Sold for 52 10 0

Answer, £.23 8 0$\frac{3}{4}$ $\frac{69}{83}$

(11) 112*lb.* at 2*s.* 11*d.* $\frac{2}{3}$=16*l.* 12*s.* 10*d.* $\frac{2}{3}$ sold for at Turkey.
Then 16*l.* 12*s.* 10*d.*$\frac{2}{3}$ ÷ 2=8*l.* 6*s.* 5*d.* ½d cost him.
Therefore, from 8*l.* 6*s.* 6*d.* $\frac{1}{3}$, take 8*l.* Rem. 6*s.* 5*d.* $\frac{1}{3}$ Loss, the Answer.

(12) First, 4*s.* 3*d.*+2*d.*=4*s.* 5*d.* prime Cost and Charges.

	s.	*d.*	*s.*	£.	*s.*	*d.*
Then—As	4	5	: 6	:: 100	: 2716	11$\frac{3}{4}$ $\frac{5}{7}$

Again, 100—12=88*l.*

£.	£	*s.*	*d.*
Also—If 100	88	:: 2716	11$\frac{3}{4}$ $\frac{5}{7}$

£. *s.* *d.*
Or, as 5088000 : 88 :: 6912000 : 119 10 11$\frac{1}{4}$ $\frac{15}{77}$ Amt.
Then, 119*l.* 10*s.* 11$\frac{1}{4}$*d.*$\frac{15}{77}$—100=19*l.* 10*s.* 11$\frac{1}{4}$*d.*$\frac{15}{77}$, gained per *lb.* the Answer.

(13) First 800×14=11200*lb.* at 12½*d.* per *lb.*
Then 11200×12½=140000*d.* Value of the Anchovies: which ÷ by 749,0, gives 18,*d.* $\frac{574}{749}$ Amount of 12½*d.*

Again, 100+17=117*l.* Amount; then,

£. *£.* *d.*

If 117 : 100 :: 18½ $\frac{5}{7}\frac{7}{4}$

Or, as 84127680 : 100 :: 5542900 : 1*s.* 3¾*d.* $\frac{20}{54}\frac{8}{1}\frac{5}{8}\frac{25}{7}\frac{}{68}$, the Answer.

(14) First, 41*l.* 3*s.* 4*d.*=9880*d.* and 3*s.* 1*d.*=37*d.*

Then, 9880÷37=267 $\frac{1}{37}$*lb.* at 3*s.* 1*d.* per *lb.* bought.

Again, 34*l.* 2*s.* 6*d.*=8190*d.* and 4*s.* 6*d.*=54*d.*

Then, 8190÷54=151½*lb.* sold at 4*s.* 6*d.* per *lb.*

Therefore, 267 $\frac{1}{37}$—151½=267 $\frac{3}{37}$—151 $\frac{40}{37}$=115 $\frac{40}{1369}$*lb.*

Spoiled at 3*s.* 1*d.*=37*d.* so that 115 $\frac{40}{1369}$×37=4268 $\frac{37}{1369}$*d.* or 17*l.* 15*s.* 8¼*d.* $\frac{37}{1369}$ prime Cost of the Goods spoiled.

(15) First, $\frac{3}{8}$ of 11*s.*=3 $\frac{3}{8}$, or, 4*s.* 1½*d.* gained per Thousand, when he sold them at 11*s.* ∴ 11*s.*—4*s.* 1½*d.*=6*s.* 10½*d.* per Thousand prime Cost; then,

As 6*s.* 10½*d.* : 11*s.* :: 100*l.* : 160*l.* Amount of 100*l.*

Again, As 11*s.* : 160*l.* :: 13*s.* 6*d.* : 196*l.* 7*s.* 3¼*d.* $\frac{1}{17}$ Amount per Cent. at 13*s.* 6*d.*

Then, 196*l.* 7*s.* 3¼*d.* $\frac{1}{17}$—100=96*l.* 7*s.* 3¼*d.* $\frac{1}{17}$ gained per Cent. the Answer.

£. *s.* *d.*

(16) 5 | $\frac{1}{20}$ | 500 16 8 at 6¼*l.* per Cent.

1	$\frac{1}{5}$	25 0 10	} C.'s Profit.
½	¼	5 0 2	
		2 10 1	

£. *s.* *d.*

Then 500 16 8—32 11 1=468 5 7 Wine Cost C.

Also, 468 5 7—38 11 6=429 14 1 —— cost B.

Again, *£.*20 | $\frac{1}{3}$ | 429 14 1 at 4$\frac{1}{3}$ per Cent.

4 | $\frac{1}{5}$ | 85 18 9¾ at 20 per Cent.

$\frac{1}{3}$ | $\frac{1}{12}$ | 17 3 9 } A.'s Profit.

1 8 7¾

Then 429 14 1—18 12 4¾=411 1 8¼ cost A.

Also, 41*l.* 1*s.* 8*d.*=8221*s.* 8¾*d.* and 15 Pipes × 126=1890 Gallons, then 1890)8221*s.* 8½*d.* (4*s.* 4*d.* $\frac{391}{1890}$ per Gal. *the Answer.*

(17) First, $\frac{3}{7}$ of 480*l.* 12*s.*=137*l.* 6*s.* 3*d.* $\frac{3}{7}$, Cost of the damaged Goods; then 137*l.* 6*s.* 3*d.* $\frac{3}{7}$—48*l.* 18*s.*=88*l.* 8*s.* 3*d.* made of the damaged Goods.

∴ As 5 6 : 1 :: 83 8 $3\frac{3}{4}$: $321\frac{1}{2}\frac{3}{6}\frac{4}{4}$ Yards, damaged $321\frac{1}{2}\frac{3}{6}\frac{4}{4}\times 7=2250\frac{2}{2}\frac{4}{6}\frac{6}{4}$ Yards, which ÷ 2 = $1125\frac{1}{2}\frac{2}{6}\frac{3}{4}$ Yards bought in all.

Then $1125\frac{1}{2}\frac{3}{6}\frac{4}{4}$—$321\frac{1}{2}\frac{3}{6}\frac{4}{4}$=$803\frac{2}{2}\frac{5}{6}\frac{3}{4}$ Yards undamaged.

Again,—from 480*l.* 12*s.* take 88*l.* 8*s.* 3*d.* $\frac{3}{7}$ Rem. 392*l.* 3*s.* 8*d.* $\frac{4}{7}$ to be made of the undamaged Goods.

∴ If $803\frac{1}{2}\frac{3}{6}\frac{4}{4}$: 392 3 $8\frac{4}{7}$ :: 1

Or, as $37\frac{1}{2}\frac{3}{6}\frac{4}{4}\frac{0}{0}$: 6 5 8 8 7 $\frac{3}{7}$ :: $\frac{5}{4}$: $1\frac{1}{1}\frac{3}{0}\frac{2}{3}\frac{1}{0}\frac{0}{6}\frac{9}{7}\frac{4}{0}\frac{3}{8}\frac{2}{8}\frac{0}{0}$ =12*s.* $2\frac{1}{4}$*d.* $\frac{1}{2}\frac{4}{1}\frac{7}{9}\frac{7}{9}\frac{7}{1}\frac{7}{7}\frac{5}{7}$ per Ell.

26. ALLIGATION MEDIAL.

(1)

Gal. at *s.*	*s.*
14 × 8	= 112
12 × 6	= 72
10 × 7	= 70
20 × 4	= 80
8 × 9	= 72
As 64 :	406 :: 1 *Gal.* : 6*s.* $4\frac{1}{8}$*d.* the Answer.

(2)

Gal. at *d.*		*£.*	*£.*	*d.*	*£.*	*s.*	*d.*	*d.*
13 × 80 = 1040	If	100	110	2600	11	18	4 or	2860
20 × 60 = 1200	then	2860—2600=260. Also 2600						
10 × 36 = 360	+260=2860.							
— *qrs.*	∴ As 172 : 2860 :: 1 : $16\frac{1}{4}$*d.* $\frac{2}{4}\frac{2}{5}$,							
Sum 43=172,2600	the Answer.							

(27) ALLIGATION ALTERNATE.

(3).

20 *Car.* { 18, 23, 19, 16 }

2+1+4=7 { 3, 3, 3 } of { 18, 23, 19, 16 } Carrats fine Answer.

(4) *Car.* *lb.* *d.*

$$6d.\left\{\begin{matrix}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{matrix}\right\}\left\{\begin{matrix}-&-&1\\-&-&1\frac{1}{2}\\-&-&4\\-&-&1\frac{1}{2}\end{matrix}\right\}\text{at}\left\{\begin{matrix}10\\7\frac{1}{2}\\5\\4\frac{1}{2}\end{matrix}\right\}\text{per } lb.\ \text{Answers.}$$

As Examples of this Nature will admit of as many different Answers as there are different Ways of linking together a larger Price and a less than the middle or mean Rate proposed, so consequently the last will admit of seven different Ways or Answers.

28. ALLIGATION PARTIAL.

(5) *d.* *Diff.* *Diff.* *lb.* *s.* *d.*

$$96\left\{\begin{matrix}144\\108\\90\\78\end{matrix}\right\}\left\{\begin{matrix}6\\18\\48\\12\end{matrix}\right\}\begin{matrix}\text{Diff. lb.}\\\text{If } 6:36::\end{matrix}\left\{\begin{matrix}18:108\\48:288\\12:72\end{matrix}\right\}\text{of}\left\{\begin{matrix}9&0\\7&6\\6&6\end{matrix}\right\}\begin{matrix}\text{per}\\lb.\\\text{An.}\end{matrix}$$

(6) *d.* *Diff.* *Diff. lb.* *Diff. lb.*

$$24\left\{\begin{matrix}30\\20\\18\\15\end{matrix}\right\}\left\{\begin{matrix}\\\\\\\end{matrix}\right\}\begin{matrix}4+6+9=19\\6\\6\\6\end{matrix}$$

If $19:120::6:37\frac{17}{19}$, so that with 120*lb.* of the given Quantity, there must be $37\frac{17}{19}$*lb.* of each sort of the other, the Answer. In all $233\frac{17}{19}$*lb.*

29. ALLIGATION TOTAL.

(7) *Diff. cz. Car. fine.*

$$20\left\{\begin{matrix}16\\18\\23\end{matrix}\right\}\left\{\begin{matrix}3\\3\\\end{matrix}\right\}\begin{matrix}3\\4+2=6\end{matrix}$$

Sum. oz.

If $12:60::\left\{\begin{matrix}3:15\\3:15\\6:30\end{matrix}\right\}$ of $\left\{\begin{matrix}16\\18\\23\end{matrix}\right\}$ Answ.

Sum 12 the whole Quan. 60 *oz.*

(8) *s.* *Diff.* *Diff. lb.* *s.*

$7s.\left\{\begin{matrix}5\\6\\8\\9\end{matrix}\right\}\left\{\begin{matrix}1\\2\\2\\1\end{matrix}\right.$ *Sum. lb.* As 6 : 168 :: $\left\{\begin{matrix}1:28\\2:56\\2:56\\1:28\end{matrix}\right\}$ at $\left\{\begin{matrix}5\\6\\8\\9\end{matrix}\right\}$ per *lb.* the Answer.

In all 168

30. EXCHANGE.

1. WITH FRANCE.

(1) *Cr. d.* *Crs.* *d.*

As 1 : 56 :: 500 : 28000 = 116*l.* 13*s.* 4*d.* the Answer.

Or by Practice, thus 4*s.* $\frac{1}{5}$ 500 at 4*s.* 8*s.*

8*d.* $\frac{1}{6}$ 100

16 13 4

Answer, £.116 13 4

d. *Cr.*

(2) As 56 : 1 :: 116*l.* 13*s.* 4*d.* = 28000

$56\left\{\begin{matrix}7)28000\\8)\ 4000\end{matrix}\right.$

Answer, 500 Crowns.

Cro. *d.* *Cro.* *sol.* *din.*

(3) If 1 : 54½ :: 640 12 8.

or, as 720 : 109 :: 460952 : 69783$\frac{7}{9}$*d.* or 145*l.* 7*s.* 7¼*d.* the Answer.

Or thus, by Practice. [Crown.

4*s.* | $\frac{1}{5}$ | 640 at 4*s.* 6½*d.* and 12 Sols, 8 Din. at 4*s.* 6½*d.* per

s. *d.*

6*d.* | $\frac{1}{8}$ | 128 — 10 *fo.* | $\frac{1}{5}$ | 4 6½

½ | $\frac{1}{12}$ | 16 — 2 — 1 — 0 9 — 2 Rem.

1 6 8 — 8 *din.* | $\frac{1}{3}$ | 0 1¾ — 1

0 0 11½ — | $\frac{1}{8}$ | 0 0½ — 1

Ans. £.145 7 7¼ $\frac{13}{15}$ as before. — 11¼ $\frac{13}{15}$

d. Cro. £. s. d.

(4) If $54\frac{1}{4}$: 1 :: 145 7 $7\frac{1}{2}$ $\frac{1}{10}$

Or, as 9810 : 1 :: 6280471 : 640 *Cro.* 12 *sol.* 8 *din.* the Answer.

2. With SPAIN.

d. Piece. £. s. d.

(5) If 56 : 1 :: 856 6 8

Or, as 56 : 1 :: 205520 : 3670 Pieces, Answer.

P. d. Piece. re. mar.

(6) If 1 : $54\frac{1}{4}$ :: 1426 4 26

Or, as 272 : 217 :: 84203378 : 309571 $\frac{33}{136}$ *qrs.* or 312*l.* 9*s.* $4\frac{3}{4}$*d.* $\frac{33}{136}$ the Answer.

By Practice, thus,

4*s.* | $\frac{1}{5}$ | 1426 at 4*s.* $6\frac{1}{4}$*d.*——then for 4 *R.* 26 *Mar.* thus,

4*d.*	$\frac{1}{12}$	285	4	0
2*d.*	$\frac{1}{2}$	23	15	4
$\frac{1}{4}$	$\frac{1}{8}$	11	17	8
		1	9	$8\frac{1}{2}$
		0	2	$8\frac{1}{4}$
		£.322	9	$4\frac{3}{4}$ as before.

		s.	*d.*	
4 *R.*	$\frac{1}{2}$	4	$6\frac{1}{4}$	
				Rem.
17 *M.*—	$\frac{1}{2}$	2	3	$\frac{2}{8}$
	$\frac{1}{2}$		$3\frac{1}{4}$	$\frac{1}{2}$
			$1\frac{1}{2}$	$\frac{1}{17}$
		2	$8\frac{1}{4}$	nearly.

3. With ITALY.

(7) 4*s.* | $\frac{1}{5}$ | 640 at 4*s.* 4*d.* per Dollar.

4*d.*	$\frac{1}{12}$	128	0	0
		10	13	4
Answer,		£.138	13	4

(8) First, 138*l.* 13*s.* 4*d.* = 33280 Pence.

Then, as 52*d.* : 1 *Dol.* :: 33280 : 640 Dollars, the Answer.

(9)

2*s.*	$\frac{1}{10}$	8644 at 3*s.* 11½*d.* per Pez.			
1	$\frac{1}{3}$	864	8		
6*d.*	$\frac{1}{2}$	432	4		
		216	2		
4	$\frac{1}{3}$	144	1	4	
1½	$\frac{1}{4}$	54	0	6	
2*s.* 6*d.*	$\frac{1}{8}$	0	0	5¾	= ⅛ of 47½*d.*

Answer, £.1710 16 3¾

(10) First, 1710*l.* 16*s.* 4*d.* = 821192 Halfpence, and 47½*d.* = 95 ditto.
Then—If 95 : 1 *Pez.* :: 821192*h.* : 8644 *Pez.* Answer.

4. With PORTUGAL.

(11)

5*s.*	¼	4760 at 5*s.* 4*d.*			
		1190	0	0	
4*d.*	$\frac{1}{60}$	79	6	8	

Answer, £.1269 6 8

(12) First, 1566*l.* 6*s.* 8*d.* = 375920*d.*
Then—If 64*d.* : 1 *Mil.* :: 375920*d.* : 5873 *Mil.* 750 *Rees*, the Answer.

5. With HOLLAND, FLANDERS, and GERMANY.

(13) First, 564*l.* 10*s.* 6*d.* = 135486 Pence, and 34*s.* 4*d.* = 412*d.*
Then, as 412*d.* : 1*l.* :: 135486*d.* : 328*l.* 16*s.* $11\frac{3}{4}d.\,{}_{\frac{1}{4}}{}^{\frac{1}{12}}{}_{\frac{1}{20}}$, the Answer.

(14)

	£.	£.	*s.*	£.	*s.*	*d.*
If	1	: 34 4	::	328*l.*	16*s.*	11¾

Or, as 960 *qrs.* : 412*d.* :: 315695 *qrs.* : $1354\frac{3}{4}d.\,{}_{\frac{1}{12}}$, or 564*l.* 10*s.* $5\frac{3}{4}d.\,{}_{\frac{1}{12}}$, the Answer.

By Practice, thus,

10*s.* ½ 328 at 1*l.* 14*s.* 4*d.*

4	⅓	164	0	
4*d.*	⅓	65	12	
		5	9	4
		1	9	1¼

Answer, £564 10 5¼

10*s.*	½	34*s.*	4*d.*	
5	¼	17	2	
1	⅕	8	7	
6*d.*	½	1	8½	
3	½	0	10¼	
2	⅓	0	5	
¾	¼	0	3¼	
		0	1¼	
£.1		9	1¼	

(15) If

s.	*d.*	*£.*	*Guil.*	*Stiv.*	*Pen.*
33	3 :	1 ::	4200	12	8
12			20		
399			84012		
			2		

399 168025(421*l.* 2*s.* 3½*d.*$\frac{270}{399}$, Answer.

(16) If

£.	*s.*	*d.*	*£.*	*s.*	*d.*
1 :	33	3 ::	421	2	3

Or, as 240*d.* : 399*d.* :: 101067*d.*

×399

24,0)40325733(168023*d.*

213*d.*
8

24,0)1704 7$\frac{24}{240}$ *Pen.*

N = 168023÷40=4200 Guil. 11 Stiv. 15 $\frac{1}{10}$ Pen. the Answer.

Or, First, 242*l.* 13*s.* 6*d.*=58242 Pence; then
(456 Guil. 1 Stiv. the Answer.

(6.) To Change CURRENT MONEY into BANCO.

(18) If 105 : 100 :: 495 18
20 20
21,00)9918,00 *G. St. Pen.* 472 5 11$\frac{1}{7}$, Answer.

Rem. 6
20
21)120(5 Stivers.

Rem. 15*d.*
×16
21)240(11$\frac{3}{7}$ Pen.
9

(19) If 100 : 105 12$\frac{1}{2}$ :: 470 8
20 20 20
2000 2112 9408
2
4225
9408
2,000)39748,800
2 19874—3$\frac{4}{5}$ St.
2,0 993,2
Answ. Guild. 496 12 3 $\frac{4}{5}$

7. With VENICE.

Du. d. Du. sol. den.

(20) If 1 : $47\frac{6}{8}$: : 4720 10 8

Or, as 240 : 382 :: 1128128 : $224450\frac{1}{4}d.$ $\frac{1}{8}$, or 935*l.* 4*s.* $2\frac{1}{4}d.$ $\frac{1}{8}$, the Answer.

Or by Practice thus,—

$\frac{4}{8}$	$\frac{1}{2}$	4700 at $47\frac{6}{8}$;	then for 10 Sol. 8 Den. at
		×47	
		220900	
$\frac{2}{2}$	$\frac{1}{3}$	2350	
		1175	
		$25\frac{1}{4}$	
12		$224450\frac{1}{4}$	
20		1870,3—9	

St.		*d.*	
5	$\frac{1}{4}$	$47\frac{6}{8}$	
		$11\frac{7}{8}$	
4	$\frac{1}{4}$	$9\frac{4}{8}$	
1	$\frac{1}{4}$	$2\frac{3}{8}$	
4*d.*	$\frac{1}{3}$	$\frac{6}{8}$	
4*d.*	$\frac{1}{3}$	$\frac{6}{8}$	
		$25\frac{1}{4}$	

Ans. £.935 3 $9\frac{1}{4}$

8. With POLAND and PRUSSIA.

G.P. £. *Flor.*

(21) If 270 1 : 4684

Or, as 270 : 1 :: 140520 : 520*l.* 8*s.* 10*d.* $\frac{3}{5}$, Flemish.

Again, If 34*s.* 4*d.* : 1*l.* :: 520*l.* 8*s.* $10d\frac{3}{5}$.

Or, as 1256 Thirds : 1 :: 374720 Thirds : 303*l.* 3*s.* $5d.\frac{4}{20}$, the Answer.

(22)

		s.	*d.*
10*s.*	$\frac{1}{2}$	390 at 33	4
3*s.* 4*d.*	$\frac{1}{3}$	195	
		65	
		650£. Flemish.	

Now 270 Gros. = 9 Florins:
Then—As 1*l.* : 9 *fl.* :: 650*l.* :
5850 Florins, the Answer.

9. With RUSSIA.

Rub. cop.
(23) 6420 42
×100

122)642042(5262 78/122 Rix Dollars.
×2 1/2

10525 34/122 } Florins Current.
2631 39/122 }

13156 73/122

Fl.Cur.Fl.B.

If 103 : 100 :: 13156 73/122 *Flor. Banco.*
Or, as 12566 : 100 :: 1605105 : 12773 2401/8183.
Now 12773 2401/8183 × 40 = 510935 5305/8183 Pence, and 34*s.* 6*d.* = 414*d.*
Then—If 114*d.* : 1*l.* :: 510935 5305/8183 *d.*
Or, as 2601162 : 1*l.* :: 3210210000 : 1234*l.* 2*s.* 10¼*d.* 2680/2601 7906/10262, the Answer.

10. With IRELAND.

£. *£.* *£.* *s.* *d.*
(24) If 112 : 100 :: 740 14. 6.
Or, as 4880 *six-d.* : 100 :: 26729 *six-d.* : 661*l.* 7*d.* 2¾*d* 1/7, the Answer.

(25) 10 | 1/10 | 651 14 11¾ at 112 per 100
2 | 1/5 | 65 3 5¾
13 0 8 7/4

Answer, £.729 19 1¾

11. With AMERICA and the WEST INDIES.

£. *£.* *£.* *s.*
(26) If 164 : 100 :: 1474 16
Or, as 3280*s.* : 100 :: 29496*s.* : £.899 5*s.* 4¼*d.* 2/41, the Sterling required.

	£.		£.	s.	d.	
(27)	50	$\frac{1}{2}$	943	17	$5\frac{1}{4}$	at 164 per 109
	10	$\frac{1}{5}$	471	18	$8\frac{1}{2}$	
	2	$\frac{1}{5}$	94	7	$8\frac{3}{4}$	
	2	$\frac{1}{5}$	18	17	$6\frac{1}{2}$	
			18	17	$6\frac{1}{2}$	

Answer, £.1547 18 $11\frac{1}{2}$ Currency.

(28) First, 100+30+5=135 Amount.
Then—If 135*l.* : 100*l.* :: 987*l.* 12*s.*
Or, as 2700*s.* : 100 :: 19752*s.* : 731*l.* 11*s.* $1\frac{1}{4}$*d.* $\frac{1}{2}$ remitted.
Consigned - - 640 16 9

Gained - - £.90 14 $4\frac{1}{4}$ $\frac{1}{2}$ Sterling.

Therefore—If 640 16 9 : 90 14 $4\frac{1}{4}$ $\frac{1}{3}$ :: 100
Or, as 153801*d.* : 261268 Thirds :: 24000*d.* : 14*l.* 3*s.* $1\frac{1}{4}d.\frac{2}{3}$, gained per Cent.

(29)	25	$\frac{1}{4}$	1470	12	8	at $136\frac{1}{2}$ per 100
			367	13	2	
	10	$\frac{1}{10}$	147	1	$3\frac{1}{5}$	
	1	$\frac{1}{10}$	14	14	$1\frac{2}{5}$	
	$\frac{1}{2}$	$\frac{1}{2}$	7	7	$0\frac{2}{5}$	

£. 2007 8 $3\frac{1}{5}$

	s.	d.	£.	d.	
(30)	If 34	4	: 1	:: 52	

Or, as 412*d.* : 240*d.* :: 52*d.* : 30*d.* $\frac{30}{103}$, for 400 Reas.
∴ $30\frac{30}{103} \times 2\frac{1}{2} = 75\frac{75}{103}$ Pence Sterling for 1000 Reas.

Crowns.

(30)	4*s.*	$\frac{1}{5}$	1200	at 4*s.* 7*d.* per Crown.
	Cd.	$\frac{1}{2}$	240	
	1	$\frac{1}{6}$	30	
			5	

£. 275 0 0

£. s. £.
As 100 : 10 :: 275 : 27*s.* 6*d.* Commission.
Therefore 275*l.*+1*l.* 7*s.* 6*d.*=2761. 7*s.* 6*d.*=66330*d.*
Then, as 56*d.* : 1 *Cr.* :: 66330*d.* : 1184$\frac{1}{16}$ *Cr.*
∴ 1200—1184$\frac{1}{16}$=15$\frac{15}{16}$ A's Gain.

d. Sols. d. Sols.
(32) As 67 : 32 :: 70 : 33$\frac{20}{67}$ Lubeck, per Florin.

(33) Receip.—If 54*d.* : 33*s.* 6*d.* :: 54$\frac{1}{2}$*d.*
Or, as 108 : 142 :: 109 : 398$\frac{34}{109}$*d.*=33*s.* 2*d.*$\frac{34}{109}$ Flemish, the Answer.

£. £. *s. d.*
(34) If 100 : 102 :: 33 4
Or, as 6000 : 102 :: 100 : 1*l.* 14*s.* Flem. per £. Sterling.

(35) As 100$\frac{3}{10}$: 100 :: 91 : 90$\frac{1730}{1003}$=90$\frac{520730}{1003}$.
Also, 100$\frac{1}{8}$: 100 :: 93 : 92$\frac{4106624}{8019224}$.
Then 92$\frac{4106624}{8019224}$—90$\frac{520730}{1003}$=1$\frac{3832557}{8019224}$, Gain per Cent. for two Months; therefore 1$\frac{3832557}{8019224}$×6=10$\frac{18520730}{8019224}$=10*l.* 19*s.* 1$\frac{3}{4}$*d.* Gain per Cent. per Ann. the Answer.

(36) First, 2000×40=80000 Flemish Pence.
Or, 160000 Halfpence, and 90$\frac{1}{2}$*d.*=182 Ditto, also 89$\frac{1}{2}$*d.*=179
181)160000(883$\frac{177}{181}$
179)160000(893$\frac{153}{179}$ } Crowns Tournois.
Or, as 104 : 100 :: 883$\frac{177}{181}$: 849$\frac{23031}{1813}$=849$\frac{865609771}{1884492171}$.
Also, as 105 : 100 :: 893$\frac{153}{179}$: 851$\frac{10941}{1879}$=851$\frac{2255214323}{1884492171}$.
851$\frac{2255214323}{1884492171}$—849$\frac{865609771}{1884492171}$=1$\frac{2455083223}{1884492171}$=1 *Cro.* 18 *Sols.* 9 *din.* in Favour of B.

31. COMPARISON of WEIGHTS and MEASURES.

Left Hand. *Right Hand.*
(1) First, 100 113, Now, 100×100×60=600000
100 81, and, 113×81=9153
60
Then, 600000÷9153=65*lb.* 8$\frac{7656}{9153}$*oz.* Answer.

(1) *lb.* *lb.*
First, 104 84$\frac{1}{2}$ | Then 104×100×64=665600
100 108 | And 108× 84$\frac{1}{2}$ =9126
64——9126)665600(72*lb.* 14$\frac{8684}{9126}$ *oz.* the Ans.

Yds. Ells.

(3) 100 78 Then $100 \times 78 \times 100 = 7800{,}00$
78 $153\frac{1}{3}$ And $133\frac{1}{3} \times 78 =$ $= 104{,}00$
100

Also, 104)7800(75 Yards, the Answer.

Canes. Ells.

(4) 100 $191\frac{1}{3}$ Then $78 \times 100 \times 100 = 780000$
78 $131\frac{2}{3}$ And $191\frac{1}{3} \times 131\frac{2}{3} = \frac{2267\,30}{9}$
100 Also $780000 \div \frac{2267\,30}{9} = \frac{76\,20\,00\,00\,0}{2267\,30} = 30\frac{4}{7}$

Canes nearly the Answer.

lb. lb.

(5) 100 92 Then $92 \times 110 \times 60 = 607600$
100 110 And $100 \times 100 = 10000$
60

Also, 1,0000)60,7200($60\frac{1}{2}\frac{8}{5}$ *lb.* the Answer.

Yds. Bra.

(6) 74 100 Then $100 \times 100 \times 30 = 300000$
100 30 And $74 \times 100 = 7400$
100 Also 74,00)300000($40\frac{3}{7}$ Canes the Ans.

32. SINGLE POSITION.

(1) Suppose A's Age to be 20
Then B's will be $20 + 10 = 30$
And C's will be $30 \times 2 = 60$

Sum 110

As $110 : 20 :: 132 : 24$ A's Age.
Then $24 + 12 = 36$ B's
And $36 \times 2 = 72$ C's

Sum, 132 Proof.

(2) Suppose she had 10 | Then $40 - 10 = 30$
Then as many 10 | As $25 : 10 :: 30 : 12$ her Flock,
One half as many 5 | the Answer.

Sum, 25, for $12 + 12 + 6 + 10 = 40$ Proof.

(3) Suppose C's Age to be 10 | Then 4+4+9=17
A's 10+4 =14 | And 45—17=28
And B's 10+14+9 =33 |

Sum 57 —17=40

As, 40 : 10 :: 28 : 7 C's Age.
And 7+4=11 A's.
Also 7+11+9=27 B's.
7 C's.

Proof, 45 D's.

(4) *Miles.*

Suppose Andrew goes - - 30 | 469
Then Ben. will go 30×3+ 3= 93 | —25
And Christopher 93×2+16=202 | ——
444

Sum, 325
Then 3×2+3+16=25

300 Diff.

M. M. M. Miles.
As, 300 : 30 :: 444 : $44\frac{2}{7}$ Andrew.
Also $44\frac{2}{7}\times 3+3$ $136\frac{1}{7}$ Benjamin.
And $136\frac{1}{7}\times 2+16=288\frac{5}{7}$ Christopher.

Proof, 469

(5)

	l.	*s.*	*l.*	*s.*
Suppose A. paid - -	200	0	45	15
Then B. 200×3½—45*l.* 15*s.*	=654	5	×	2
And C. 200+624 5+26 10	=880	15	——	
			91	10
Sum,	1735	0	26	10
			Difference,	65 0

Now 2000+65=2065, and 65+1735=1800

	£.	£.	£.	£.	s.	
Then—As 1800 : 200 :: 2065 : 229					$8\frac{8}{9}$	A.
∴ 229 $8\frac{8}{9}$ × $3\frac{1}{3}$ — 45				15 = 757	$6\frac{2}{9}$	B.
And 229 $8\frac{8}{9}$ + 757 $6\frac{1}{9}$ +			26	10 = 1013	5	C.
				Proof 2000	0	

(6) Suppose they will empty the Cistern in 30 Minutes, then the first will empty $\frac{1}{2}$ or 30 Pipes.
The second - - $\frac{1}{4}$ or 15
And the third - $\frac{1}{6}$ or 10

Sum 55

Pipes. Min. Pipes. M. Sec.
∴ As 55 : 30 :: 60 : 32 $43\frac{7}{11}$ the Time sought.

(7) First, $\frac{1}{4}$ of $\frac{1}{2}=\frac{1}{8}$
Suppose he had 240, then $240 \div 8 \times 3 = 90$
∴ As 90 : 260 :: 900 : 2400 Men, the Answer.
Or, thus, $900 \div \frac{3}{8} = \frac{7200}{3} = 2400$ Men, as before.

(8)

Suppose he had	12	If 37 : 12 :: 333
Then as many	12	333
$\frac{1}{2}$ as many	6	
$\frac{1}{3}$ Ditto	4	37)3996(108 Scholars, the Ans.
$\frac{1}{4}$ Ditto	3	
Sum	37	

33. DOUBLE POSITION.

(1)

	£.
Suppose the Value of the first Horse to be	20
To which add the Value of the Saddle,	50
	2)70 Double the Val. of the second.
Then the Value of the Second is	35
To which add the Value of the Saddle,	50
The Sum should have been triple the Value of the first Horse.	85 = 25 too much.

	£.
Again, Suppose the first Horse to be worth	24
Then proceeding as above	50
	2)74
Also the Value of the second is	37
	50
	87=15 too much.

Sup.	*Er.*	
20	25	Then 25—15=10 Difference of Errors.
24	15	Also 600—300=300 the Difference of the Prod.
600	300	∴ 300÷10=30 the Value of the first Horse.

£.30 Value of the first Horse.
50 Value of the Saddle.

2)80 Double the Value of the second Horse.

£.
40 Value of the second Horse.
50 Value of the Saddle.

Proof, 90=30×3=90

(2)

d.	*d.*
Suppose he had 8	Again, suppose he had 7
Then 8+ 8=16	Then 7+ 7=14
Also 16— 6=10	14— 6= 8
10+10=20	8+ 8=16
20— 6=14	16— 6=10
14+14=28	10+10=20
28— 6=22	20— 6=14
too much.	too much.

Sup.	*Er.*	
8	22	Then 22—14=8, and 154—112=42.
7	14	∴ 42÷8=5¼*d.* the Answer.
154	112	

(3.)

	Crs.		*Crs.*
Suppose	28 A's	Again, suppose A had	34
Then 28+16 :	=44 B's	Then B. 44+16	=50
And 28+44÷6	=12 C's	C. 84÷ 6	=14
Sum,	84=28	Sum,	98=14
too little.		too little.	

Sup.	*Er.*
28	28
34 ×	14
952	392

Then 952—392=560, and 28—14=14 Diff. of Errors.
∴ 14)560 - 40 A's
And 40+16 - =56 B's } Crowns.
Also 40+56(96)÷6=16 C's

(4 Suppose he had 60
Tom took 30—10=20
Rem. 40
Ned took 20—4=16
Rem. 24
Jack took 12—1=11
Rem. 13=5
too little.

Again, suppose he had 76
Then, 38—10=28
Rem. 48
24—4=20
Rem. 28
14—1=13
Rem. 15=3
too little.

Sup.	*Er.*
60	5
76 ×	3
380	180

Then 380—180=200, and 5—3=2
∴ 200÷2=100, what he had at first.

(5) Suppose the Father's age to be 40
Then the Son will be 40÷5= 8
And 4 Years ago was 8—4= 4
which is 12 too little.

Again, suppose the Father was 60
Son will be 60÷5= 12
And 4 Years since was 12—4=8 which is 4 too little.

Sup.		*Er.*	
40	×	12	Then 720—160=560, and 12—4=8 ∴ 560
60		4	÷8=70 Years, the Father's Age; and 70
			÷5=14 the Son's.
720		160	For 14—4=10=$\frac{1}{7}$ of 70 (the Age of the Son
			4 Years ago) and 14×5=70, the Proof.

(6) Suppose the Body to be 24 Inches.
Then 24÷2+9=21 Tail; also 21+9=30, which is 6 too much.
Again, suppose the Body to be 26, then 26÷2+9=22 Tail; also 22+9=31, which is 5 too much.

Sup.		*Er.*	
24	×	6	Then 156—120=36, and 6—5=1
26		5	∴ 36 Inches is the Length of the Body.
			And 36÷2+9=27 Tail.
156		120	Therefore, 36+27+9=72 In. the Answ.

(7)

Suppose the No. to be	4	Again, suppose the No. to be	5
Then 4+4= -	8	Then 5+5= -	10
And 8×4= -	32	And 10×5= -	50
Also 32—4= -	28	Also 50—5= -	45
Likewise 28÷4=	7	45÷5= -	9
Therefore 13—7=	6	13—9= -	4
too little.		too little	

Sup.		*Er.*	
4	×	6	Then 30—16=14, and 6—4=2.
5		4	∴14÷2=7, the Number sought. For
			7+7×7—7=91, which÷7=13, the Proof.
30		16	

(8)

Suppose she	14	Again, suppose she	16	
Then he 14×3=	42	Then he will be 16×3=	48	
14+10+5=	29	16+10+5=	31	
42+10+5=	57 }	48+10+5=	63 }	sub.
29× 2=	58 }	31× 2=	62 }	
too little by	1	too much by	1	

Sup. Er.

14 ×	1	Then 16+14=30, and 1+1=2
16 ×	1	∴ 30÷2=15 Years, her Age.
		And 15×3=45 his, Answer. For
16	14	15+10+5=30 her, when married 15 Years.
		And 45+10+5=60 his,
		For as 8 : 16 :: 30 : 60 Proof.

(9)

	Beg.	*Beg.*
Sup.	8	8
	×4	×6
	32	48
	+16	−12
	48	36=

12 too little.

Again, suppose

	Beg.	*Beg.*
	10	10
	×4	×6
	40	60
	×16	−12
	56	−48=8 too

little.

Sup. Er.

8 ×	12	Then 120−64=56, and 12−8=4 ∴ 56÷4=
10 ×	8	14, the Number of Beggars. For 14×4+16
		=14×6−12=72*d.* what he gave, Proof.
120	64	

(10)

Suppose the Number to be 15

Then	15 × 3=45
And	45− 5=40
Also	40÷ 2=20
	20+15=35
	40−35= 5

too little.

Again, suppose 25

Then	25 × 3=75
And	75− 5=70
Also	70÷ 2=35
	35+25=60
	60−40=20

too much.

Sup. Er.

15 ×		Then 125+300=425, and 20+5=25.
		∴ 425÷25=17, the Number required.
35		or 17×3−5=46, and 46÷2+17=40 Proof.
12	5	

(11)

Suppose B. had 6	Again, suppose B. had 8
Then A. must have 4	Then A. must have 6
For 6—2=4+1=5	For 8—2=6+1=7
Again 4—1 =3	Again, 6—1 =5
And 6+1 =7	And 8+1 =9
Also, 3×2 =6	Also, 5×2 =10
7—6 =1 too lit.	19—9=1, too much.

Sup. Er.

6 × 1	Then 8+6=14, and 1+1=2 ∴ 14÷2=7 B's
8 × 1	Guineas, and 5 A's; for 7—1=5+1=6 A's
— —	when he had received 1 of B. Also, 5—1 2
8 6	=7+1=8 B's when he had received 1 of A's
— —	Proof.

(12)

Suppose Father's Age 37	Again, suppose Father's 45
Then 37—5 =32	Then 45—5 =40
And 32÷8 =4	And 40÷8 =5
∴ 4×3=12 Son's Age	∴5×3=15 Son's Age.
Then 12+2 =14	Then 15+2 =17
And 14×3 =42	And 17×3 =51
Also, 42—7 =35	Also, 51—7 =44
∴37—35=2 too little.	∴ 45—44=1 too little.

Sup. Er.

37 × 2	Then 90—37,=53, and 2—1=1
45 × 1	∴ 53 is the Father's Age; then 53—5=48, and
— —	48÷8=6=⅓ of the Son's Age. ∴ 6×3=
90 37	18, the Son's Age.
— —	

(13)

	£.
Suppose B. had	360
Then 360÷3×2=240	
And 1200—240=960 A's.	
Then 960÷4×3=720	
And 360+720=1080 A's and B's.	
∴ 1200—1080=120 too little.	

	£.
Again, suppose B. had	420
Then 420÷3×2	= 280
And 1200—280	= 920 A's.
Then 920÷4×3	= 690
And 420+690	= 1110
∴ 1200—1110	= 90 too little.

Sup.	*Er.*
360 ×	120
420 ×	90
151200	32400

Then 50400—32400=18000
And 120—90=30 ∴ 18000
÷30=£.600 B's Money; and
600÷3×2=400. Also 1200—400=800.
A's money, the Answer.

For 600+800÷4×3=800+600÷3×2=1200, the Proof.

(14) *greater, lesser.*

Suppose 28 and 22
28÷7=4 Also 22×3=66
4+66=70, then 70—
—50=20 too much.

gr. less.

Again, suppose 42 and 8
42÷7=6; also 8×3=24,
6+24=30 ∴ 50—30=
20 too little.

Sup.	*Er.*
28 ×	20
42 ×	20
840	560

Then 840+560=1400, and 20+20=40 ∴
1400÷40=35, the greater Number; and
50—35=15 the lesser; for 35÷7+15×3
=50, the Proof.

(15) Suppose he worked 140 Days, ∴ 390—140=150 Days he was idle.

Then 140×12=1680
And 250×8 =2000
} Subt.=320 too little.

Again, suppose he worked 150 Days, ∴ 390—150=240 Days idle.

Then 150×12=1800
And 240× 8=1920
} Subt.=120 too little.

Sup.	*Er.*
140 ×	320
150 ×	120
48000	16800

Then 48000—16800=31200
And 320—120=200. ∴ 31200÷200=156
Days worked, and 390—156=234 Days
idle, the Answer.
For 156×12=234, and 234×8=1872 Proof.

(16)

Suppose his Age was	24	Suppose he was	30
Then $24\div3\times2\times4$	$=64$	Then $30\div3\times2\times4$	$=80$
And $64+12+50$	$=126$	And $80+15+50$	$=145$
Then $126-100$	$=26$	Then $145-100$	$=45$
And $100-24$	$=76$	And $100-30$	$=70$
$\therefore 76-26=50$ too little.		$\therefore 70-45=25$ too little.	

Sup.	*Er.*
24 ×	50
30 ×	25
1500	600

Then $1500-600=900$, and $50-25=\therefore$
$900\div25=36$ Years, his Age required.
For $36\div3\times2\times4+18+50-100=164-100$
$=64$ Proof.

(17)

	£.
Suppose the first Horse to be worth	24
Then the Trappings must be $50-24$	$=26$
And the second Horse must be $24+26$	$=50$

Now $50+26=76$, and $24\times2=48$; also $76-48=28$, too little.

Again, suppose the first Horse to be worth	28
Then the Trappings must be $50-28$	$=22$
And the second Horse must be $28+22$	$=50$

Now $50+22=72$, and $28\times2=56$; also, $72-56=16$, too little.

Sup.	*Er.*
24 ×	28
28 ×	16
784	384

Then $784-384=400$
And $28-16=12.\therefore 400\div12=33\frac{1}{3}$£. Value of the first Horse; and $50-33\frac{1}{3}$£. $=16\frac{2}{3}$£. Value of the Trappings.
Also, $33\frac{1}{3}+16\frac{2}{3}=50$£. Value of the second Horse for $50-16\frac{2}{3}=33\frac{1}{3}$ and $33\frac{1}{3}\times2=66\frac{2}{3}$ the Proof.

(18)

	£.
Suppose he left	5000
Interest for 11 Years	$=2750$
Amount	7750
Education, &c. $100\times11=1100$	
Neat,	6650

$\therefore$ $7400-6650=750$ too little.

	£.
Again, suppose he left	6000
Interest for 11 Years	3300
Amount	9300
Education, &c. $100 \times 11 = 1100$	
Neat	8200

$\therefore$ $8200 - 7400 = 800$ too much.

Sup.	*Er.*	
5000	750	Then, $4500000 + 4000000 = 8500000$
6000 ×	800	and $750 + 800 = 1550$ $\therefore$ $8500000 \div 1550 = £.5483$ 17*s.* 5*d.* $\frac{3}{11}$ the Answer required.
4500000	4000000	

34. ARITHMETICAL PROGRESSION.

(1) First, $1 + 24 = 25$ Sum of the Extremes.
Then per Rule I. $25 \times 24 \div 2 = 300$ Strokes, the Answer;
or per Rule IId. $25 \times 12 = 300$.

(2) Thus, $1 + 12 \times 6 = 13 \times 6 = 78$ Strokes, the Answer.

(3) First $2 + 188 = 190$ Sum of the Extremes.
And $94 \div 2 = 47$ = half the Number of Terms.
Product 8930 Feet a Mile = 5280 Feet.
Then 5280)8930(1 Mile, 5 Fur. 21 P. $3\frac{1}{2}$ Feet, the Answer.

(4) First $2 + 200 = 202$ Sum of the Extremes.
And $100 \div 2 =$ × 50 half the Number of Terms.
A Mile = yds. 1760)10100(5 m. 5 fur. 36 p. 2 yds. the Answer.

(5) First 9*l.* 19*s.* = 199 Shillings.
Then $1 + 199 = 200$ Sum of the Extremes.
And $100 \div 2$ × 50 half the Number of Terms.
2,0)1000,0(500*£.* the Answer.

(6) Here $43 - 5 = 38$ Difference of the Extremes.
And $20 - 1 = 19$ Number of Terms, less one.
Then $38 \div 19 = 2$ Common Difference.

Which added to the Age of the youngest, and so on to the rest, will give their several Ages, viz. 5+2=7 Age of the Second.

And 7+2=9 Age of the third, and so on for the rest.

(7) First 60—6=54 Difference of the Extremes.
Then 19—1=18 Number of Terms, less one.
Then 54÷18=3 the common Difference sought.
Then 3 added to 6 and every other Term respectively, gives each Day's Journey as follows, 6+9+12+15+18+21+24+27+30+33+36+39+42+45+48+51+54+57+60=627 Miles from London.

(8) First 50—5=45 Difference of the Extremes.
Then 10—1=9 Number of Terms, less one.
Then 45÷9=5 common Difference.
Then each Payment and the whole Debt will be as follow, viz. 5+10+15+20+25+30+35+40+45+50=275£. the whole Debt.

(9) First 43—5=38 Difference of the Extremes.
Then 38÷2=19 Number of Terms less one.
∴ 19+1=20 Children he had.

(10) First 60—6=54 Difference of the Extremes.
And 54÷3=18 ∴ 18+1=19 Days, the Answer.

(11) First 19—1=18 Number of Terms, less one.
Then 18×3 (the common Diff.)=54.
And 60—54=6, the first Day's Journey.

(12) First 10—1=9 Number of Terms, less one.
And 9×2=18. Then 23—18=5, the Answer.

(13) First, 275÷10=27£. 10s.
And 10—1=9, also 9×5=45, which÷2=22£. 10s.
Then from 27£. 10s. take 22£. 10s.
Remains 5£. Q.E.F.

(14) First 100÷2=50 what each travelled.
And 50÷5=10. Also 5—1=4. Then 4×2=8. Likewise 8÷2=4 ∴ 10—4=6 his first Day's Journey, and 6+2=8 the 2d. 10 the 3d. 12 the 4th, and 14 the 5th. Sum of which =50.

Again, 4)50(12½. Also 4—1=3. Then 3×3=9. And 9÷2=4½ ∴ 12½—4½=8 his first Day's Journey, and 8+3=11 his 2d. 14 his 3d. and 17 his 4th. Sum of which=50 Leagues.

(15) First, $19\times3=57$. And $57-3=54$. Then $54+6=60$, the last Term required.

(16) First $100\times2=200$. Then $200-2=198$. And $198+1=199$ the last Term required.

(17) First $30\times2=60$. Also $60-2=58$.
Again $58+6=64$. And $64\times30=1920$. $\therefore$ $1920\div2=960s.=48£$. due, Answer

(18) First $\overline{200\times2}\div8=50$. And $50-12=38$. Also $8-1=7$. Then $38\div7=5\frac{3}{7}$ Miles, the common Diff. required. $\therefore$ $5\frac{3}{7}$ added to 6, and every other Term respectively gives for the 1st. 2d. 3d. 4th. 5th. 6th. 7th. and 8th.
6, $11\frac{3}{7}$, $16\frac{6}{7}$, $22\frac{2}{7}$, $27\frac{6}{7}$, $33\frac{5}{7}$, $38\frac{4}{7}$, and 44, which, added together, gives 200 Miles, Proof.

(19) First $200\times2=400$ and $400\div9=44\frac{4}{9}$, also, $44\frac{4}{9}-4\times2\,(8)=36\frac{4}{9}$ $\therefore$ $36\frac{4}{9}\div8=4\frac{5}{9}$ common Difference. Then the Miles he travelled each Day will be as follow, viz. 1st. 2d. 3d. 4th. 5th. 6th. 7th. 8th. 9th.
4, $8\frac{5}{9}$, $13\frac{1}{9}$, $17\frac{6}{9}$, $22\frac{2}{9}$, $26\frac{7}{9}$, $31\frac{3}{9}$, $35\frac{8}{9}$, and $40\frac{4}{9}$, which added together=200, the whole Distance, Proof.

(20) First $9\times2=18$ Leagues, double the Space gone each Day by the pursued, which + to the common Difference of the Pursuer's Day's Journey, viz. 2=20 the Sum from which take double the Space gone the first Day by the Pursuer, viz. 6=14 remains, this ÷ by the common Difference 2, gives 7 Days, the Answer.

(21) For A's Race thus,
First $4+40=44$, Sum of the first and last Terms. Then $44+5$, half the Number of Terms=220 Yards. A's first Race per Prop. the first.
Then $901\times3=3604$ Yds. A's first Term of the last Race.
Also $910\times4=3640$ A's last Term.

Their Sum 7244 which $\times5=36220$ A's last Race.
Then to find his whole Ground, put 220 first Term. And 36220 the last Term, which (by Prob. 1.) is $36440\times5=182200$ Yards gone by A.

For B's, thus,
First $11\times4=44$ first Term. And $20\times4=80$ last Term.

Then 44+80=124 their Sum, which ×5=620 Yards, B's first Race.

∴ 911×4=3644 yds. B's first Term of the last Race.
920×4=3680 B's last Race.
Their Sum 7324 which ×5=36620 yds. B's last Race.
Then 620+36620=37240 Sum of the first and last Terms, which ×5=186200 Yards gone by B.
Deducted 182200 ditto by A.

4000 Yards, common Difference.

Which added continually to each of their Shares, show that

	Yds.	*Miles.*	*Fur.*	*Yds.*
A in all ran	182200	= 103	4	40
B	186200	= 105	6	80
C	190200	= 108	0	120
D	194200	= 110	2	160
E	198200	= 112	4	200
F	202200	= 114	7	20
G	206200	= 117	1	60
H	210200	= 119	3	100
I	214200	= 121	5	140
K	218200	= 123	7	180
Sums	2002000	=1137	4	0

If 2002000 : 300*l.* :: 182200

2002,000)54660,000(27*l.* 6*s.* 0½*d.* $\frac{590}{1001}$ A's Part.

If 2002,000 : 300*l.* :: 186200

	l.	*s.*	*d.*	
2002,000)55860,000(	27	18	0½	$\frac{14}{1001}$ B's Part.
	27	6	0½	$\frac{590}{1001}$ A's——
Common Difference		11	11¾	$\frac{425}{1001}$

Which by continually adding to each we shall have

First	27	6	$0\frac{1}{2}$	590	= A's	
Then	27	18	$0\frac{1}{2}$	14	= B's	
	28	10	$0\frac{1}{4}$	439	= C's	
	29	2	0	864	= D's	
	29	14	0	288	= E's	} Part.
	30	5	$11\frac{3}{4}$	713	= F's	
	30	17	$11\frac{3}{4}$	137	= G's	
	31	9	$11\frac{1}{2}$	562	= H's	
	32	1	$11\frac{1}{4}$	987	= I's	
	32	13	$11\frac{1}{4}$	411	= K's	
Proof	300	0	0			

From London to York suppose 180 Miles, which $\times 4 = 720$ Miles, twice a-back.

Their Sum Total 1137 M. 4 Fur.
Distance of York 720

Short of the Undertaking $417\frac{1}{2}$ Miles.

35. GEOMETRICAL PROGRESSION.

(1) First { 0. 1. 2. 3. 4. 5. Indices.
1. 2. 4. 8. 16. 32. Terms.

Now, $5+5=10$. Then $32\times 32=1024=10$th Term.
Again $10+5=15$. $\therefore$ $1024\times 32=32768$ Qrs. the 16th Term. As the Indices are less than the Terms by one.
And 32768 Qrs $=$ £. 34. 2. 8. Q.E.F.

(2) First { 0. 1. 2. 3. 4. 5. 6.
1. 2. 4. 8. 16. 32. 64.

Then $64\times 64=4096=12$th Term.
And $4096\times 4096=16772\,16=24$th Term.
Also $64\times 32=2048=5+6=11$th Term.
$\therefore$ $16777216\times 2048=34359738368$ Farthings, or 36th Term, which reduced to Pounds, will give 35791394£. 2s. 8d. the Answer.

(3) First { 0. 1. 2. 3. 4. 5. 6. Indices.
10. 30. 90. 270. 810. 2430. 7290. Terms.

Now 5+6=11. Then, 2430×7290=17714700, which ÷ 10 the first Term gives 1771470*£*. the eldest Child's Fortune.

0 1 2 3 4 5
(4) 500. 750. 1125 1687,5. 2531,25. 3796,875.

5+4=9 Number of Terms less 1.
Then 3796, 875×2531,25=9610839, 84375.
Which÷500 the first Term=*£*.19221 13*s*. 7*d*. 125.

(5) First { 0. 1. 2. 3. 4. 5. 6. Indices.
1. 2. 4. 8. 16. 32. 64. Terms.

Now 5+6=11 Number of Terms less 1.
Then 64×32=2048 last Term.
And 2047—1÷2—1=2047.
Also 2048+2048=4095 Guineas, or *£*.4299 15*s*.

(6) First { 0. 1. 2. 3. 4. 5. 6.
1. 2. 4. 8. 16. 32. 64.

Then 6+6+6=18 Number of Terms less 1.
64×64×64=262144 last Term.
262144—1÷2—1=262143, +262144=524287 Farthings: or, *£*.546 2*s*. 7¾*d*. what 20 Horses came to.
∴ 20 546 2 7¾*d*.(27*l*. 6*s*. 1¼*d*. $\frac{7}{20}$ per Head the Answer.

(7) First { 0. 1. 2. 3. 4. 5.
2. 6. 18. 54. 162. 486.

Then 486×486=236196=5+5=10th Term.
And 486×162= 78732=5+4= 9th Term.

Product 18596183472 last Term.
—2 First.

Ratio 3—1=2 18596183470

929809173; this added to the last Term,

gives 1,00)278942752,07 the Sum of all the Terms at ¾ per Ct.

Value of the Pins $27894275\frac{2}{100}$ Qrs. = 290565 7 4
Lace comes to £.8 1*s*. 8*d*. × 5 × 4 (20 = 161 13 4

Answer gained 29043 14 0

(8) 0 1 2 3 4 5
10. 100. 1000. 10000. 100000. 1000000.
Then 1000000 × 1000000 = 1000000000000
—10

10 Ratio—=9)999999999990
Pints.
768,0)111111111111,0(14467592 $\frac{456}{768}$
A Bushel=pts. 64)14467592(226056$\frac{1}{8}$ Bushels at 4*s*.
226056$\frac{1}{8}$
2 See Case 4 P. 85 G.

45211.4.—
6=$\frac{1}{8}$ of 4*s*.

Answer, £.45211 4*s*. 6*d*.

(9) First 4. 2. 1. $\frac{1}{2}$, $\frac{1}{4}$, and ad infinitum.
Then 4 × 4=16 Square of the first Term.
And 16 ÷ 2=8 Leagues, the Answer.

(10) First 12 × 12=144 Square of the first Term.
And 12—10=2 Difference of the first and second Term.
Then 2) 144(72 Miles, the Answer.

36. PERMUTATION.

(1) First 1 × 2 × 3 × 4 × 5 × 6 × 7=5040 Days.
Then 365) 5040(13 Years, 295 Days, the Answer.

(2) First 1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 × 11 × 12=
479,001600 Rounds. Which × 3=1437004800 Se-

And in a Year there
are 815576,00)1437004800(45 Years.

A Day =864,00)169128,00(184 Days.

An hour=36,00)52,00(1 Hour.

A Minute =6,0)160,0(26⅔ Minutes.

$\frac{40}{60}=\frac{2}{3}$

45 Years, 184 Days, 1 Hour, 26⅔ Min. the Anſwer.

(3) Firſt 1×2×3×4×5×6×7×8×9=362880 Days.
And 20 Guineas =5040 Pence.
Then, as 362880 Days : 5040*d.* :: 365D. : $5\frac{315}{4536}$*d.* per Year.

38. REDUCTION OF VULGAR FRACTIONS.

CASE I.

1) 336)896(2

224)226(1

112)334(2

Then 112)$\frac{336}{896}$(=$\frac{3}{8}$, the Term required,

(2) 2832)12848(4 Then $16)\frac{2832}{12848}(=\frac{177}{803}$ the Anſwer.
1520)2832(1
1312)1520(1
208)1312(6
64)288(3
16)64(4

(3) 144)560(3 Then $16)\frac{144}{560}(=\frac{9}{35}$ the Anſwer.
128)144(1
16)128(8

(4) 192)336(1 Then $48)\frac{192}{336}(=\frac{4}{7}$ the Anſwer.
144)192(1
48)144(3

(5) 1476)1938(1 Then $6)\frac{1476}{1938}(=\frac{246}{323}$ the Anſwer.
462)1476(3
90)462(5
12)90(7
6)12(2

(6) Thus $\frac{18}{27}$ | $\frac{000}{000}$ becomes $\frac{19}{27}$ Terms required.
(7) $6)\frac{42}{84}$ | $\frac{00}{00}(=\frac{7}{9}$ the Anſwer.
(8) Thus $8)\frac{24}{112}(=\frac{3}{14}$ $12)\frac{96}{324}(=\frac{8}{27}$ $12)\frac{60}{144}(\frac{5}{12}$
And $12)\frac{12}{72}0(=\frac{1}{6}$.
Anſwer, $\frac{3}{14}$, $\frac{8}{27}$, $\frac{5}{12}$, and $\frac{1}{6}$, the Fractions required.

CASE II.

(9) Thus $\frac{4}{9}\times\frac{5}{7}\times\frac{9}{10}=\frac{180}{630}$ or $=\frac{2}{7}$ the Fraction.
Or thus $\frac{4}{\cancel{9}}$ of $\frac{5}{7}$ of $\frac{\cancel{9}}{10}=\frac{2}{7}$ as before.

(10) Thus $\frac{2}{9}\times\frac{3}{4}\times\frac{4}{1}=\frac{24}{36}$ or $\frac{2}{3}$, the Fraction required.

Or thus $\frac{2}{\cancel{9}}$ of $\frac{\cancel{3}}{4}$ of $\frac{\cancel{4}}{1}=\frac{2}{3}$ as before.

(11) Thus $\frac{3}{7}\times\frac{1}{3}\times\frac{7}{8}=\frac{21}{168}$, or $\frac{1}{8}$, the Fraction required.

Or thus $\frac{\cancel{3}}{\cancel{7}}$ of $\frac{1}{\cancel{3}}$ of $\frac{\cancel{7}}{8}=\frac{1}{8}$ as before.

CASE III.

(12) Thus $\frac{12}{1}$, $\frac{27}{1}$, and $\frac{176}{1}$, the Fractions required.

(13) Thus $\overline{27\times12}=324$. Then $\frac{324}{11}$, the Fraction required.

(14) Thus $\overline{7\times4}+2=30$. Then $\frac{30}{7}$, the Fraction required.

(15) Thus $\overline{16\times2}+1=33$. Then $\frac{33}{2}$, the Fraction required.

(16) Thus $\overline{142\times23}+17=3283$. Then $\frac{3283}{23}$, the Anſwer.

(17) Thus $\overline{146\times37}+21=5423$. Then $\frac{5423}{37}$, the Anſwer.

CASE IV.

(18) Thus $30\div7=4\frac{2}{7}$, the Anſwer.

(19) Thus $33\div2=16\frac{1}{2}$, the Anſwer.

(20) Thus $3283\div23=142\frac{17}{23}$, the Anſwer.

(21) Thus $5423\div37=146\frac{21}{37}$, the Anſwer.

CASE V.

(22) $\frac{3}{4}$, $\frac{11}{12}$, and $\frac{7}{10}$

First $\left\{\begin{array}{l}3\times12\times10=360\\11\times\ 4\times10=440\\7\times12\times\ 4=336\end{array}\right\}$ New Numerators.

And $4\times12\times10=480$ Common Denominator.

Anſwer $\frac{360}{480}$, $\frac{440}{480}$, and $\frac{336}{480}$.

(23) First, $\frac{1}{6}$ of $\frac{7}{8}=\frac{7}{48}$. $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{7}{48}$.

Then $\left\{\begin{array}{l}1\times1\times5\times48=\ 960\\1\times3\times5\times48=\ 720\\1\times3\times4\times48=\ 576\\7\times5\times4\times\ 3=\ 420\end{array}\right\}$ New Numerators.

And $3\times4\times5\times48=2880$ Common Denominator.

Anſwer $\frac{960}{2880}$, $\frac{720}{2880}$, $\frac{576}{2880}$, and $\frac{420}{2880}$.

(24) $\frac{1}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{5}$ and $\frac{1}{6}$.

First $\left\{\begin{array}{l}1\times3\times4\times5\times6=360\\1\times2\times4\times5\times6=240\\1\times2\times3\times5\times6=180\\1\times2\times3\times4\times6=144\\1\times2\times3\times4\times5=120\end{array}\right\}$ New Numerators.

And $2\times3\times4\times5\times6=720$ Common Denominator.
Anſwer $\frac{360}{720}$, $\frac{240}{720}$, $\frac{180}{720}$, $\frac{144}{720}$, and $\frac{120}{720}$.

(25) Firſt $\frac{3}{4}$ of $\frac{3}{4}=\frac{9}{4}$ or $2\frac{1}{4}$.
Then we have the Fractions $\frac{5}{6}$, $\frac{7}{10}$, $\frac{3}{4}$, and $\frac{1}{4}$.

Now $\left\{\begin{matrix}5\times10\times\ 4\times4=800\\7\times\ 6\times\ 4\times4=672\\3\times\ 6\times10\times4=720\\1\times\ 4\times10\times6=240\end{matrix}\right\}$ New Numerators.

And $6\times10\times\ 4\times4=960$ Common Denominator.
Anſwer $\frac{800}{960}$, $\frac{672}{960}$, $\frac{720}{960}$, and $\frac{240}{960}$.

CASE VI.

(26) Thus $\frac{3}{4\times21}=\frac{1}{28}$, the Fraction required.

(27) Thus $\frac{1\times20\times12}{620}=\frac{12}{31}$, the Fraction required.

(28) Thus $\frac{3}{4\times4\times12\times27}=\frac{1}{1728}$, the Fraction required.

(29) Thus $\frac{5}{7\times12\times20}=\frac{1}{336}$, the Fraction required.

(30) Thus $\frac{1\times21\times12\times4}{1344}=\frac{1008}{1344}$ or $\frac{3}{4}$, the Fract. req.

(31) Thus $\frac{5}{6\times20\times12}=\frac{1}{288}$, the Fraction required.

(32) Thus $\frac{3\times112}{448}=\frac{3}{4}$, the Fraction required.

(33) Thus $\frac{3}{4\times16\times16\times112}=\frac{3}{114688}$, the Fraction req.

(34) Thus $\frac{5\times12\times20}{144}=\frac{25}{3}$, the Fraction required.

(35) Thus $\frac{7\times7\times8\times40}{9}=\frac{2240}{3}$, the Fraction required.

(36) Thus $\frac{7\times4\times4}{45}=\frac{112}{45}$, the Fraction required.

(37) Thus $\frac{11}{12\times63}=\frac{11}{756}$, the Fraction required.

(38) Thus $\frac{7\times48\times8}{9} = \frac{896}{3}$, the Fraction required.

(39) Thus $\frac{7\times36}{142} = \frac{126}{71}$, the Fraction required.

(40) Thus $\frac{3}{5\times8\times36} = \frac{1}{480}$, the Fraction required.

(41) Thus $\frac{7\times7\times24\times60\times60}{142} = \frac{2116800}{71}$, the Fraction required.

(42) Thus $\frac{1ø}{11\times60\times24} = \frac{1}{1584}$, the Fraction required.

CASE VII.

(43) 367 Moidores.

$9\times3=27$

1296)9909(7*s.*

Remains 837*s.*

×12

1296)10044(7*d.*

Remains 972*d.*

×4

1296)3888(3 Qrs.

Answer 7*s.* 7¾*d.*

(44) 5 Guineas.

=21

12)105

Answer 8*s.* 9*d.*

(45) 5*s.*

×12

8)60

Answer 7½

(46)

£.	s.
3	12
	4
5)14	8

Answer 2 17 7 ²

(47) 5£.

×20

7)100

Answer 14*s.* 3¼*d.* ⅔.

(48) *lb.*

411
×12

729)4932(6 oz.

Remains 612
×20

72,0)1224,0(17dwts.

504

Anſwer 6 oz. 17 dwts.

(49) *cwt.*

7
×4

9)28

Anſwer 3 qrs. 3 lb. 1 oz. $12\frac{4}{9}$ drs.

(50) *Tons.*

63
×20

124)1260(10 cwt.

Remains 20
×112

2240(18 lb.

1000

Remains 8
×16

124)128(10

Remains 4

Anſwer 10 C. 18lb. 10 $\frac{1}{31}$ oz.

(51)

7
×8

8)56

Anſwer 7 Furlongs.

(52) *Ell Eng.*

5
5

8)25

Anſwer $3\frac{1}{8}$ Qrs.

(53) *Acres.*

7
×4

8)28

Anſwer 3 Rds. 20 Pol.

(54) *Hhd. W.*

11
×63

252)693(2 Galls.

Remains 189
×4

252)756(3 Qts.

Anſwer 2 Galls. 3 Qts.

(55)

7 Ba. B.
×36

9)152

Anſwer 28 Galls.

(56) *Chal.*

25
× 36

72)900(12 Buſh.

Remains 36 Buſh.
× 4

72)144(2 Pecks.

Anſwer, 12 Buſh. 2 Pecks,

(57) *Months.*

7
× 4

8)28

Anſwer, Weeks 3. 3. 12.

(58) 5 *Days.*

× 24

7)120

Anſwer, Hours 17. 8. $34\frac{2}{7}$.

CASE VIII.

(59) Firſt 6*s.* $8\frac{1}{2}$*d.*=161 Halfpence and 1*l.*=480 ditto.
Then $\frac{161}{480}$ is the Fraction required.

(60) Firſt $4\frac{1}{2}$*d.*=9 Halfpence; and 1*s.*=24 Anſ $\frac{9}{24}$ or $\frac{3}{8}$.

(61) Firſt 2*l.* 17*s.* 7*d* $\frac{1}{5}$=3456 Fifths. Anſwer $\frac{3456}{4320}$ or $\frac{4}{5}$.
And 3*l.* 12*s.*— =4320 ditto.

(62) Firſt 8*s.* 2*d.*=98 Pence. Anſwer $\frac{98}{258}$ or $\frac{7}{18}$.
And 1*l.* 1*s.*=252 ditto.

(63) Firſt 6 oz. 17 dwts. 21 grs.=3309 grs.
And 1 lb. =5760 Anſ. $\frac{3309}{5760}$ or $\frac{1103}{1920}$.

(64) Firſt 3 qrs. 3 lb. 1 oz. $12\frac{5}{9}$=200705 Ninths.
And 1 Cwt. =258048. Anſ. $\frac{200705}{258048}$.

(65) Firſt 10 Cwt. 18 lb. $1\frac{1}{31}$ oz.=564480 or $\frac{63}{124}$ Anſwer.
And 1 Ton. =1111040

(66) Firſt 3 Qrs. $\frac{1}{8}$=25 Eighths. or $\frac{5}{8}$, the Anſwer.
And 1 Ell. Eng.=40

(67) Firſt 2 Feet 6 Inch.=30 Inches. or $\frac{5}{6}$, the Anſwer.
And 1 Yard =36

(68) Firſt 4 Fur. 32 p.=192 Poles. or $\frac{3}{5}$, the Anſwer.
And 1 Mile =320

(69) Firſt 3 Roods 2 p.=122 Poles or $\frac{61}{80}$, the Anſwer.
And 1 Acre =160

(70) Thus $\frac{42}{63}$, or $\frac{2}{3}$, the Fraction required.

(71) Thus $\frac{28}{36}$ or $\frac{7}{9}$, the Fraction required.

(72) Firſt 14 Buſh. 2 Pks.=58 Pecks. or $\frac{29}{72}$, the Anſwer.
And a Chaldron =144

(73) Firſt 1 W. 3 d. 12 h.=252 Hours. or $\frac{3}{8}$, the Anſwer.
And 1 Month. =672

(74) Firſt 243 D. 8. h.=5840 Hours. or $\frac{2}{3}$, the Anſwer.
And 1 Year =8760

39. ADDITION of VULGAR FRACTIONS.

(1) Thus $\frac{3}{7}+\frac{1}{4}+\frac{5}{6}=\frac{72}{168}+\frac{42}{168}+\frac{140}{168}+\frac{254}{168}$ or $1\frac{43}{84}$ Anſ.

(2) Thus $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=\frac{360}{720}+\frac{480}{720}+\frac{540}{720}+\frac{576}{720}+\frac{600}{720}=\frac{2556}{720}$ or $3\frac{11}{20}$, the Sum required.
Firſt $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$.

(3) Then $\frac{8}{15}+\frac{13}{15}=\frac{21}{15}$ or $1\frac{2}{5}$, the Sum required.

(4) Firſt $\frac{3}{6}$ of $\frac{5}{6}=\frac{5}{14}$, and $\frac{5}{9}$ of $\frac{3}{8}=\frac{5}{12}$.
Then $\frac{5}{14}+\frac{5}{12}=\frac{60}{168}+\frac{70}{168}=\frac{130}{168}$ or $\frac{65}{84}$, the Sum required.

(5) Firſt $\frac{1}{2}$ of $\frac{5}{6}=\frac{1}{6}$, (ſee Note Caſe II.)
Then $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}=\frac{72}{144}+\frac{48}{144}+\frac{36}{144}+\frac{24}{144}=\frac{180}{144}$ or $1\frac{1}{4}$ the Sum required.

(6) Firſt $\frac{1}{2}+\frac{3}{8}=\frac{4}{8}+\frac{3}{8}$ or $\frac{2}{4}+\frac{3}{8}=\frac{4}{4}$ or $1\frac{1}{4}$. Then $4+17+1\frac{1}{4}=22\frac{1}{4}$, the Sum required.

(7) Firſt $\frac{2}{3}+\frac{4}{5}+\frac{7}{10}=\frac{100}{150}+\frac{120}{150}+\frac{105}{150}=\frac{325}{150}$ or $2\frac{1}{6}$. Then $7+9+6+2\frac{1}{6}=24\frac{1}{6}$, the Sum required.

(8) Firſt $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=\frac{360}{720}+\frac{480}{720}+\frac{540}{720}+\frac{576}{720}+\frac{600}{720}=\frac{2556}{720}$ or $3\frac{11}{20}$.
Then $8+9+10+11+12+3\frac{11}{20}=53\frac{11}{20}$, the Sum req.

(9) Firſt $\frac{1}{8}$ of $1s.=\frac{1}{160}$ of £1. per Caſe VI.
Then $\frac{201}{460}+\frac{1}{160}=\frac{32160}{73600}+\frac{460}{73600}=\frac{32620}{73600}$ or $\frac{1631}{3680}$£. which by Caſe VII.=8*s.* $10\frac{1}{4}$*d.* $\frac{1}{23}$.

(10) Firſt $\frac{3}{8}$ of an oz.=$\frac{3}{96}$ or $\frac{1}{32}$ of a lb.
Then $\frac{1}{4}+\frac{1}{32}=\frac{32}{128}+\frac{4}{128}=\frac{36}{128}=\frac{9}{32}$lb. or 3 oz. 7 dwts. 12 grs. the Sum required.

(11) Firſt $\frac{1}{8}$ of a lb.=$\frac{1}{336}$ of a Cwt.
Then $\frac{5}{8}+\frac{1}{336}=\frac{1680}{2688}+\frac{8}{2688}=\frac{1688}{2688}=\frac{211}{336}$ Cwt. or 2 qrs 14 lb. 5 oz. $5\frac{1}{3}$ drs. the Anſwer.

(12) Firſt $\frac{2}{5}$ of an Ell Eng.=$\frac{2}{4}$ of a Yard.
Then $\frac{3}{4}+\frac{2}{4}=\frac{5}{4}$ yd. or 1 yd. 1 qr. the Sum required.

(13) First $\frac{2}{4}$ of a Yard $= \frac{2}{3\,2\,8\,0}$ or $\frac{1}{2\,6\,4\,0}$ of a Mile.
Then $\frac{5}{6} + \frac{1}{3\,6\,0} = \frac{1\,3\,0\,0\,5}{1\,5\,8\,4\,0} + \frac{6}{1\,5\,8\,4\,0} = \frac{1\,3\,2\,0\,6}{1\,5\,8\,4\,0} = \frac{4\,2\,0\,1}{2\,6\,4\,0}$ Mile.
or 6 Fur. 26 p. 4 yds. 1 ft. the Sum required.

(14) First $\frac{1}{9}$ of a Peck $= \frac{1}{2\,8\,8}$ of a Chaldron.
Then $\frac{3}{8} + \frac{1}{2\,8\,8} = \frac{8\,5\,4}{2\,1\,6\,0} + \frac{8}{2\,1\,6\,0} = \frac{8\,6\,2}{2\,1\,6\,0} = \frac{1\,0\,9}{2\,8\,8}$ or 13 Bush. 2 pks. 1 gall. the Sum required.

(15) First $\frac{3}{4}$ of a Week $= \frac{3}{16}$ of a Month.
Then $\frac{3}{16} + \frac{5}{6} = \frac{18}{96} + \frac{80}{96} = \frac{98}{96} = \frac{49}{48}$ Mon. or 1 m. 14 hrs. the Sum required.

(16) First $\frac{3}{4}$ of an Hour $= \frac{3}{6\,7\,2}$ or $\frac{1}{2\,2\,4}$ of a Week.
Then $\frac{1}{224} + \frac{5}{8} = \frac{8}{1792} + \frac{1120}{1792} = \frac{1128}{1792}$ or 4 D. 9h. 45 m. the Sum required.

(17) First $\frac{3}{5}$ of $12\mathcal{L}. = \frac{24}{5}$ or $4\frac{4}{5}\mathcal{L}.$ and $\frac{1}{8}$ of $\frac{9}{10} = \frac{9}{80}$.
Also $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{1}{20} = \frac{1}{40}$.
Then $\frac{4}{5} + \frac{3}{7} + \frac{9}{80} + \frac{1}{40} = \frac{860000}{700000} + \frac{300000}{700000} + \frac{122400}{700000} + \frac{17850}{700000}$
$= \frac{1003250}{700000}$ or $1\frac{303250}{700000}$.
Then $4+4+1\frac{303250}{700000} = 9\frac{303250}{700000}\mathcal{L}.$ or $9\mathcal{L}.\ 8s.\ 8d.\ \frac{2}{35}$, the Sum required.

40. SUBTRACTION OF VULGAR FRACTIONS.

(1) Thus $\frac{11}{12} - \frac{3}{8} = \frac{88}{96} - \frac{24}{96} = \frac{75}{108}$, or $\frac{25}{36}$, the Diff. required.
(2) Thus $\frac{19}{65} - \frac{2}{13} = \frac{247}{845} - \frac{130}{845} = \frac{117}{845}$, the Diff. required.
(3) Thus $\frac{14}{17} - \frac{7}{11} = \frac{154}{187} - \frac{119}{187} = \frac{35}{187}$, the Diff. required.
(4) First $\frac{3}{4}$ of $\frac{7}{8} = \frac{21}{32}$.
Then $\frac{21}{32} - \frac{2}{5} = \frac{105}{160} - \frac{64}{160} = \frac{41}{160}$, the Difference required.
(5) First $\frac{5}{6}$ of $\frac{7}{10} = \frac{7}{12}$, and $\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3}$.
Then $\frac{7}{12} - \frac{1}{3} = \frac{21}{36} - \frac{12}{36} = \frac{9}{36}$ or $\frac{1}{4}$, the Diff. required.
(6) Thus $10\frac{3}{4} - 6\frac{5}{6} = 10\frac{18}{24} - 6\frac{20}{24} = 3\frac{22}{24}$ or $3\frac{11}{12}$, the Diff.
(7) Thus $17\frac{11}{12} - 16\frac{5}{7} = 17\frac{77}{84} - 16\frac{72}{84} = 1\frac{5}{84}$, the Diff.
(8) First $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{3}{4} = \frac{35}{64}$.
Then $12 - \frac{35}{64} = 11\frac{29}{64}$, the Difference.
(9) First $\frac{3}{4}$ of a Penny $= \frac{3}{48}$ or $\frac{1}{16}$ of 1*s*. per Case VI.
Then $\frac{2}{3} - \frac{1}{16} = \frac{22}{48} - \frac{3}{48} = \frac{19}{48}s.$ or $7\frac{1}{2}d.$ per Case VII. Anſ.
(10) First $\frac{3}{4}$ of an oz. $= \frac{3}{7168}$ of a Cwt.
Then $\frac{1}{3} - \frac{3}{7168} = \frac{7168}{21504} - \frac{9}{21504} = \frac{7159}{21504}$ C. or 1 qr. 9 lb. 4 oz. $9\frac{1}{8}$ drs. the Difference required.
(11) First $\frac{3}{8}$ of an Inch $= \frac{3}{288}$ or $\frac{1}{96}$ of a Yard.
Then $\frac{3}{4} - \frac{1}{96} = \frac{288}{384} - \frac{4}{384} = \frac{284}{384} = \frac{71}{96}$ Yd. or 2 ft. $2\frac{5}{8}$ in. the Difference required.

(12) First $\frac{3}{4}$ of a Peck $= \frac{3}{4}\frac{0}{8}$ or $\frac{1}{192}$ of a Chaldron.
Then $\frac{3}{8} - \frac{1}{192} = \frac{570}{1846} - \frac{8}{1536} = \frac{568}{1536} = \frac{71}{192}$ Chal. or 13 Bush. 1 Pk. 2 Qts. the Difference.

(13) First $\frac{3}{4}$ of an Hour $= \frac{3}{60}$ or $\frac{1}{20}$ of a Day.
Then $\frac{5}{6} - \frac{1}{32} = \frac{160}{192} - \frac{6}{192} = \frac{154}{192}$ or $\frac{77}{96}$ Day; or 19 h. 15 min. the Difference.

MULTIPLICATION OF VULGAR FRACTIONS.

(1) Thus $\frac{6}{11} \times \frac{7}{12} = \frac{7}{22}$ (see Note in Case II.) the Product required.

(2) Thus $12\frac{1}{3} \times 6 = 75$, the Product required.

(3) First $17\frac{3}{8} = \frac{88}{8}$.
Then $\frac{88}{8} \times \frac{7}{8} = \frac{77}{8}$ or $15\frac{3}{8}$, the Product required.

(4) First $2\frac{3}{4} = \frac{11}{4}$, and $\frac{3}{4}$ of $\frac{7}{8} = \frac{21}{32}$.
Then $\frac{11}{4} \times \frac{21}{32} = \frac{231}{128}$ or $1\frac{103}{128}$, the Product required.

(5) First $12\frac{3}{4} = \frac{51}{4}$, and $\frac{8}{7}$ of $\frac{18}{1} = \frac{21}{2}$.
Then $\frac{51}{4} \times \frac{21}{2} = \frac{1071}{8}$ or $133\frac{7}{8}$, the Product required.

(6) First $\frac{3}{4}$ of $\frac{10}{11} = \frac{15}{22}$.
Then $\frac{15}{22} \times \frac{6}{1} = \frac{45}{11}$ or $4\frac{1}{11}$, the Product required.

(7) First $\frac{7}{8}$ of $\frac{3}{4} = \frac{21}{32}$ and $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{14}{1} = \frac{4}{1}$.
Then $\frac{21}{32} \times \frac{4}{1} = \frac{21}{8}$, or $2\frac{5}{8}$, the Product.

(8) First $3\frac{2}{3} = \frac{11}{3}$
Then $\frac{11}{3} \times \frac{1}{7} = \frac{11}{21}$. Also $\frac{3}{5}$ of $\frac{3}{4} = \frac{9}{20}$.
Again $\frac{11}{21} \times \frac{9}{20} = \frac{33}{140}$, the Product required.

42. DIVISION OF VULGAR FRACTIONS.

(1) Thus $\frac{6}{7} \div \frac{3}{5}$ or $\frac{6}{7} \times \frac{5}{3} = \frac{10}{7}$ or $1\frac{3}{7}$, the Quotient required.

(2) First $\frac{2}{3}$ of $\frac{7}{8} = \frac{7}{12}$.
Then $\frac{2}{11} \div \frac{7}{12}$ or $\frac{2}{11} \times \frac{12}{7} = \frac{24}{77}$, the Quotient required.

(3) First $12\frac{1}{3} = \frac{25}{3}$, and $17\frac{2}{3} = \frac{53}{3}$.
Then $\frac{25}{2} \div \frac{53}{3}$ or $\frac{25}{2} \times \frac{3}{53} = \frac{75}{106}$, the Quotient required.

(4) First $12\frac{3}{4} = \frac{51}{4}$, and $3\frac{7}{8} = \frac{31}{8}$,
Then $\frac{51}{4} \div \frac{31}{8}$ or $\frac{51}{4} \times \frac{8}{31} = \frac{102}{31}$ or $3\frac{9}{31}$, the Quotient req.

(5) First $\frac{7}{8}$ of $\frac{3}{4} = \frac{21}{32}$, and $\frac{2}{5}$ of $\frac{12}{1} = \frac{24}{5}$.
Then $\frac{21}{32} \div \frac{24}{5}$ or $\frac{21}{32} \times \frac{5}{24} = \frac{35}{256}$, the Quotient required.

(6) Firſt $7\frac{2}{7}=\frac{51}{7}$, then $\frac{51}{7}\div\frac{9}{1}$ or $\frac{51}{7}\times\frac{1}{9}=\frac{17}{21}$, the Quot.

(7) Thus $\frac{96}{1}\div\frac{146}{1}$ or $\frac{96}{1}\times\frac{1}{146}=\frac{48}{73}$, the Quot. required.

(8) Firſt $14\frac{3}{7}=\frac{101}{7}$, and $\frac{3}{1}$ of $\frac{12}{1}=\frac{9}{1}$.
Then $\frac{101}{7}\div\frac{9}{1}$ or $\frac{101}{7}\times\frac{1}{9}=\frac{101}{63}$ or $1\frac{38}{63}$, the Quot. req.

(9) Firſt $142\frac{7}{12}=\frac{1711}{12}$, and $12\frac{3}{5}=\frac{63}{5}$.
Then $\frac{1711}{12}\div\frac{63}{5}$ or $\frac{1711}{12}\times\frac{5}{63}=\frac{8555}{756}$ or $11\frac{239}{756}$, the Quotient required.

(10) Firſt $\frac{7}{8}$ of $\frac{6}{7}=\frac{21}{4}$, and $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{11}{12}=\frac{33}{56}$.
Then $\frac{21}{4}\div\frac{33}{56}$ or $\frac{21}{4}\times\frac{56}{33}=\frac{1176}{132}$ or $8\frac{10}{11}$, the Quot. req.

If the Diviſor and Dividend have both the ſame Denominator, the Quotient may be found by dividing one Numerator by the other.

Thus $\frac{7}{11}\div\frac{4}{11}$; thus $7\div4=\frac{4}{7}$ the Quotient required.

2. If the Diviſor and Dividend have each the ſame Numerator, divide one of the Denominators by the other, which will give the Quotient required.

Divide $\frac{8}{17}$ by $\frac{8}{11}$; thus $17\div11=\frac{11}{17}$ the Quotient.

3. If a Number can be found that will divide both the Numerators, or both the Denominators, (viz. thoſe of the Diviſor and Dividend) without a Remainder; uſe thoſe Quotients inſtead of the given Numerators and Denominators, which will give the reſult in its loweſt Terms.

Divide $\frac{8}{15}$ by $\frac{9}{16}$; thus $\frac{8}{15}\div\frac{9}{16}=\frac{1}{5}\div\frac{3}{2}=\frac{3}{10}$, the Quotient required.

For 8 and 16 divide by 8, alſo 15 and 9 divide by 3.

43. THE RULE OF THREE DIRECT IN VULGAR FRACTIONS.

(1) If $2\frac{2}{3}$ yds. : $3\frac{3}{4}$*l.* . $4\frac{4}{5}$ yds. Or
As $1\frac{2}{3}$: $1\frac{5}{4}$: : $2\frac{4}{5}$: $1\frac{5}{2}$*l.*
For $\left\{\begin{matrix}2+\times15\times5\\12\times\ 4\times5\end{matrix}\right\}=1\frac{5}{4}$*l.* or 7*l.* 10*s.* the Anſwer.

(2) Firſt 5*s.* 6*d.*$=\frac{11}{40}$*l.* Then
As $\frac{3}{4}$: $\frac{11}{40}$: : $33\frac{3}{8}$: $12\frac{43}{80}$*l.*
For $\left\{\begin{matrix}339\times11\times4\\3\times40\times8\end{matrix}\right\}=12\frac{43}{80}$*l.* or 15*l.* 10*s.* 9*d.* the An.

(3) Firſt 14*s.* 8*d* or $14\frac{2}{3}=\frac{11}{15}$*l.* Then
As $\frac{7}{8}$: $\frac{11}{15}$: : $\frac{8}{1}$: $2\frac{64}{77}$.

For $\left\{\begin{matrix}9\times11\times8\\7\times15\times1\end{matrix}\right\}=\frac{264}{35}$*l.* or 7*l.* 10*s.* $10\frac{1}{4}$*d.* $\frac{1}{7}$ the Anſ.

(4) Firſt $10\frac{1}{2}=\frac{21}{2}$*l.* and $2700\frac{1}{3}=\frac{5401}{2}$
As $\frac{100}{1}:\frac{21}{2}::\frac{5401}{2}:\frac{113421}{400}$*l.*
For $\left\{\begin{matrix}5401\times21\\100\times2\times2\end{matrix}\right\}=\frac{113421}{400}$*l.* or 283*l.* 11*s.* $0\frac{1}{2}$*d.* $\frac{3}{5}$ the Anſwer

(5) Firſt $112\frac{5}{8}=\frac{901}{8}$*l.* Then
As $\frac{901}{8}:\frac{100}{1}::\frac{1270}{1}:\frac{1016000}{901}$.
For $\left\{\begin{matrix}1270\times8\times100\\901\times1\times1\end{matrix}\right\}=\frac{1016000}{901}$ or 1127*l.* 12*s.* $8\frac{1}{2}$*d.* $\frac{470}{901}$, the Anſwer.

(6) Firſt $4\frac{3}{4}=\frac{19}{4}$ Piece[s] And $22\frac{3}{4}=\frac{179}{8}$ Yards in each Piece. Then $\frac{179}{8}\times\frac{19}{4}=\frac{3401}{32}$ Yards bought. Alſo 8*s.* 9*d.* $=\frac{7}{10}$*l.* Therefore,
As $\frac{1}{1}:\frac{7}{10}::\frac{3401}{32}:\frac{23807}{320}$.
For $\left\{\begin{matrix}3401\times7\\32\times16\end{matrix}\right\}=\frac{23807}{512}$*l.* or 46*l.* 9*s.* $11\frac{1}{2}$*d.* $\frac{1}{8}$ the Anſwer.

(7) Firſt 100*l.* 10*s.* 6*d.* $=\frac{4021}{40}$*l.* and 5*s.* $6\frac{3}{4}$*d.* $=\frac{39}{320}$*l.*
Alſo 1 Ell Fl. $=\frac{3}{5}$ Ell Eng. Therefore,
As $\frac{8039}{320}:\frac{3}{5}::\frac{4021}{40}:\frac{965504}{4435}$.
For $\left\{\begin{matrix}4021\times320\times3\\89\times5\times40\end{matrix}\right\}=\frac{965504}{4435}$ Ells Eng.
Now $\frac{965504}{4435}\div\frac{12}{1}=\frac{80042}{4435}$ or $18\frac{32}{443}$ Ells Eng. in each Piece.

44. THE RULE OF THREE INVERSE.

(1) Firſt $25\frac{1}{4}=\frac{76}{3}$*l.* and $6^{3}=\frac{27}{4}$ Months.
Alſo $10^{3}=\frac{43}{4}$*l.* Then
As $\frac{76}{3}:\frac{27}{4}::\frac{43}{4}:\frac{6843}{43}$ M.
For $\left\{\begin{matrix}76\times27\times4\\43\times4\times3\end{matrix}\right\}=\frac{6843}{43}$ M. or 15 M. $27\frac{9}{43}$ Days, the Anſwer.

(2) Firſt $12\frac{3}{4}=\frac{51}{4}$ Hours. Then
As $\frac{4}{1}:\frac{51}{4}::\frac{12}{1}:\frac{51}{12}$ or $\frac{17}{4}$ Hours,
For $\left\{\begin{matrix}51\times4\\12\times4\end{matrix}\right\}=\frac{17}{4}$ Hours, or 4 Hours, the Anſwer.

(3) Firſt $12\frac{2}{3}=\frac{38}{8}$ Oz and $8\frac{3}{4}=\frac{35}{4}$ ditto. Then
As $\frac{18}{8}:\frac{4}{1}::35:\frac{21}{152}$*s.*
For $\left\{\begin{matrix}38\times5\times4\\35\times3\times1\end{matrix}\right\}=\frac{21}{152}$ or 7*s.* $2\frac{3}{4}$*d.* $\frac{4}{7}$ the Anſwer.

(4) First $100\frac{2}{3}=\frac{302}{3}$£. and $6\frac{4}{5}=\frac{20}{3}$ M. or $\frac{20}{3}$ of $\frac{1}{12}=\frac{5}{9}$ Yr. Alſo $3\frac{5}{8}=\frac{23}{8}$ Yrs. Then

As $\frac{5}{9}$: $\frac{302}{3}$ :: $\frac{23}{8}$: $\frac{30820}{207}$.

For $\left\{\begin{matrix}302\times5\times6\\23\times3\times9\end{matrix}\right\}=\frac{30820}{207}$£. or 14£. 11*s*. $9\frac{1}{4}$*d*. $\frac{55}{414}$, the Anſwer.

(5) First $8\frac{1}{2}=\frac{17}{2}$*s*. and $26\frac{5}{8}=\frac{213}{8}$ Yards. Alſo 5*s*. 7*d*.$=\frac{17}{12}$*s*. Then

As $\frac{17}{12}$: $\frac{213}{8}$ :: $\frac{17}{2}$: $\frac{14823}{810}$.

For $\left\{\begin{matrix}213\times67\times2\\12\times17\times8\end{matrix}\right\}=\frac{47847}{8772}$ Yds. or $17\frac{1323}{2924}$ Yards, the Anſwer.

45. THE DOUBLE RULE of THREE.

(1) First 4£. 15*s*.$=4\frac{3}{4}$£. or $\frac{19}{4}$£. and 9 M.$=\frac{3}{4}$ Yr. Then

100£. : 1 Yr. : 6£.	Now $\frac{3}{4}\times\frac{6}{1}=\frac{9}{2}$ Diviſor.
—: $\frac{3}{4}$: $\frac{19}{4}$	And $\frac{100}{1}\times\frac{19}{4}=\frac{475}{1}$ Dividend.

Then $\frac{475}{1}\div\frac{9}{2}=\frac{950}{9}$£. or 105£. 11*s*. $1\frac{1}{4}$*d*. $\frac{1}{9}$, the Anſwer.

Or by two Statings, thus:

1ſt. As 1 : 100 :: $\frac{3}{4}$: $\frac{400}{3}$.

2d. As $\frac{6}{1}$: $\frac{400}{3}$ :: $\frac{19}{4}$: $\frac{950}{9}$ or 105£. 11*s*. $1\frac{1}{4}$*d*.$\frac{1}{9}$

(2) First 14£. 6*s*. 8*d*.$=14\frac{1}{3}$ or $\frac{43}{3}$£. Then

12 Stu. : 16 D. : $\frac{43}{3}$£.	Now $12\times16=192$ Diviſor.
18 : 34 : —	And $\frac{43}{3}\times\frac{34}{1}\times\frac{18}{1}=\frac{8772}{1}$ Divid.

Then $8772\div192=45$£. 13*s*. 9*d*. the Anſwer.

Or thus, 1ſt. As $\frac{12}{1}$: $\frac{43}{3}$ :: $\frac{18}{1}$: $\frac{43}{2}$£.

2d. As $\frac{18}{1}$: $\frac{43}{2}$ :: $\frac{34}{1}$: $\frac{731}{16}$ or 45£. 13*s*. 9*d*.

(3) First 16£. 13*s*. 4*d*.$=16\frac{2}{3}$ or $\frac{50}{3}$£. And 6£. 17*s*. 6*d*.$=6\frac{7}{8}$ or $\frac{55}{8}$£. Then

40 Cwt. : 30 M. $\frac{50}{3}$£.	Now $\frac{50}{3}\times\frac{80}{1}=\frac{4000}{3}$ Diviſor.
— : 80 : $\frac{55}{8}$	And $\frac{40}{1}\times\frac{30}{1}\times\frac{55}{8}=\frac{8250}{1}$ Div.

Then $\frac{8250}{1}\div\frac{4000}{3}=\frac{99}{16}$ Cwt. or 6 Cwt. 21 lbs. the Anſ.

Or, thus, 1ſt. As 30 : 40 :: 80 : 15 Cwt.

2d. As $\frac{50}{3}$: $\frac{15}{1}$ :: $\frac{55}{8}$: $\frac{99}{16}$ or 6 Cwt. 21 lbs.

(4) First 26£. 19*s*. 4*d*.$=26\frac{29}{30}$ or $\frac{809}{30}$£. And 14£. 15*s*.$=14\frac{3}{4}$ or $\frac{59}{4}$£.

12 Per. 3 M. : $\frac{809}{30}$£.	Now $\frac{809}{30}\times\frac{36}{1}=\frac{4854}{5}$ Diviſor.
36 : — : $\frac{59}{4}$	And $\frac{59}{4}\times\frac{12}{1}\times\frac{3}{1}=\frac{531}{1}$ Divid.

Then $553\frac{1}{10} \div 36\frac{3}{5} = \frac{24455}{1854}$ M. or $16\frac{3331}{8089}$ Days, the Anſwer.
Or thus, 1ſt. As 12 : 3 :: 36 : 1 Month.
2d. As $\frac{809}{30} : \frac{1}{1} :: \frac{5}{4} : \frac{884}{1618}$ Mo. or $16\frac{3331}{8089}$ Days.

(5) 30 M. : 11 D. 1 W. | Now $1\frac{1}{2} \times \frac{1}{4} = 1\frac{1}{8}$ Diviſor.
— : $1\frac{1}{2}$: 4 | And $30 \times 11 \times 4 = 1320$ Divid.
Then $\frac{1320}{1} \div 1\frac{1}{8} = \frac{6600}{11}$ or 600 Men the Anſwer.
Or thus, 1ſt. As 1 : 30 :: 4 : 120 Men.
2d. As 11 : 120 :: $1\frac{1}{8} : \frac{6600}{11}$ or 600 Men.

(6) Firſt $2\frac{1}{2} = \frac{5}{2}$ Tons. And $3 - \frac{1}{10} = 2\frac{9}{10}$ or $\frac{29}{10}$ Miles. Alſo $\frac{1}{8}$ of $\frac{3}{5}$ of $2\frac{1}{2} = \frac{63}{40}$ s. Likewiſe 1 Cwt. $= \frac{1}{20}$ Ton.
Then
$\frac{5}{2}$ T. : $\frac{29}{10}$ M. : $\frac{63}{40}$ s. | Now $\frac{29}{10} \times \frac{5}{2} = \frac{29}{4}$ Diviſor.
$\frac{1}{20}$: 1 : — | And $\frac{63}{40} \times \frac{1}{20} = \frac{63}{800}$ Dividend.
Then $\frac{63}{800} \div \frac{29}{4} = \frac{252}{23200}$ s. or $\frac{378}{725}$ Qr. the Anſwer.
Or thus, 1ſt. As $\frac{5}{2} : \frac{63}{40} :: \frac{1}{20} : \frac{63}{2080}$.
2d. As $\frac{29}{10} : \frac{63}{2000} :: \frac{1}{1} : \frac{252}{23240}$ s. or $\frac{378}{725}$ Qr.

46. QUESTIONS IN VULGAR FRACTIONS.

(1) Thus $99\frac{9}{9}$ the Anſwer. For $\frac{9}{9} = 1$, and $99 + 1 = 100$

(2) Firſt $\frac{3}{10}$ of $\frac{18}{7}$ of $\frac{141}{213} = \frac{1269}{4970}$.
$\therefore 1 - \frac{1269}{2485} = \frac{1216}{2485}$ Anſw. For $\frac{1269}{2485} + \frac{1216}{2485} = 1$, Proof.

(3) Firſt $\frac{1}{25}$ of $\frac{7}{8} = \frac{7}{200}$ and $\frac{1}{16}$ of $\frac{47}{19} = \frac{47}{304}$.
$\therefore 3\frac{7}{200} - \frac{47}{304} = 3\frac{2126}{60400} - \frac{9400}{60800} = 2\frac{58538}{60800}$ or $2\frac{4691}{7600}$ the Anſwer.
For $2\frac{58538}{60800} - \frac{58138}{60800} = 2\frac{51400}{60800}$ which added to $\frac{9400}{60800} = 3$, the Proof.

(4) Firſt $\frac{1}{11}$ of $\frac{12}{1} = \frac{12}{11}$, and $\frac{1}{19}$ of $\frac{27}{1} = \frac{27}{19}$.
Then $\frac{12}{11} + \frac{27}{19} = \frac{228}{209} + \frac{297}{209} = \frac{525}{209} = 2\frac{107}{209}$.
Again $7\frac{1}{2} = \frac{15}{2}$; then $\frac{1}{4}$ of $\frac{15}{2} = \frac{5}{8}$; alſo $1\frac{1}{4} = \frac{5}{4}$, and $\frac{20}{30}$ of $\frac{5}{4} = \frac{29}{24}$; then $\frac{5}{8} - \frac{29}{24} = \frac{31}{24}$ or $1\frac{7}{24}$, which added to $8 = 9\frac{7}{24}$.
$\therefore 9\frac{7}{24} - 2\frac{107}{209} = 9\frac{1463}{5016} - 2\frac{2508}{5016} = 6\frac{3911}{5016}$, the Anſwer.

(5) Firſt $\frac{3}{4}$ of $\frac{1}{8}$ of $\frac{11}{1} = \frac{33}{128}$.
$\therefore \frac{1}{4} \div \frac{33}{128} = \frac{128}{132}$ the Number; therefore, $\frac{128}{132} \times \frac{128}{132} \times \frac{128}{132} = \frac{2097152}{2299968}$, the Cube of that Number.

(6) First $\frac{3}{5}$ of $\frac{3}{16}=\frac{1}{8}$, and $9\frac{38}{37}=\frac{361}{37}$.
$\therefore \frac{361}{37}\times\frac{1}{8}=\frac{361}{296}$, the Number.
And $\frac{361}{296}\times\frac{361}{296}=\frac{130321}{87616}$ or $\frac{142705}{87616}$ Sqr.

(7) First $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}=\frac{3}{10}$; alſo $\frac{1}{9}$ of $\frac{4}{7}$ of $\frac{11}{13}=\frac{22}{819}$.
Then, as $\frac{3}{10}$: $\frac{22}{819}$ :: $\frac{12000}{1}$: $\frac{8800000}{273}=3223\frac{1}{273}$. 8*s*. $10\frac{1}{4}$*d*. $\frac{45}{91}$, the Value of the Ship.
$\therefore$ 12000+3223$\frac{1}{273}$. 8*s*. $10\frac{1}{4}$*d*. $\frac{45}{91}$=15223$\frac{1}{273}$. 8*s*. $10\frac{1}{4}$*d*. $\frac{45}{91}$, the Anſwer.

(8) First $\frac{3}{8}=\frac{9}{24}$ ſold. Then, as $\frac{9}{24}$: $\frac{1710}{1}$ :: $\frac{20}{20}$ or $\frac{1}{1}$: $\frac{11400}{3}=3800$. the Anſwer.

(9) His whole Eſtate $\frac{83}{83}-\frac{34}{83}=\frac{49}{83}$. Then $\frac{34}{83}$ of $\frac{49}{83}=\frac{1666}{6889}$ youngeſt Son's Part; and $\frac{34}{83}=\frac{2822}{6889}$ eldeſt.
$\therefore \frac{2822}{6889}-\frac{1666}{6889}=\frac{1156}{6889}=157$. 3*s*. 4*d*. or $157\frac{3}{6}$.
Alſo $\frac{2822}{6889}+\frac{1666}{6889}=\frac{4488}{6889}$ both the Son's Part.
So that $\frac{6889}{6889}-\frac{4488}{6889}=\frac{2401}{6889}$, Widow's Part.
Then by rejecting the Common Denominator we ſhall have the following Proportion.

		l.	*s*.	*d*.
As 1156 : $154\frac{3}{6}$: : 2401 : $370\frac{4743}{6936}=534$		2		$7\frac{3}{4}\frac{588}{3468}$
Widow had the Uſe of.				
As 1156 : $154\frac{3}{6}$: : 2822 : $217\frac{1173}{3468}=627$		15		$9\frac{1}{4}\frac{2244}{3468}$
eldeſt Son had.				
As 1156 : $154\frac{3}{6}$: : 1666 : $123\frac{531}{3468}=370$		15		$4\frac{198}{3468}$
His whole Eſtate	1532	13		$9\frac{3024}{3468}$

(10) First $\frac{19}{30}-\frac{11}{19}=\frac{741}{1170}-\frac{330}{1170}=\frac{411}{1170}$.
Then, As $\frac{411}{1170}$: $108\frac{1}{2}$. : : $\frac{1170}{1}$ or : $\frac{1}{1}$: $632\frac{385}{411}=153\frac{8}{411}$.
12*s*. $11\frac{3}{4}$*d*. $\frac{267}{411}$, the Anſwer.

(11) First 4000 his whole Stock.
Then $\frac{5}{6}$ of $\frac{2}{3}$ of 4000 = $1666\frac{2}{3}$ Mad Tom took.

$2333\frac{1}{3}=\frac{7000}{3}$ left.
$\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{7000}{3}$ = $583\frac{1}{3}$=Raving Ned took.

1750 left.
$\frac{7}{10}$ of $\frac{17}{20}$ of 1750 = $1041\frac{1}{4}$ Poſitive Jack took.

$708\frac{3}{4}=708\frac{48}{64}$ left.
$\frac{3}{4}$ of $\frac{1}{4}$ of $2835\frac{4}{4}$ = $132\frac{57}{64}$ Dolly had.

Anſwer $575\frac{55}{64}$ left.

(12) As $\frac{4}{15}$: 2000 :: $\frac{1}{2}\frac{2}{3}$ or : $\frac{1}{1}$ = 5280*l*. Brother's Fortune.
And $5280 \times 3\frac{1}{3} = 16500$; alſo $1\frac{1}{3} = \frac{3}{2}$.
$\therefore 16500 \div \frac{3}{2} = \frac{33000}{3} = 11000$*l*. Father's Part.

(13)

	l.	*s*.
First 17 Hhds. at 34*l*. each = 34×17 =	578	0
As $\frac{4}{5} : \frac{57}{1} :: \frac{7}{7}$ or $\frac{1}{1} : \frac{4048}{4}$ =	1011	10
13 Guns at 18*l*. 10*s*. each = 18*l*. $10s. \times 13$ =	240	10
Value of what was caſt overboard	1830	0

Then $\frac{3}{7}$ of $\frac{9}{13} = \frac{27}{91} = 1830$*l*. and $\frac{27}{91} + \frac{64}{91} = \frac{91}{91}$.
$\therefore$ As $\frac{27}{91} : \frac{1830}{1} :: \frac{64}{91} : \frac{4771\frac{20}{27}}{27} = 4337$*l*. 15*s*. $6\frac{2}{3}d$. the Value which came into Port.

(14) Firſt $1613\frac{17}{10} = \frac{16131}{10}$.
Then $\frac{7}{8}$ of $\frac{3}{5}$ of $\frac{1}{2}$ of $\frac{16131}{10} = \frac{338751}{800} = 423$*l*. 11*s*. $5\frac{1}{10}d$. A, had at firſt.
And $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{338751}{800} = \frac{1016253}{1000} = 203$*l*. 11*s*. $10\frac{1}{2}d$. $\frac{54}{125}$ ſold B.
$\therefore \frac{16131}{10} - \frac{338751}{800} = \frac{1290536}{1000} - \frac{338751}{800} = \frac{981760}{800} = \frac{98176}{80}$
Alſo $\frac{9}{11}$ of $\frac{98176}{800} = \frac{8568}{8800} = 973$*l*. $4\frac{1}{4}d$. $\frac{19}{25}$ Couſin P. paid.

(15) Firſt $\frac{1}{3} = \frac{12}{408}$ X, $\frac{1}{25} = \frac{17}{408}$ Z.
Then $\frac{12}{408} + \frac{17}{408} = \frac{29}{408}$ performed in one Day by X and Z.
And $\frac{1}{12} = \frac{34}{408}$ performed in one Day by all three working together.
$\therefore \frac{34}{408} - \frac{29}{408} = \frac{5}{408}$ done in one Day by Y.
Therefore, as, $\frac{5}{408}$: 1 Day :: 1 Work : $\frac{408}{5} = 81\frac{3}{5}$ Days, the Anſwer.

(16) Firſt $\frac{16}{16} - \frac{3}{16} = \frac{13}{16}$ left at 6 Months' End.
Then $\frac{2}{3}$ of $\frac{13}{16} = \frac{13}{24}$; and $\frac{11}{10} - \frac{13}{24} = \frac{312}{384} - \frac{208}{384} = \frac{104}{384}$, or $\frac{13}{48} = 348$ left.
$\therefore$ As $\frac{13}{48} : 348 :: \frac{48}{48}$ or $\frac{1}{1} : 1284$*l*. 18*s*. $5\frac{1}{2}d$. $\frac{2}{13}$, the Anſwer.

(17) As $\frac{3}{5} : \frac{19312}{1} :: \frac{5}{5} : \frac{96560}{3} = 32186$*l*. 13*s*. 4*d*. two Years Rent; which $\div$ by $2 = 16093$*l*. 6*s*. 8*d* the Yearly Income required.

(18) Firſt 880 Guineas = 924*l*.
Then as $\frac{3}{20} : \frac{924}{1} :: \frac{20}{20}$ or $\frac{1}{1} : \frac{18480}{3} = 6160$*l*.
$\therefore 2200 + 6160 = 8360 = \frac{4}{5}$ of his whole Fortune.
Now as $\frac{4}{5} : \frac{8360}{1} :: \frac{5}{5}$ or $\frac{1}{1} : \frac{41800}{4} = 10450$*l*. the Anſwer.

(19) Here as the Son was to have twice as much as the Mother, and the Mother twice as much as the Daughter, ſo we will ſuppoſe the Eſtate to be divided as follows, viz. $4+2+1=7$ the whole Eſtate; then as ſhe had both a Son and a Daughter, the Mother muſt have but $\frac{2}{7}$ of the Eſtate; whereas had it been only a Daughter ſhe would have had $\frac{2}{3}$.

Then $\frac{2}{3}-\frac{2}{7}=\frac{14}{21}-\frac{6}{21}=\frac{8}{21}=2000£$.

$\therefore$ As $\frac{8}{21} : \frac{2000}{1} :: \frac{8}{8} : \frac{42000}{24}=1750£$. the Anſwer.

(20) Firſt Cock runs off 3 Galls. $=\frac{3}{103}$ of the Ciſtern in a Minute; and $1\frac{1}{4}$ Min. $=\frac{5}{4}$.

Then, as $\frac{5}{4} : \frac{3}{103} :: 1 : \frac{12}{515}$ runs off by the ſecond in a Minute.

And $\frac{3}{103}+\frac{12}{515}=\frac{15}{515}+\frac{12}{515}=\frac{27}{515}$ runs off by both in a Minute.

$\therefore$ As $\frac{27}{515} : 1 :: 1 : \frac{515}{27}=19$ Min. $4\frac{4}{9}$ ſec. the Anſwer.

(21) Firſt $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{18}{72}+\frac{24}{72}+\frac{12}{72}=\frac{54}{72}=\frac{3}{4}$.

Then as $\frac{3}{4} : \frac{45}{1} :: \frac{1}{1} : \frac{180}{3}=60$ Crowns the Anſwer.

(22) Firſt $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{24}{48}+\frac{12}{48}+\frac{8}{48}=\frac{44}{48}$ or $\frac{11}{12}$ and $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}=50$ Cherries.

$\frac{1}{2} = 50\times 6=300$ Apples.

$\frac{1}{4} = 50\times 3=150$ Pears.

$\frac{1}{6} = 50\times 2=100$ Plumbs.

50 Cherries.

Anſwer, in all 600 Trees.

(23) Firſt $1=\frac{4}{4}$, and $\frac{1}{2}=\frac{2}{4}$.

Then $\frac{4}{4}+\frac{4}{4}+\frac{2}{4}+\frac{1}{4}=\frac{11}{4}=99$ by the Queſtion.

$\therefore$ As $\frac{11}{4} : 99 :: 1 : \frac{396}{11}=36$ Scholars.

(24) Firſt $\frac{2}{3}+\frac{1}{3}+\frac{1}{5}=\frac{16}{30}+\frac{10}{30}+\frac{6}{30}=\frac{31}{30}$.

Then $\frac{31}{30}+\frac{30}{30}=\frac{61}{30}$; and $74-\frac{1}{2}=73\frac{1}{2}=\frac{36}{5}=\frac{2196}{30}$.

$\therefore \frac{61}{30} : \frac{2196}{30} :: 1 : \frac{2196}{61}=36$ Years, the Anſwer.

(25) Firſt $\frac{1}{8}$ of $\frac{2}{3}=\frac{1}{4}$ B.'s
And $\frac{2}{10}$ of $\frac{3}{3}=\frac{1}{5}$ C.'s
} Firſt Acquiſition, their Sum $=\frac{9}{20}$.

Then $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}=\frac{91}{420}$ left.

$\therefore \frac{1}{7}$ of $\frac{13}{60}=\frac{13}{420}$ E.'s firſt Acquiſition.

Alſo $\frac{91}{420}-\frac{13}{420}=\frac{78}{420}$ D.'s, thus ended the firſt Heat.

Again $\frac{1}{2}$ of $\frac{1}{4}$ = $\frac{1}{8}$ B.'s
Retained $\frac{1}{8}$ C.'s
And $\frac{13}{10}+\frac{1}{16}=\frac{139}{160}$ D.'s
Alſo $\frac{13}{20}+\frac{1}{16}=\frac{157}{1680}$ E.'s
} Part at the end of the ſecond Scuffle.

Proceeding $\frac{1}{2}$ of $\frac{1}{2}=\frac{1}{20}$ A.'s
$\frac{1}{8}$ of $\frac{1}{2}+\frac{1}{8}=\frac{23}{120}$ B.'s
$\frac{3}{7}$ of $\frac{1}{2}+\frac{130}{300}=\frac{171}{300}$ D.'s
Then $\frac{1}{20}+\frac{1}{15}+\frac{9}{35}=\frac{7}{420}$.
$\frac{1}{8}-\frac{73}{420}=\frac{11}{420}$ and $\frac{1}{2}$ of $\frac{11}{420}=\frac{11}{840}$ C.'s
$\frac{157}{868}=\frac{11}{820}$ $=\frac{179}{1680}$ E.'s
} Part of the third Scuffle.

Further $\frac{1}{20}+\frac{1}{15}=\frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{80}=\frac{7}{80}$, loſt by A. and B.
Then $\frac{5}{10}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{20}=\frac{51}{1280}$ A.'s.
Alſo $\frac{6}{10}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{23}$ of $\frac{1}{8}=\frac{649}{38400}$ B.'s
$\frac{1}{8}$ of $\frac{7}{80}+\frac{11}{840}$ $=\frac{323}{13440}$ E.'s
} Part after the laſt Scuffle.

$\frac{1}{8}$ of $\frac{7}{80}+\frac{171}{840}=\frac{1417}{4480}$ D.'s
$\frac{1}{8}$ of $\frac{7}{80}+\frac{179}{1680}=\frac{1579}{13440}$ E.'s
} Part after the Scuffle.

Then $\frac{51}{1280}+\frac{1}{15}=\frac{2863}{26880}$ A.'s
And $\frac{649}{38400}+\frac{1}{15}=\frac{6335}{26880}$ B.'s
Alſo $\frac{328}{13440}+\frac{1}{15}=\frac{2438}{26880}$ C.'s
$\frac{1417}{4480}+\frac{1}{15}=\frac{10294}{26880}$ D.'s
$\frac{1579}{13440}+\frac{1}{15}=\frac{4940}{26880}$ E.'s
} Share carried off at the laſt.

So that if the Number of Sugar Plumbs were 26880,

A got	2863	
B	6335	
C	2438	=26880, the Sum.
D	10294	
E	4950	

47. ADDITION of DECIMALS.

(1)		(2)		(3)	
	,0476		,427		,274
	21,476		64,075		,076
	,0067		27,6421		,64762
	,64		10,8		,0706
	17,6		,0074		,47
	,20764		104,046842		,007
					968,42
Sum	39,97794		206,998342		969,96522

(4)		(5)		(6)	
	47,674444		11,4444444		14,2764210
	4,026420		6,1427400		7,1444444
	32,866666		91,7888888		21,6488666
	6,144444		37,671111		9,2777777
	27,064666		146,4767410		31,1474444
	117,576642		293,5239254		83,7927543

(7)		(8)	
	14,14712471		,14882462
	768,14624624		3,8787672
	7,86406406		27,0427694
	26,086,0620		9,8489469
	815,56364122		40,9127298

48. SUBTRACTION of DECIMALS.

(1)		(2)		(3)	
From	176,		647,		74,6407
Take	10,764		,00764		69,5
Diff.	165,236		646,99254		5,1407

(4)		(5)		(6)	
From	41,744444		24,1466		16,1888262826
Take	21,946480		19,8999		4,1942764
Diff.	19,797964		4,1468		11,9319862848

49. MULTIPLICATION of DECIMALS.

(1)		(2)		(3)	
	,17504		27,42		8,04704
	×76		×3,56		×,2575
Prod.	13,30304		97,6152		2,0721128

(4)		(5)		(6)		(7)	
	5745		,4		,047		,000476
	×,0675		×,2		×,046		×,00078
Prod.	387,7875		,08		,00,2162		,0000037128

(8) ,47 × by ,0008=,000376, the Product.

(9) 1,076 × 1000=1076 (10) ,42768 × 1000=427,68.

(11)
3,47678
×2876,72 Mul. inverted
69536
24337
2086
243
28
1
96,231

(12)
47,689464
×49671,62
95378928
28613678
476895
333826
28613
4292
191
1248,36423

(13)
147,6#
×,7
Prod. 103,351

(14)
42,64#
×,276
25585#
2984955
8528#44
11,769253

(15)
46,2762
8
9)370,2096
Prod. 41,134#

(16)
261,276
,4#
9)18,28932
2,03214#
104,5104
Prod. 106,542548

(17)
141,14
8,47

9)98801

109778
564577
11291555

1196,5912

(18)
24,6072
,498

9)1476433

1640481
22146500
98428888

12,2215870

(19)
7,942
6

45,852

(20)
64,942
27,9

583828
453899
129888555

1809,1742

(21)
46,74
4,267

32718
28044
9348
18696

19943958
19943
19

199,63842

(22)
764,742
,6074

3058968
5353194
4588452

515436108
51543
5

515951595

(23)

47,847
3,874

190578
334988
28585555
142927777

175,038897
175038
175

175,214106

(24)

76,78
8,498

38141
686584
305213
61025555

648,01574
64801
64

648,66440

50. DIVISION OF DECIMALS.

Divis. Divid. Quot.

(1) 6,5)1735,5(267

(2) ,0084),8332(99,19047

(3) 7,476)186,900(25

(4) ,008)6,000(750

(5) 604,25)246,1476(,40736+

(6) 119)7,268401(,061079

(7) 647)6,7258(,010395+

(8) 678),0008136(,0000012

(9) 1000)487,67(,48767

(10) 1,0000)4746(.4746

(11) 2137,2)913,08(427
85488
5820
4274
1546
1496
50

(12) 240,649)6109,2674(25,3866
481298
129628
120325
9303
7220
2083
1925
158
144
14
14

(13) 6,84)14,44(2,11176+ (14) 46,849)1694,647(36,172546

764 289177

804 80836

1204 339874

5204 119317

4164 256196

60 219514

321187

400993

(15) Divide 42,86 by ,8

428

,8)3858

48,225

(16) Divide 6,428 by ,68

642

,68)5,784(8,505882+

(17)

7414,864) 81869,8694(

741486 8186,9869

6673,378 73682,8825(11,0413

(18)

64,842)9,46948(

6,484 94694

58,358 8,52252,1460385+

(19) Divide 147,42683 by 8,467.

8,467)147,42683(

8 14742683

8,459) 147,2794058 5(17,41097+

(20) Divide 4176,42∦6262 by 37,26974.

37,26974)4176,42∦6262

37 4176426

37,26602)4176,0089842(112,06 nearly.

51. REDUCTION of DECIMALS.

CASE I.

(1) Thus 1,0÷4=,25; and 1,0÷2=,5; alſo 3,0÷4=,75.
(2) Thus $\frac{6}{25}$=5,000÷24=,208∦—the Decimal required.
(3) Firſt $\frac{1}{12}$ of $\frac{3}{4}$=$\frac{1}{16}$ per Caſe III. Sect. 41.
Then $\frac{1}{16}$=5,0000÷16=,3125 the Decimal required.
(4) Thus $\frac{4}{7}$=4,0÷7=,5∦14285∦ the Decimal required.
(5) Thus $\frac{2}{3}$=2,0÷3=,∦ the Decimal required.
(6) Firſt $\frac{1}{4}$ of $\frac{1}{8}$ of $\frac{1}{8}$=$\frac{1}{64}$:
Then $\frac{21}{64}$=21,000000÷64=,328125, the Decimal req.
(7) Firſt $\frac{1}{4}$ of $\frac{6}{13}$=$\frac{6}{48}$:
Then, 5,00000÷48=,1041∦, the Decimal required.

CASE II.

(8) By Rule 1ſt, thus 17*s*. 6¾*d*.=843 qrs.
And 1£. =960 ditto. Then $\frac{843}{960}$=$\frac{281}{320}$ or 281÷320=,878125 the Decimal required.

By Rule 2d thus;

4)3,00

12)6,75

2,0)17,5625

,878125

By Rule 3d thus;

17*s*. 6¾*d*,

5 4

85 27+1=28

28

,878

(9) Firſt 6*s*. 9*d*.=$\frac{27}{80}$£. per Caſe VIII. Sect. 38.
For 6*s*. 9*d*.=27 Three-pences, and 1£.=80 ditto.
Then 80)27,0(,3375 the Decimal required.

(10) Thus $\frac{9}{11}$ or $\frac{3}{7}$=3,00000÷7=,42857+, Anſwer.

(11) First, 14*s.* 6½*d.*=349 Halfpence, and a Moidore=648 ditto.
Then 648)349,0(,53858+, the Decimal required.
(12) First, 18*s.* 4½*d.*=441 Halfpence, and 1£.=480 ditto.
Then 480)441,0(,91875, the Decimal required.
(13) First ¾ of a Penny=$\frac{1}{1\,2\,0}$£. per Case VI. Sect. 38.
Then 320)1,000(,003125, the Decimal required.
(14) First 1lb.=240 dwts. 240)11,00(,04583 the Dec. req.
(15) First 1lb.=256drs. then 256)10,00(,03906+the Dec.
(16) First 3 qrs. 14lb.=98lb. then 112)98,0(,875, the Dec.
(17) A yd.=36 Inches, then $\frac{6}{3\,6}$ or $\frac{1}{6}$=1,00÷6=,1ϕ, Ans.
(18) First a League, or 3 Miles=24 fur. then $\frac{6}{2\,4}$ or $\frac{1}{4}$=1,00÷4=,25, Answer.
(19) First 18gall. 2 qts.=74 qts. and 1 hhd.=252 ditto.
Then 252)74,0(,29365+, the Decimal required.
(20) First, 3 qts. 1 pt.=7 pts. and a bar.=256 do.
Then 256)7,00(,02734+, the Decimal required.
(21) First an Acre=160Perches, then $\frac{8}{1\,6\,0}$ or $\frac{1}{2\,0}$=1,00÷20 =.05, Answer.
(22) First 4 bush. 2 pks.=18 pks. and a chald.=144 do.
$\frac{18}{144}$ or $\frac{1}{8}$=1,000÷8=,125, Answer.
(23) Thus $\frac{1\,2}{6\,0}$ or $\frac{1}{5}$=1,0÷5=,2, the Decimal required.
(24) First 12 Days=288hrs. and 365 d. 6 h.=8766 do.
Then 8766)288,00(,03285+, the Decimal required.

CASE III.

(25)
£. ,878125
20
s. 17,56250ϕ
12
d. 6,750ϕ
4
qrs. 3,00
Ans. 17*s.* 6¾*d.*

(26)
£. ,3375
×20
s. 6,7500
×12
d. 9,00
Answer, 6*s.* 9*d.*

(27) *Guinea.*

45
×7×3=21

3,15
3

s. 9,45
12

d. 5,40
4

1,6

Answer, 9*s.* 5*d.*¼,6.

(28) *Moidore.*

,72708
×9×3=27

6,54372
×3

s. 19,63116
12

d. 7,57392
4

2,29568

Answer, 19*s.* 7*d.*½,29+

(29) *lb.*

,00243
12

,02916
20

,58320
4×6=24

2,3328
6

13,9968

Answer, 14 Grs.

(30) *Ton.*

,3375
20

cwt. 6,7500
4

qrs. 3,00

Ans. 6 Cwt. 3 Qrs.

(31) *lb.*

,0396
16

6336
16

drs. 10,1376

Ans. 10 drs. 1376

(32) *Cwt.*

,875
4

qrs. 3,500
28

lb. 14,0

Ans. 3 Qrs. 14 lb.

(33) *Yard.*

,16669
3

,50007
×12

6,00084

Ans. 6 Inches+

Leagues.

(34) ,259
3

,777
8

Fur. 6,216
40

8,640

Ans. 6 Fur. 8,64 Poles.

Hhd.

(35) ,29265
×63

Galls. 18,5 nearly.

Ans. 18½ Galls.

Bar.

(36) ,875
×32

28,000

Answer, 28 Galls.

Acre.

(37) ,05
4

,20
40

8,00

Answer, 8 Poles.

Chaldrons.

(38) ,125
×36

Bush. 4,500
4

Pks. 2,0

Ans. 4 Bush. 2 Pks.

Days.

(39) ,4765
×24

Hrs. 11,4360
60

Min. 26,1600
60

Sec. 9,60
60

Thirds, 36,00

Answer, 11 Hrs. 26 Min. 9 Sec. 36 Th.

52. EXTRACTION of the SQUARE ROOT.

(1) 74770609(8647 the Root
64

166)1077
996=166×6

1724)8106
6896=1724×4

17287)121009
121009=17287×7

Proof thus, 8647×8647=74770609.

(2) 60516(246 the Root required.
4

44)205
176

486)2916
2916

(3) 7658)87,5099, or rather 87,51+, the Root.

167)1258

1745)8900

175009)1750000

1750189)17491900

Rem. 1740199

(4) 39342864(6272,389+, the Root.

122)334

1247)9028

12542)29964

125443)488000

1254468)11167100

12544769)113135600

Rem. 232679

(5) 8209667940,5290(90607,21792+, the Root.

1806)10966

181207)1307940

1812142)3949152

18121441)32486890

181214427)1436544900

1812144349)16804391100

18121443582)49509195900

Rem. 13266308736

(6) ,000729(,027 the Root

4

47)329

(7) 2)1,41421+, the Root.
4

24)100
281)400
2824)11900
28282)60400
282841)383600
Rem. 100759

(8) 2,27109570(1,50701+
1
25)127
3007)21095
301401)467000
Rem. 165599

(9) 36,00000625(6,0000005+, the Root.
12000005(625000000
Rem. 24999975

53. VULGAR FRACTIONS.

(1) Thus $\sqrt{\frac{4}{81}}=\frac{2}{9}$, the Root required.
(2) Thus $\sqrt{\frac{225}{144}}=\frac{15}{12}$, or $\frac{5}{4}$, the Root.
(3) First $10\frac{10}{49}=\frac{500}{49}$. Then $\sqrt{\frac{500}{49}}=\frac{23}{7}$, or $3\frac{2}{7}$, the Root.
(4) First $27\frac{9}{16}=\frac{441}{16}$. Then $\sqrt{\frac{441}{16}}=\frac{21}{4}$, or $5\frac{1}{4}$, the Root.
(5) First $8\frac{2}{7}=8,4285714$. Then $\sqrt{8,4285714}=2,9032$, the Root.
(6) First $\frac{507}{16}=,8831360946$. Then $\sqrt{,8831360946}=,93975$ +the Root.
(7) First $76\frac{14}{17}=76,8235294$. Then $\sqrt{76,8235294}=8,7649$ +the Root.

54. The USE of the SQUARE ROOT.

(1) First $9\times4=36$. Then $\sqrt{36}=6$, the mean Proportion. For as $4:6::6:9$.
(2) First $36\times16=576$. Then $\sqrt{576}=24$ the mean Proportion. For as $16:24::24:36$.

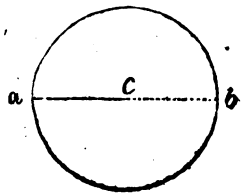
(3) Thus, $\sqrt[2]{4276,5} = 65,4$ nearly.

(4) First 9A. 2r. 15p. = 1535 Perches. And $5,5 \times 5,5 = 30,25$ Yards in 1 Square Perch.
$\therefore 1535 \times 30,25 = 46433,75$ Yards the superficial Cont.
Then $\sqrt{46433,75} = 215,484918$ Yards $+ = 215$ Yards 1 Foot 5,467 inc. the Side required.

(5) Thus, $\sqrt[2]{160} = 12,649+$, the Side required.

(6) First, As 355 : 452 :: 4840 : 616,2479 Sq. of the Diam.
$\therefore \sqrt{616,2479} = 24,82+$ the Diameter required.

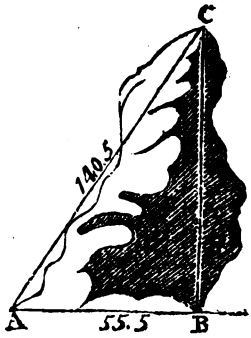
(7) First $4840 \times 2 = 9680$ Yards. Content of the two Acres.
Then as 355 : 452 :: 9680
12324 Yards $= {}^2ab$.
$\therefore \sqrt{12324} = 111 = ab$.
Therefore $111 \div 2 = 55\frac{1}{2}$ Yards $= cb$, the Length of the Cord required.

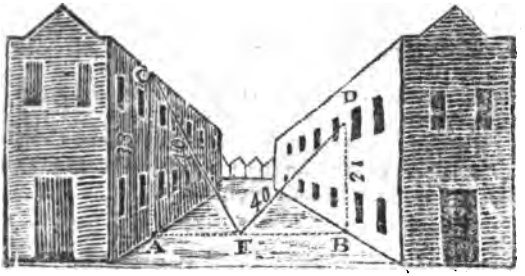


(8) First $140,5 \times 140,5 = 19740,25 = {}^2AC$.
And $55,5 \times 55,5 = 3080,25$ ${}^2AB$.

${}^2BC = 16660$

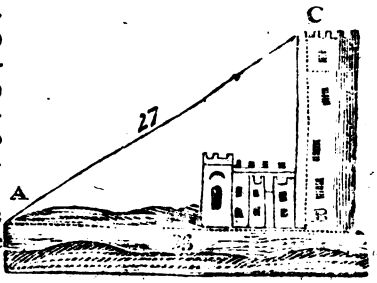
$\therefore \sqrt{16660} = 129,07$ Yds. $= BC$, the Height required.

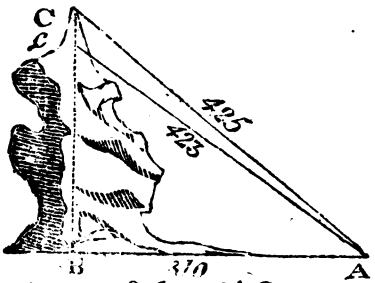




(9) First $40 \times 40 = 1600 = {}^2$C E, or ^{2}D E.
And $33 \times 33 = 1089 = {}^2$A C.
$\therefore \quad \sqrt{\quad 51 \quad} = 22{,}6 =$ A E.
Again $21 \times 21 = 441 = {}^2$ B D.
Then $1600 - 441 = 1159 = {}^2$E B.
$\therefore \sqrt{1159} = 34{,}04 = {}^2$E B.
Therefore $22{,}6 + 34{,}04 = 59{,}64 +$ Ft. $=$ A E $+$ E B $=$ A B, the Answer.

(10) First $27 + 27 = 729 = {}^2$A C.
And $23 \times 23 = 529 = {}^2$A B.
Diff. $200 = {}^2$B C.
$\therefore \sqrt{200} = 14{,}142 +$, or 14 Yds. 5,112 Inches $=$ B C. the Height required.





(11) First, $425 \times 425 = 180625 = {}^2$A C.
And $310 \times 310 = 96100 = {}^2$A B.
Diff. $84525 = {}^2$B C.

∴ $\sqrt{84525}=290,72183=$ B C, Height of the Light-house and Rock.

Again $423\times423=178029=^{2}$ A c.
$-\ 96100=^{2}$ A B.

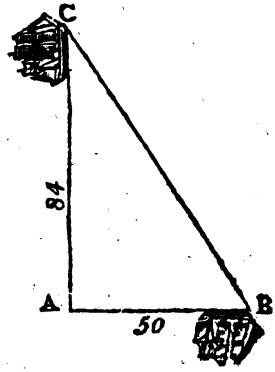
Diff. $82829=^{2}$ B C.

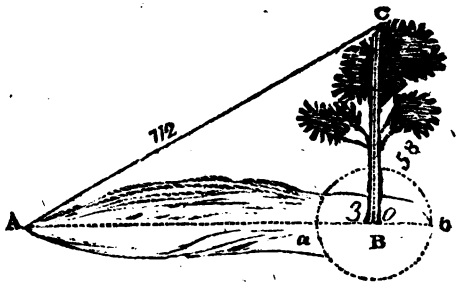
∴ $\sqrt{82829}=287,80027=$ B c, Height of the Rock.
Therefore $290,73183-287,80027=2,93156$ Fathoms.
∴ Or 17 Ft. 7 In.=C c, the Height of the Light-house.

(12) First $50\times50=2500=^{2}$ A B.
And $84\times84=7056=^{2}$ A C.

Sum $9556=^{2}$ B C.

∴ $9556=97,75$ or $97\frac{3}{4}$ Leagues=B C, the Distance required.

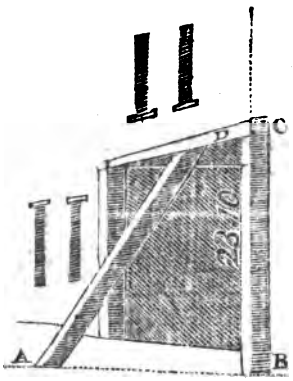




(13) First $112\times112=12544=^{2}$ A C.
And $53\times53=2809=^{2}$ B C.

Diff. $9735=^{2}$ A B.

$\therefore \sqrt{9735} = 98,6 = AB$. Now $ab = 30$. Which $\div 2 = 15 = aB$. And $98,6 - 15 = 83\frac{2}{3} = Aa$, Breadth of the Mt. required.

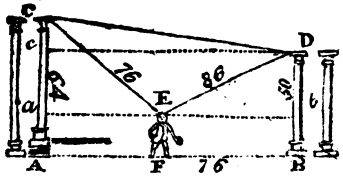


11

(14) First $11 \times = 121 = {}^{2}AB$.
And $23,83 \times 23,83 = 568,02\not{7} = BC$.

Sum $689,027 = {}^{2} = AC$.

$\therefore \sqrt{689,127} = 26,249$, or 26 Feet 29 Inches $= AC$, the Length required.



(15) First $86 \times 86 = 7396 = {}^{2}ED$.
And $76 \times 76 = 5776 = {}^{2}FB$. or Eb.

Diff. $1620 = {}^{2}Db$.

$\therefore \sqrt[2]{1620} = 40{,}24922 = D\,b.$
Also $50 - 40{,}24922 = 9{,}75078 = E\,F$, Height of Statue.
Likewise $64 - 9{,}75078 = 54{,}24922 = A\,C - A\,a = a\,C.$
Again $97 \times 97 = 9409 = {}^2E\,C.$
And $54{,}24922 \times 54{,}24922 = 2942{,}97787 = a\,C.$

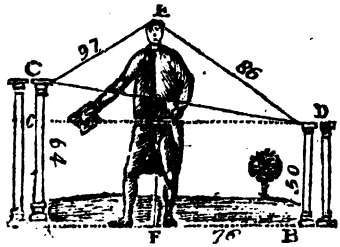
Diff. $6466{,}02213 = a^2E$, or ${}^2A\,F.$

$\therefore \sqrt{6466{,}02213} = 80{,}41157 = a\,E$, or $A\,F.$
Now $76 + 80{,}41157 = 1564115 = A\,B$, or $c\,D$, Distance of the Columns.
And $64 - 50 = 14 = c\,C$ diff. of their Heights.
Now $156{,}41157 \times 156{,}41157 = 24464{,}5792 = {}^2C\,D.$
And $14 \times 14 = 196 = {}^2C\,c.$

Sum $24660{,}5792 = {}^2C\,D.$

$\therefore \sqrt{24660{,}5792} = 157{,}03 + = C\,D$, Distance of the Top of the Columns.

But if the Statue be higher than the Columns, then, by working as before, we shall find the Statue to be 40,24922 Feet higher than the lower Column.



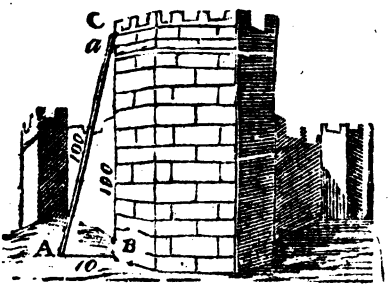
Therefore $50 + 40{,}24922 = 90{,}24922 = E$, F, Height of the Statue.
Also, $90{,}2492 - 64 = 26{,}24922$ higher than the highest Column.
Then $97 \times 97 = 9409 = {}^2C\,E.$
And $26{,}24922 \times 26{,}24922 = 689{,}02135$

Diff. $8719{,}97845$

∴ ✓ 8719,97845=93,3808 A F.
So that 76+93,3808=169,3808=A F+F B Distance of the Columns.
Again 1693,808 × 169,3808=28689,8554064=²*c* D.
And 14×14 = 196 =²*c* C.

Sum 28885,85540864=²*c* D.

∴ ✓28885,85540864=169,95839 Ft.=C D, Distance required.



(16) First 100×100=10000=²A *a*.
And 10× 10= 100=²A B.

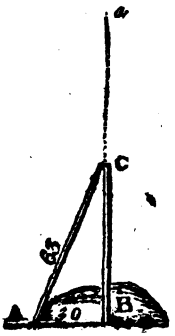
Diff. 9900=²B *a*.

Then ✓9900=99,49874=B *a*.
∴ 100—99,49874=,50125=*a* C, which is 6 Inches nearly. Q. E. F.

(17) First 63×63=3969=²A C, or ²C D.
And 30×30= 900=²A B.

Diff. 3069=²B C.

Then ✓ 3069=55,3985 Yds. = B C, Piece standing
∴ 63+55,3985=118,3985 Yds. or 118 Yd. 1 Foot 2¼ Inches=B *a*, Height of the Pole.



(18) Thus $\sqrt[3]{23716} = 154$ Men, the Answer.

* 54. the EXTRACTION of the CUBE ROOT.

(1) 21024576(276
8

$3rr = 1200)13024$

8400 = $3rre$
2940 = $3ree$
343 = eee

Here $r = 20$, and $e = 7$.

11683 Subtrahend.

$3rr = 218700)1341576$ Dividend.

1312200 = $3rre$
29160 = $3ree$
216 = eee

Here $r = 270$, and $e = 6$.

1341576 Subtrahend.

(2) 92398647(452,08 + the Root.
64

$3rr = 4800)28398$ Dividend.

24000 = $3rre$
3000 = $3ree$
125 = eee

Here $r = 40$, and $e = 5$.

27125 Subtrahend.

$3rr = 607500)1273647$ Dividend.

1215000 = $3rre$
5400 = $3ree$
8 = eee

Here $r = 450$, and $e = 2$.

1220408

$3rr=612910000)532390000000$ Dividend.

$490329600000=3rre$
$8678400=3ree$
$512=eee$
Here $r=45200$, and $e=8$.

$490416389 12$ Subtrahend.

4197361088 Rem.

Now $452,08\times452,08\times452,08+4197361088=92398647$, the Proof.

(3) $2716243264)1395,26+$the Root.

$3rr=300)1716$ Dividend.
1197 Subtrahend. } Here $r=10$, and $e=3$.

$3rr=50700)519243$ Dividend.
488619 Subtrahend. } Here $r=130$, and $e=9$.

$3rr=5796300)30624264$ Dividend.
29085875 Subtrahend. } Here $r=1390$, and $e=5$.

$3rr=583807500)1538389000$ Dividend.
1167782408 } Here $r=13950$, and $e=2$.

$3rr=583974912 00)3706065920 00$ Dividend.
$3504000155 76$ Subtrahend. } Here $r=139520$, and $e=6$.

$2020657642 4$ Remains.

(4) $91)4,497+$the Root.
64

$3rr=4800)27000$ Dividend.
21184 Subtrahend. } Here $r=40$, and $e=4$.

$3rr=580800)5816000$ Dividend.
5334849 Subtrahend. } Here $r=440$, and $e=9$.

$3rr=60480300)48151000$ Dividend.
424022473 Subtrahend. } Here $r=4490$ and $e=7$.

57128527 Remains.

(5) 67527834239(4072,18+ the Root
64

$3rr=480000)3527834$ Dividend.
3419143 Subtrahend. } Here $r=40$, and $e=7$.

$3rr=49694700)108691230$ Dividend.
99438248 Subtrahend. } Here $r=4070$, and, $e=2$.

$3rr=4974352009)252991000$ Divid.
4974477361 Subtr. } Here $r=40720$, and $e=1$.

$3rr=497459952300)4278513639000$ Divid.
397975780323 2 Subtr. } Here $r=407210$, and $e=8$.

2987558357 68 Remains.

6) 4764,750(16,827+ the Root.

$3rr=300)3764$ Dividend.
3096 Subtrahend. } Here $r=10$, and $e=6$.

$3rr=76800)668750$ Dividend.
645632 Subtrahend. } Here $r=160$, and $e=8$.

$3rr=8467200)23118000$ Dividend. } Here $r=1680$, and $e=2$.
16954568 Subtrahend. }

$3rr=84873700)6163432000$ Dividend. } Here $r=16820$, and $e=7$.
5943633293 Subtrahend. }

219798717

(7) 36155,027576(33,0665

$3rr=2700)9155$
8937

$3rr=32670000)218027576$ Dividend. } Here $r=3300$, and $e=6$
196376616 Subtrahend. }

$3rr=32788908000)21650960000$ Dividend. } Here $r=33060$, and $e=6$
19676915496 Subtrah. }

$3rr=32800810680000)1974044504000$ Divid. } Here $r=330660$, and $e=5$.
1640065333625 Subtr. }

333979170375 Remains.

(8) 67667,921875(40,75 the Root.

$3rr=480000)3667,921$ Dividend. } Here $r=408$, and $e=7$.
3419143 Subtrahend. }

$3rr=49694700)248778875$ Dividend. } Here $r=4070$, and $e=5$.
248778875 Subtrahend }

(9) 219365329(603,1+the Root.

$3rr=1080000$)3365329 Dividend.
3256227 Subtrahend. } Here $r=600$, and $e=3$.

$3rr=109082700$)109102000 Divid.
109100791 Subtr. } Here $r=6030$, and $e=1$.

1209 Remains.

(10) 3105926,917(145,9+ the Root.

$3rr=300$)2105 Dividend.
1744 Subtrahend. } Here $r=10$, and $e=4$.

$3rr=58800$)361926 Dividend.
304625 Subtrahend. } Here $r=140$, and $e=5$.

$3rr=6307500$)57301917 Divid.
57120579 Subtra. } Here $r=1450$, and $e=9$.

181338 Remains.

(11) ,000421875(,075 the Root.

$3rr=14700$ 78875 Dividend.
78875 Subtrahend. } Here $r=70$, and $e=5$.

(12) 28022810,390625(303,7472+the Root.

$3rr=270000$)1022810 Dividend.
810837 Subtrahend. } Here $r=300$, and $e=3$.

$3rr=27542700$)204683390 Dividend.
193244653 Subtrahend. } Here $r=3030$, and $e=7$.

$3rr=27670107$00)11438737625 Divid.
11082607350 Subtr. } Here $r=30370$, and $e=4$.

$3rr=2767739628$00)36923700100 Div.
276774874021 Subtr. } Here $r=303740$, and $e=1$

$3rr=27677578524300$)9246212697900 0 Div.
830328175829997 Sub. } Here $r=3037470$, and $e=2$.

9429309396003 Remains.

(13) First $\sqrt[2]{4857532416}=69696$.
Then $\sqrt[2]{69696}=264$ the biquadatic Root required.

(14) First $\sqrt[2]{49656}=216$, then $\sqrt[3]{216}=6$, the Square-cubed Root required.

(15) First $\sqrt[2]{43046721}=6561$, again $\sqrt[2]{6561}=81$, also $\sqrt[2]{81}=9$ the Root.

(16) First $\sqrt[3]{387420489}=729$. Then $\sqrt[3]{729}=9$, the Roots at the 9th Power.

To EXTRACT the CUBE ROOT of VULGAR FRACTIONS.

(1) First $\frac{3}{1}\frac{2}{3}\frac{4}{0}\frac{0}{0}=\frac{27}{125}$, then $\sqrt[3]{\frac{27}{125}}=\frac{3}{5}$ the Root required.

(2) First $\frac{3}{1}\frac{5}{1}\frac{2}{3}=\frac{8}{27}$, then $\sqrt{\frac{8}{27}}=\frac{2}{3}$, the Root.

(3) First $5\times 125+104=729$, then $\sqrt[3]{\frac{7}{1}\frac{2}{2}\frac{9}{5}}=\frac{9}{5}$, or $1\frac{4}{5}$, Root.

(4) First $405 \times 125 + 28 = 50653$, then $\sqrt[3]{50653} = \frac{37}{5}$ or $7\frac{2}{5}$, the Root.

(5) First $5\frac{1}{7} = 5{,}6$, then $\sqrt[3]{5{,}600} = 1{,}775 +$ the Root.

(6) First $7\frac{5}{7} = 7{,}\overline{714285}$, then $\sqrt[3]{7{,}\overline{714285}} = 1{,}975$, the Root required.

55. The USE of the CUBE ROOT.

(1) First $\sqrt[3]{21952} = 28$ Side of the Cube.
Then $28 \times 28 = 784$, the Content required.

(2) First $125 \times 125 \times 125$
And $25 \times 25 \times 25$
Also $15 \times 15 \times 15$
$\times 3 =$ 5859375, 46875, 10125
$\therefore \sqrt[3]{5859375} = 180{,}28$ Keel.
And $\sqrt{46875} = 36{,}05$ Midship-beam.
Also $\sqrt{10125} = 21{,}6$ Depth in the Hold.

(3) First $\frac{125 \times 125 \times 125}{}$
And $\frac{25 \times 25 \times 25}{}$
Also $\frac{15 \times 15 \times 15}{}$
$\div 2 =$ 976562,5, 7812,5, 1687,5
Which Numbers extracted will give 99,21; 19,84 and 11,905 the Dimensions required.

(4) First $75 \times 75 \times 75 = 421875$, and $100 \times 100 \times 100 = 1000000$.
Then as 421875 C. K. : 300 Tons, :: 1000000 C. K. 711,1 Tons, or 711 Tons, 2 cwt. 24, 64lb. the Burden required.

(5) First $4 \times 4 \times 4 = 64$. Cube of the Diameter.
Then, as 18lb. : 64 inc. :: 114lb. : 405,3 inc. Cube of the Diameter.
$\therefore \sqrt[3]{405{,}3} = 7{,}4+$, the Diameter required.

(6) First $11{,}5 \times 11{,}5 \times 11{,}5 = 1520{,}875$, and $20{,}83 \times 20{,}83 \times 20{,}83 = 9037{,}905787$.
Then, as 1520,875 : 1000lb. :: 9037,905787 : 5942,5697lb. +, the Weight required.

(7) First $189 \div 7 = 27$, whose Cube Root is 3.

Then $3 \times 7 = 21$, the lesser Mean, and $21 \times 3 = 63$, the greater Mean.

For, as 7 : 21 :: 63 : 189, the Proof.

(8) First $256 \div 4 = 64$, then $\sqrt[3]{64} = 4$, which $\times$ by $4 = 16$, the lesser Mean, and $16 \times 4 = 64$, the greater.

For, as 4 : 16 :: 64 : 256, the Proof.

To EXTRACT THE FIRST SURSOLID ROOT.

RULE.

1. Having pointed the given Resolvend into Periods of five Figures, seek such a Sursolid Number in the Table, or otherwise, as comes nearest to the first Period of the Resolvend, whether greater or less; and call the respective Root, either more than just, or less than just, as it falls out; annexing so many Ciphers to it as there are remaining Periods of whole Numbers in the Resolvend.

2. Find the Difference between the Resolvend and the Sursolid Number, so taken, by subtracting the lesser from the greater.

3. Find the Cube of the aforesaid Sursolid Root, with its annexed Ciphers, which also may be done by the Table, and multiply that Cube into five, the Index of the Sursolid, and divide the difference between the Resolvend and the Sursolid Number by that Product; by which it will be depressed to a Square: and when pointed into Periods of two Figures each, call it the new Resolvend.

4. Make the first Root without Ciphers a Divisor, inquiring how often it may be found in the first Period of the new Resolvend; with this Consideration, if the Root, now a Divisor, be less than just, annex twice the Quotient Figure to it; but if more than just, subtract twice the Quotient Figure from a Cipher, either annexed, or supposed to be annexed, to that Divisor or Root, multiplying it so increased or diminished with the said Quotient Figure: setting down the Unit's Place of the Product under the pointed Figures of that Period, subtracting it as in Division.

TABLE.

1	2	3	4	5	6	7	8	9
1	32	343	1024	3125	7776	16807	32768	59049

EXAMPLES.

(1) Extract the Sursolid Root of 12309502009375, the Resolvend pointed.

The nearest Sursolid Number to 1230, the first Period of the Resolvend, is 1024, whose Root is 4 (by the Table) less than just.

Therefore 12309502009375
—1024

2069102009375 their Difference.

Now as there are two Periods remaining in the Resolvend, place two o's to 4, the Root of the first Period.

Next the Cube of 400=64000000.

And 64000000 × 5=320000000 the Divisor.

Then 320000000)2069502009375(6497, &c.

First Root = 400
+2 × 10 = 20

First Divisor = 420)6467(15 last Root.
+20 × 2 × 5=30 42 400 First.

450)2267(415 Root (true) required.
2250

Rem. 17 to be rejected.

Now 415 × 415 × 415 × 415 × 415=12309502009375, the Proof.

(2) Extract the Sursolid Root of 2327134559873.

Now the nearest Sursolid Number to 232 is 243 (per Term) whose Root is 3, being more than just.

Therefore 2430000000000
—2327834559873

Rem. 102165440127 for a Dividend.

300 Cubed=27000000, which × 5=135000000 Divisor.

Then 135000000,192'165440127'756,7810 new Resolvend.
First Root. 300
—2 × 2 =—4

300 first Root.

1 Divisor =296;756,7810(2,566 last ditto, too much,
—4—2×,5=5,0 592

2 97·434 the Root only too
2 Divisor = 291,0)164,78(little by 2 in the
—1—2×00=—1,12 145,50 lowest Figure.

3 Divisor =289,88)19,2810(
&c. &c.

To EXTRACT the ROOT of the SECOND SURSOLID, or SEVENTH POWER.

RULE.

Having pointed the Resolvend into Periods of seven Figures, seek out such a Number by the Table as comes nearest to the first Period of the Resolvend, whether greater or lesser, calling its Root more than just, or less than just, annexing a proper Number of Ciphers.

2. Find the Difference between the Resolvend and that Number of the seventh Power, by subtracting the lesser from the greater.

3. Find the Sursolid or fifth Power of that Root, with its annexed Ciphers, by that Table; and multiply that Sursolid Number into seven, the Index of the Resolvend.

4 Make that Product a Divisor, by which the aforesaid Difference must be divided; so that it may be depressed to a Square, and pointed as such.

5. Make the first Root without a Cipher, a Divisor, working with it and the new Resolvend, as in Sursolid; only here you must increase or diminish the Divisor with three Quotient Figures.

Table.

1	2	3	4	5	6	7	8	9
	128	2187	16384	78125	279936	823543	2097152	4782969

EXAMPLE.

What is the second Sursolid Root of
382786553955078125 Resolvend pointed,
—2187 the nearest of the seventh Power.
16428655395507812 5 their Difference.

The first Root is 300, being less than just, and the fifth Power of 300=24300000000000, which being multiplied by 7=170100000000000 for a Divisor, by which the aforesaid Difference must be divided, which contracted may stand thus (1701) 16428655(9658,23, &c.

First Root= 300
3 × 20=+ 60

1 Divisor = 360 | 9658 | 25
300

60+3×,05=+75 72 325 the true Root.

2 Divisor=435)2478
2175

Rem. 283 to be rejected.

56. THE SINGLE RULE OF THREE IN DECIMALS.

(1) First 6*s.* 3*d.* =,3125*l.* and $4\frac{3}{4}$=4,75 Yards. Also $48\frac{1}{2}$ =$48\frac{1}{2}$=48,5 Yards.
Then, as 4,75 : ,3125 :: 48,5 : 3,19078 or £.3 $3\frac{1}{2}$. $9\frac{1}{4}$*d.* the Answer.

(2) First $2\frac{1}{4}$*lb.*=2,5. And £.1 5*s.*=1,25£. Also $14\frac{3}{4}$= 14,75 lb.
Then, as 2,5*lb.* : 1,25£. :: 14,75*lb.* : 7,375=£7. 7*s.* 6*d.* the Answer.

(3) First 1*lb.*=,008928*C.* and $11\frac{1}{2}$*d.*=,0489583£.
also 4*bhds.* each 4*C.* *qrs.* 24*lb.*=4,625×4=18,5*C.*
Then, as ,0089285 : ,0489583£. :: 18,5*C.* : 101,4417£. =.£101 8*s.* 10*d.* the Answer.

(4) First 4 Chests each, 2*C.* 3*qrs.* 14*ll.*=2,875 *C.* × 4= 11,5*C.* And £.906 10*s.*=906,5£.
Then, as 11,5 : 906,5 :: ,00892857 : 7038=14*s.* 0¾*d.* ,6. the Answer.

(5)

As *D.*	*M.*	*D.*
3	: 12 :	: 9,6
	×	12
	116,0	See Case 4, p. 166 G.
	116	
	3)1044	

Answer 348 Miles.

(6) First from 4 Tuns 201½ gal. take 24½ gal. remain 4 Tuns, 177 gal. or 1185 gal.
And .240 16*s.* 6*d.*=240,825£.
As, 1185 : 240,825 :: 1 : ,20322, or 4*s.* 0¾*d.* nearly.

(7) First 4 $\frac{4}{7}$=,4592, which÷12=,3826 ft. this added to 10 ft.=10,3826 ft.=to 6,5 Cubits.
Then, as 6,5 cub. : 10,3826 :: 1 cub. : 1,5973=1 ft. 7,168 Inches, the Answer.

(8) First 78,4÷8=9,8, the second Number.
Then, as 5 : 9,8 :: 815,68 the fourth Number.
∴ 15,68—9,8=5,88, the Answer.

(9) First 13$\frac{1}{3}$=13,3. And 13,3×282=3760 Cubic Inches, the Contents of the Cask.
Then 3760×,52835=1986,596 oz. Weight of the Oil.
Which ÷ 16=124,16225lb. Also 7½=7,5lb. Weight of 1 gall.
Therefore, as 7,5lb. : 1 gall. :: 124,16225lb. : 16,555=16 gall. 2 qts. 4 pts. the Answer.

(10) First 4,63—1,5688=3,0612 oz. Difference.
And a solid Foot and Half,=1728+864=2592 solid In.
Then, as 1 inc. : 3,0612 oz. :: 2592 inc. : 7934,6304 oz. =4,4278 cwt or 4 cwt. 1 qr. 19 lb. 14 oz. 9,88 drs. Answer.

(11) Now, as 5 : 8 :: 75 : 120, the greater Number.
Then 75+120=195, their Sum.

And 120—75=45, their Difference.
Also 195×45=8755, Prod. of their Sum and Diff.
120×120=14400, Square of the gr. and 75×75=5626 of the lesser.
14400—5625=8775, Diff. of those Squares.
14400+5625=20025, Sum of ditto.
120÷75=1,6, Quot. of the greater divided by the lesser.
75÷120=,625 Quot. of the lesser div. by the greater.
1,6×1,6,=2,56, Square of the greater Quot.
625×,625=—,390625, Square of the lesser Quot.

Answ. 2,950625, Sum of those Squares.

(12) Now, as 8 : 7 :: 224 : 196, the lesser Number.
Then 224+196=420, their Sum.
And 420×420=176400, Square of their Sum.
Also 240—196=28 Diff.
28×28=784. Square of that Diff.
224÷196=1,148256, Quot. of greater÷lesser.
196÷224=,875, Quot. of the lesser÷greater.
1,148256×1,148256=1,30612+ } Square of those
,875×,875=,765625 } Quotes.
420×28+28=11788, Prod. of their Sum and Difference, and the Diff. added.
Lastly, 11788×11788=138956944, Square of the Sum of their Diff. added to the Prod. of their Sum and Diff.

(13) He that rows towards } London goes { 7,5 } per Ho.
And he that rows from } { 2,5 } *Miles.*

Sum 10

∴ 34÷10=3,4 Hours before they will meet.
Then, as 1 h. : 2,5 m. :: 3,4 h. : 8,5 or 8½ Miles from London.
And 34—8½=25½ Miles from Chertsey, the Answer.

(14) First 13,5—7,5=6 Inches, Diff.
Then, as 4 : 9 :: 6 : 13,5lb. the Answer.

(15) First 18200*l*. at 3 per Cent.=546*l*. spent on the Funeral and Monument.

Then 18200—546=17564£. left.
Again 17654£. at 9£. per Cent=1588,86 gave to his Cousins.
Then 17654—1588,86=16065,14£. what he had then left.
Again $\frac{3}{7}$ of 16065,14=4590,04£. paid for the Seat.
Then 16605,14—4390,04=11475,1 left.
Again 11476,1÷=81434,3875 paid for Horses.
Then 11475,1—1434,3875=10040,7125 left.
Also 10040,7125—550£.=9490,7125 left, after he had spent 550£. on his Mistress.
Now, as 12 : 2000 :: 29 : 3166,6£. spent in riotous Living.
∴ 9490,7125—3166,6=6324,0458£. or £.6324 11*d.* the Answer.

(16) First 6×6=36. And 3×3=9 Square of their Distances.
Then Receip. as 36 : 1 :: 9 : 4, so that A's Place is four Times as hot as B's.

(17) First 81000000×81000000=6561000000000000.
Then, Receip. as
1 : 6561000000000000 :: 2 : 32805000000000000.
∴ $\sqrt[2]{32805000000000000}$=57275649 Miles the Answ.
Squares of their Distances; the Ciphers being omitted.

(18) First 81×81=6561. And 424×424=179776 Squares of their Distances; the Ciphers being omitted.
Then Receip. as 179776 : 1 :: 6561 : 27,4; so that the Sun's Influence on the Earth to that on the Planet Jupiter is as 27,4 to 1.

(19) First 32×32=1024. And 777×777=603729, the Squares of the Distances, Ciphers omitted.
Then, Receip. as 603729 Sat. : 1 :: 1024 : 589,584 Merc. nearly.
∴ The Solar Influence on Mercury to that of Saturn is as 589,584 to 1, nearly.

(20) First 115×115=13225 Square.
As 13225 : 1 :: 2 : 13225 Degrees hotter.

(21) The Square Roots of the Distances being as the Times, viz. as the $\sqrt[2]{1}$: $\sqrt[2]{2}$ :: is the Time of falling through the whole required Height.

Now the $\sqrt[2]{1}$=1, and $\sqrt[2]{2}$=1,4142, from which take 1. *Remains* ,4142.

∴ As, 4142 : 1,4142 :: 1 : 3,414 Secs. the Descent. Its Square of which is 11,6574.

Then as 1 : 16,083 :: 11,6574 : 187,48 ft. the Tower's Height.

(22) First $7970 \times 7970 \times 7970 = 506261573000$.

And $2170 \times 2170 \times 2170 = 10218313 13000$.

Then, as 10218313 13000 : 506261573000 :: 1 : 49,5445 Times bigger than the Moon.

∴ As 132,5 : 100 :: 49,5445 : 40,117, Q. E. F.

That is, the Earth contains 40,117 Times more Matter than the Moon.

(23) First $81000000 + 240000 = 81240000$, Sun from a full Moon.

And $81000000 - 240000 = 80760000$, Sun from a new Moon.

Now $\left\{\begin{array}{l} 8124 \times 8124 = 65999376 \\ 8076 \times 8076 = 65221776 \end{array}\right\}$ Squares of their Distances, Ciphers omitted.

Receip. As 65221776 : 1 :: 65999376 : ,9832

So that the Proportion of Light and Heat a new Moon has to that of a full one, is

As 1 to ,9882, or as 458329 to 452929 in whole Numbers.

(24) First $11 \times 11 = 121$, and the Square of 1 is 1.

Then, As 1 : 16,083 :: 121 : 1946,083. Q.E.F.

Or

$$16{,}083 \times \left\{\begin{array}{l} 1 = 16{,}083 \\ 3 = 48{,}25 \\ 5 = 80{,}416 \\ 7 = 112{,}583 \\ 9 = 144{,}75 \\ 11 = 176{,}916 \\ 13 = 219{,}083 \\ 15 = 241{,}25 \\ 17 = 273{,}416 \\ 19 = 305{,}583 \\ 21 = 337{,}75 \end{array}\right\} \text{ in the } \left\{\begin{array}{l} 1\text{st.} \\ 2\text{d.} \\ 3\text{d.} \\ 4\text{th.} \\ 5\text{th.} \\ 6\text{th.} \\ 7\text{th.} \\ 8\text{th.} \\ 9\text{th.} \\ 10\text{th.} \\ 11\text{th.} \end{array}\right\} \text{ Sec. of Time.}$$

Answ. 1946,083 as before.

(25) First $10 \times 10 = 100$, and $6 \times 6 = 36$, Square of their Descents.

Then, as 1 : 16,083 :: $\left\{\begin{array}{l} 100 : 1608{,}3 \\ 36 : 579 \end{array}\right\}$ their Depths.

Answer, 1029,3 Difference.

(26) First 19,5 × 19,5 = 380,25 Sq. of the Descent.
Then, As ^{2}1 : 16,083 :: 380,25 : 6115,6875 Feet.
Which ÷ 6 = 1019 faths. 1 ft. 8¼ inc. Q. E. F.

(27) Thus, As, ,16,083 : ^{2}1 :: 400 : 24 ,8702.
$\therefore \sqrt[2]{24,87002} = 4,987+$, or 5 secs. nearly the Answer.

(28) The Semi-diameter of the Earth = 3980 Miles, or 21014400 ft.
Then, as 16,083 : 1^2 :: 21014400 : 1306594,82.
$\therefore \sqrt{1306594,82} = 1135,554$ sec. = 18 Mn. 55″ 33‴ Q. E. F.

57. The DOUBLE RULE of THREE in DECIMALS.

(1) First, as 1,1£. : 12 Per. :: 1,6£. : 17,45 Persons.
Then Receip. as 2,52£. : 27,45 :: 18,9£. : 23 Persons, nearly the Answer.

(2) As 19lb. : 4,25lb. :: 13lb. : 2,908lb.
Also 2,908 : 6 :: 10 : 20,6327lb. worth 63lb. at 5*d.* per lb. = 63 × 5 = 315*d.* or 1,3125£.
Then, as 20,6327 : 1,3125£. :: 112lb. : 7,12467£. = £.7 2*s.* 5¾*d.* 9 (nearly) the Answer.

£.	P.	yr.	I.
(3) 100	.	1 .	3,5
——		1,25	38,5

Then $\frac{100 \times 1 \times 38,5}{3,5 \times 1,25} = \frac{3850}{4,375}$ £. = 880£. the Answer.

(4) First from December 11th, to May 10th. = 150 Days.
And from Sept. 3d to Christmas-Day = 113.
Also 91 Guin. = 95,55£. and 100 Mar. = 66,6£.
Then, As 95,55£. : 150 Days, :: 66,6£. : 215 Days.
$\therefore$ 215 — 113 = 102 Days.
Receip. as 66,6 : 102 :: 40£. : 169,13 Days, or rather 170 Days, the Answer.

(5) *Days.* *w*

As { 12, 14, 15, 18 } *Work.* *Day.* : 1 :: 1 : { ¹⁄₁₂ = ,0833333 = A.B.C., ¹⁄₁₄ = ,0714285 = B.C.D., ¹⁄₁₅ = ,0666666 = A.C.D., ¹⁄₁₈ = 0,555557 = A.B.D. } Will do Part of the whole Work.

All working three Days will do ,276984 Part of the Work.

Then, As 276984 : 3 :: 1 : 10,8309505 Days, all working.
14=B. C. D.
—10,83095 A. B. C. D.

As 3,16905 : 10,8395 :: 14 : 47,848 Days by A.

15=A. C. D.
—10,83095

As, 4,16905 : 10,8395 :: 15 : 38969 Days by B.

18=A. B. D
—10,83095

As 7,16905 : 10,8395 :: 18 : 27,194 Days by C.

12=A. B. C.
—10,83025

As 1,16905 : 10,8395 :: 11 : 111,1766 Days by D.

Alone.

(6) First 34″=,009444 ho. and 5 Rods,=,015625 Miles.
Then, as 1 ho. : 12 Miles, :: ,009444 ho. : ,113328 m.
∴ ,113328+,015625=,128953 m. or 680,9 ft. the Hare had started.
Now 20—12=8, Dog gained in running 20.
Again, as 8 : 20 :: ,12895 : 2,57906 fur.=1702$\frac{1}{4}$ ft. run by the Greyhound.
∴ As 8 : ho. :: ,128953 m. : ,016119 ho.=58″,0284 run by the Greyhound.

I. VIBRATION OF PENDULUMS.

(7) Receip. As 3600 : 39,2 :: { 900 : 156,8. / 14400 9,8. }

Answer, Inches 147

(8) Receip. As 39,2 : 3600 :: { 12 : 11760 / 6 : 23520 }

$\therefore \sqrt[2]{11760} = 108,444$, and $\sqrt[2]{23520} = 153,362$.
Then $153,362 \times 60 = 9201,72$
And $108,444 \times 60 = 6506,64$

Answer, 2695,08 the Difference.

(9) A Pendulum which vibrates 60 Times in a Minute will vibrate $60 \times 60 = 3600$ Times in an Hour; the Square of which $= 12960000$, and the Square of $30 = 900$; also of $100 = 10000$. Then Receip.

As 12960000 : 39,2 inc. :: { 900 : 564480 / 10000 : 50803,2

12)513676,8 Inches.

Answer Diff. 42806,4 Feet.

(10) First, a Pendulum which vibrates Thirds will vibrate 60 Times in a Second.

$\therefore$ As 1^2 : 39,2 :: 60^2 (3600) : ,0108 inc. the Length of that which vibrates Thirds.

Secondly, a Pendulum which vibrates Seconds will make 60 Vibrations in a Minute.

$\therefore$ As 60^2 (3600) : 39,2 :: ^{21} : 11760 inc. $= \frac{5}{12}$ Miles, the Length of a Pendulum which vibrates once in a Minute.

Thirdly, a Pendulum which vibrates Seconds will make $(60 \times 60) = 3600$ Vibrations in an Hour.

$\therefore$ As 3600^2 (12960000) : 39,2 :: ^{21} : 508032000 inc. $= 8018\frac{9}{12}$ Miles, the Length of the Pendulum, which vibrates once in an Hour.

Lastly, a Pendulum which vibrates Seconds will make $(60 \times 60 \times 24) 86400$ Vibrations in a Day, or 24 Hours.

$\therefore$ As 86400^2 (7464960000) : 39,2 :: ^{21} : 292626432000 inc. $= 4612818\frac{1}{7}\frac{5}{3}\frac{1}{6}\frac{1}{0}$ Miles, the Length of a Pendulum which vibrates once in a Day.

(11) First $60 \times 60 = 3600$.

Then, As 39,2 inc. : 3600 sec. :: 18 inc. : 7840 sec.

$\therefore \sqrt[2]{7840} = 88,54370$ Vibrations in one Minute.

Then 60)88,54378(1,475729 Vibrations in a Second.

Also 1,475729)8,000000(5,421 sec. in eight Vibrations.

Now 5,421 × 5,421 = 29,387.
Then, As 21 : 16,083 ft. :: 29,387 : 472,64091β ft. Sound was returning.
Again, As 1150 ft. : 1 sec. :: 472,64091β : 41099″, Time Sound was returning.
∴ 5,421 — ,411 = 5,01 Seconds, Time of the Body's Descent.
And 5,01 × 5,01 = 25,1. Also 25,1 × 16,083 = 403,69 ft. the Depth of the Well.

58. FELLOWSHIP.

(1) First, 750+450+300 = 1500 the whole Stock.
And 300£. ÷ 1500 = ,2 the Quotient. Then,

$$\left.\begin{matrix}750\text{ A's}\\450\text{ B's}\\300\text{ C's}\end{matrix}\right\}\text{Stock} \times ,2 = \left\{\begin{matrix}150\text{ A's}\\90\text{ B's}\\60\text{ C's}\end{matrix}\right\}\text{Gain.}$$

Proof 300£.

(2) First $\left\{\begin{matrix}120\\250\\100\end{matrix}\right\} \times \left\{\begin{matrix}8\\4\\5\end{matrix}\right\} = \left\{\begin{matrix}960\text{ A's}\\1000\text{ B's}\\500\text{ C's}\end{matrix}\right\}$ Stock and Time.

Sum 2460)184,5(,075 the Quot.

Then $\left\{\begin{matrix}960\\1000\\500\end{matrix}\right\} \times ,075 = \left\{\begin{matrix}72\\75\\37,5\end{matrix}\right\}\begin{matrix}\text{A's}\\\text{B's}\\\text{C's}\end{matrix}\Big\}\text{Gain.}$

Proof 184,5 = £.184 10s.

(3) First £.229 13s. 4d. = 2296 : $5\frac{5}{9} = 5,5$ :: $4\frac{3}{7} = 4,\dot{4}2857\dot{1}$ × $4,4\frac{1}{9} = 4,\dot{4}$; and $3\frac{2}{3} = 3,\dot{6}$.
Then Nell takes up 3,6, as often as Anne takes up 4,4; then as often as Moll takes up 5,5, Anne takes up 4,428571; and Nell must take

$$\frac{3,6 \times 4,428571}{4,1} = 3,949809$$

For, As 4,1 : 3,6 :: 4,428571 : 3,949809.

Hence Moll takes 5,555555
Anne 4,428571 } Ratio of their Share.
And Nell 3,949809

Sum 13,933935

		£.	s.	d.
∴ As 13,933935 : 229,6£. ::	5,555555 : 91,569 =	91	11	$4\frac{1}{4}$ M.
	4,428571 : 72,995 =	72	19	$10\frac{3}{4}$ A.
	3,949809 : 65,102 =	65	2	$\frac{1}{4}$ N.

59. SIMPLE INTEREST.

Theorem, I. P t r = I. the Interest.

(1) Here is given P=260,875, t=5,5 and r=,045.
Then per Theo. 260,875 × 5,5 × ,045 = 64,5665625 = £.64 11*s.* $3\frac{3}{4}$*d.* ,9, the Interest required.

(2) First, from May 12, 1784, to Nov. 24, 1789, is 5 yrs. 126 ds. or 5,536986 yrs. = t, r =, 0375, and P=500.
Then per Theo. 5,536986 × ,0375 × 500 = 103,8184875 = £.103 16*s.* $4\frac{1}{4}$*d.* ,48 the Answer.

When the Interest is for Days.

(3) Here r =,00012328767, t =, 220 and P=370,5.
Then per Rule, ,00012328767 × 370,5 × 220 = 10,049 + = £.10 $11\frac{3}{4}$*d.* the Interest required.

(4) First, from July 1, to Feb. 24, following, is 238 Days = t, P=600, and r =,00016438356.
Then 00016438356 × 238 × 600 = 23,4739723628 = £.23 9*s.* $5\frac{1}{4}$*d.* the Interest.

Thorem II. P, t. r. + P = A, the Amount.

(5) Here P=284,5, t=7, and r=,035.
Then per Theo. 284,5 × 7 × ,035 + 284,5 = 354,2025 = £.354 4*s.* $0\frac{1}{2}$*d.* ,4, the Amount required.

(6) Here P=672,25, t=5,5 and r=,045.
Then per Theorem, 672,25 × 5,5 × ,045 + 672,25 = 838,631875 = £.838 12*s.* $7\frac{1}{2}$*d.* ,6, the Answer.

(7) Here P=500, t=6,328767, and r=,0475.
Then per Theorem VI. 328767 × ,0475 × 500 + 500 = 650,30821625 = £.650 6*s.* $1\frac{3}{4}$*d.* 8+, the Amount.

Theorem III. $\frac{I}{tr}$ = P, the Principal.

(8) Here I=69,675, t=3, and r=,05.
Then per Theo. $\frac{69,675}{3\times,05}=\frac{69,675}{,15}=464,5=464l.$ 10*s.* the Principal.

(9) Here I=64,35, t=5,5, and r=,045.
Then per Theo. $\frac{64,35}{5,5\times,045}=\frac{64,3500}{,2475}=260£.$ the Answ.

(10) Here I=67,790625, t=4 and r=,04.
Then per Theo $\frac{67,790625}{4\times,04}=\frac{67,790625}{,16}=423l.$ 13*s.* $9\frac{1}{2}d.$,75+, the Principal required.

Theorem IV. $\frac{a}{tr+1}$ = P, the Principal.

(11) Here a=354,2208$\frac{3}{4}$, t=7, and r=,035.
Then per Theo. $\frac{354,2208\frac{3}{4}}{,035\times7+1}=\frac{334,22083}{1,245}=284,5147$
=284*l.* 10*s.* $3\frac{1}{2}d.$,112, the Answer.

(12) Here A=500,460416, t=6,41β, and r=,05.
Then per Theo. $\frac{500,460416}{6,41\beta\times05+1}=\frac{500,460416}{1,32083}=$
378,89826+=378£. 17*s.* $11\frac{1}{2}d.$,2296, the Answer.

(13) Here A=100 t=7,6027397, and r=,0475.
Then per Theo. $\frac{100}{7,6027397\times,0475+1}=\frac{100}{1,36113}=$
73,4675+=73*l.* 9*s.* 4*d.* ,2, the Principal.

Theorem V. $\frac{I}{pr}$ = t, the Time.

(14) Here I=69,675, P=4645, and r=,05.
Then per Theo. $\frac{69,675}{464,5\times,05}=\frac{69,675}{23,225}=3$ yrs. the Time required.

(15) Here I=64,45 P=260, and r=,045
Then per Theo. $\frac{64,35}{260\times,045}=\frac{64,35}{11,7}=5\frac{1}{2}$ yrs. the Ans.

(16) Here I=130,4751℔, P=500, and r=,065.
Then per Theo. $\frac{130,4791℔}{,065\times500}=\frac{130,47516}{32,5}$=4,0144+=4
Years, 5,25 Days, the Answer.

Theorem VI. $\frac{a-P}{Pr}=t$, the Time.

(17) Here a=354,22083, P=284,5, and r=,0325.
Then per Theo. $\frac{354,22083-284,5}{284,5\times,0325}=\frac{70,72083}{9,24625}$=7,65405=
7 Years, 238,7 Days, the Answer.

(18) Here a=847,875, P=672,25, and r=,0475.
Then per Theo. $\frac{847,875-672,25}{672,25\times,0475}=\frac{175,625000}{31,931875}=5\frac{1}{2}$ ys.
the Answer.

(19) Here a=500,464583, P=378,9, and r=,05.
Then per Theo. $\frac{500,464583-378,9}{378,9+,05}=\frac{121,564583}{18,945}$=
6,41467=6 Years, 5 Months (nearly) Answer.

Theorem VII. $\frac{I}{pt}=r$, the Rate.

(20) Here I=69,675, P=464,5, and t=3.
Then per Theo. $\frac{69,675}{494,5\times3}=\frac{69,675}{1393,5}$=,05, or 5 per Cent.
the Rate required.

(21) Here I=64,35, P=260, and t=5,5.
Then per Theo. $\frac{64,35}{260\times5,5}=\frac{64,35}{1430}$=,045, or $4\frac{1}{2}$ per Cent.
the Answer.

(22) Here I=235,4℔, P=560,63541℔, and t=7.
Then per Theo. $\frac{225,2℔}{560,63541℔\times7}=\frac{235,4℔666666}{3924,44791℔}$=,06,
or 6 per Cent. the Answer.

Theorem VIII. $\frac{a-P}{Pt}=r$ the Rate

(23) Here $a=354,2208\mathfrak{z}$, $P=284,5$ and $t=7$.
Then per Theo. $\frac{354,2208\mathfrak{z}-284,5}{284,5\times 7}=\frac{69,7208\mathfrak{z}}{1991,5}=,035$, or $3\frac{1}{2}$ per Cent. the Answer.

(24) Here $a=500,463541\mathfrak{h}$, $P=378,9$, and $t=6$.
Then per Theo. $\frac{500,463541\mathfrak{h}-378,9}{378,9\times 6}=\frac{121,563541\mathfrak{h}}{2273,4}$,05372+, or £.5 6*s.* $11\frac{1}{4}d.$,512, per Cent. the Answ.

(25) Here $a=847,875$, $P=672,25$, and $t=5,5$.
Then per Theo. $\frac{847,875-672,25}{672,25\times 5,5}=\frac{175,62500}{3697,375}=,475$ or $4\frac{3}{4}$ per Cent the Answer.

60. Of ANNUITIES, PENSIONS, &c. in ARREARS, at SIMPLE INTEREST.

Theorem 9. $\frac{ttu-tu}{2}\times r:+tu=A$, the Amount.

(26) Here $u=250$, $t=6$, and $r=,03$.
Then per Theo. $\frac{250\times 6\times 6-250\times 6}{2}\times,03:+250\times 6=$
$\frac{9000-1500}{2}\times,03:+5100=\overline{3750\times,03}+1500=1125,$
$+1500=1612,5$, or £.1612 10*s.* the Answer.

(27) Here $u=125$, $t=12$, and $r=,015$, per Note.
Then per Theo. $\frac{125\times 12\times 12-125\times 12}{2}\times,015:+125\times$
$12=\frac{18000-1500}{2}\times,015:+1500=\overline{8250\times,015}+$
$1500=123,75+1500=1623,75=1623$£. 15*s.* the Ans.

(28) Here $u=62,5$, $t=24$, and $r=,0075$, per Note.

Then per Theo. $\frac{62,5\times 24\times 24-62,5\times 24}{2}\times,0075 : + 62,5\times 24=\frac{36000-150}{2}\times,0075 : + 1500 =$ $17250\times,0075+1500=129,375+1500=1629,375=$ 1629*l*. 7*s*. 6*d*. the Answer.

Theorem X. $\frac{2a}{ttr-tr+2t}=$ U, the Annuity.

(29) Here $a=1612,5$, $t=6$, and $r=,03$.

Then per Theo. $\frac{1612,5\times 2}{6\times 6\times,03-6\times 2}=\frac{3225,0}{2,9}=$ 250*l*. the Answer.

Theo. for half yearly Payment. $\frac{4a}{ttr-tr+2t}=$U.

(30) Here $a=1623,75$, $t=12$, and $r=,015$.

Then per Theo. $\frac{1623,72\times 4}{12\times 12\times,015-12\times,05+12\times 2}=$ $\frac{6495}{1,98+24}=\frac{6495,00}{25,98}=250$*l*. the Answer.

Theo. for quarterly Payment. $\frac{8a}{ttr-tr+2t}=$U.

(31) Here $a=1629,375$, $t=24$, and $r=,0075$.

Then per Theo. $\frac{1629,375\times 8}{24\times 24\times,0075-24\times,0075+24\times 2}=$ $\frac{13035}{4,14\times 48}=\frac{13035,00}{52,14}=250$*l*. the Answer

Theo. XI. $\frac{2a-2ut}{utt-ut}=r$, the Rate.

(32) Here $a=1612,5$, $u=250$, and $t=6$.

Then per Theo. $\frac{1612,5\times 2-250\times 2\times 6}{250\times 6\times 6-250\times 6}=\frac{3225-3000}{9000-1500}=$ $\frac{225,00}{7500}=,03$, or 3 per Cent. the Answer.

Theo. for half yearly Payments, $\frac{4a-4ut}{utt-ut}=r$.

(33) Here $a=1623{,}75$, $u=125$, and $t=12$, per Note.

Then per Theo. $\frac{1623{,}75\times4-125\times4\times12}{125\times12\times12-125\times12}=\frac{6495-6000}{18000-1500}$

$=\frac{495{,}00}{16500}=,03$ or 3 per Cent. the Answer.

Theo. for quarterly Payments, $\frac{8a-8ut}{utt-ut}=r$.

(34) Here $a=1629{,}375$, $u=62{,}5$, and $t=24$.

Then per Theo. $\frac{1629{,}375\times8-62{,}5\times8\times24}{62{,}5\times24\times24\times-62{,}5\times24}=$

$\frac{13035-12000}{36000-1500}=\frac{1035{,}00}{34500}=,03$ or 3 per Cent. Answer.

Theorem XII. First $\frac{2}{r}-1=X$. Then $\sqrt{\frac{2a}{ur}+\frac{xx}{4}}:\frac{x}{2}=T$.

(35) Here $a=1612{,}5$, $u=250$, and $r=,03$.

Then per Theorem I. $\frac{2}{,03}-1=65{,}\not{6}=X$.

Then $\frac{\sqrt{1612{,}5\times2}}{250\times,03}+\frac{65{,}\not{6}\times65{,}6}{4}:\frac{65{,}\not{6}}{2}=$

$\frac{\sqrt{3225}}{7{,}5}+\frac{5{,}307{,}73}{4}:-32{,}8=\sqrt{1506{,}93}-32{,}8=38{,}8-$

$32{,}8=6$ Years, the Answer.

(36) Here $a=1623{,}75$, $u=125$, and $r=,015$.

Then per Theorem I. $\frac{2}{,015}-1=132{,}\dot{3}=X$.

Then $\sqrt{\frac{1623{,}75\times2}{125\times,015}+\frac{132{,}\dot{3}\times132{,}\dot{3}}{4}}:\frac{132{,}\dot{3}}{2}=$

$\sqrt{\frac{3247{,}5}{1{,}875}+\frac{17507{,}68}{4}}:-66{,}=\sqrt{1752+4370{,}92}$

$:-66{,}1=\sqrt{6108{,}92}-66{,}1=78{,}1-66{,}1=12$ Half yrs. or 6 yrs. the Time required.

(37) Here $a=1629,375$, $u=62,5$, and $r=,0075$.
Then p Theorem I. $\frac{1}{,007}-1=265,\not{6}=X$.

Then $\frac{\sqrt{1629,375\times 2}}{62,5\times ,0075}+\frac{265,\not{6}\times 265,\not{6}}{4}:\frac{2\not{6}5\not{6}}{2}=$
$\frac{\sqrt{3258,5}}{,46875}+70\ 61,0\not{6}:132,8=\sqrt{6952+17690,26}$
$-132,\not{8}=\sqrt{24642,\ 26}-132,\not{8}=1569,-132,\not{8}=24$
Quarterly Payments, or 6 Years the Answer.

61. PRESENT WORTH OF ANNUITIES, &c.

Theorem XIII. $\frac{ttr-tr+2t}{2tr+2}:\times u=$ P. the present Worth.

(38) Here $t=6$, $r=,03$, and $u=250$.
Then per Theo. $\frac{6\times 6\times ,03-6\times 03+6\times 2}{2\times 6\times ,03+2}:\times 250=$
$\frac{1,08-,18+12}{,36+2}\times 250=\frac{12,9}{2,36}\times 250=5,4661017\times$
$250=1366,52675$, or $1366\pounds$. 10*s*. 6*d*. ,42, the present Worth required.

(39) Here $t=12$, $r=,015$, and $u=125$.
Then per Theo. $\frac{12\times 12\times ,015-1+,015+12\times 2}{12\times 2\times ,015+2}:\times 125$
$=\frac{2,16-18+24}{,36+2}:\times 125=\frac{1,98+24}{2,36}:\times 125\times$
$\frac{25,98}{2,36}\times 12=11,008474\times 125=1376,05925=1376\pounds$.
1*s*. 2*d*. ,22, the present Worth.

(40) Here given $t=24$, $u=62,5$ and $r=,0075$.
Then per Theo. $\frac{24\times 24\times ,0075-24\times ,0075+24\times 2}{24\times 2\times ,0075+2}:\times$

$6,25 = \frac{4,32 - ,18 + 48}{2,36} : \times 62,5 = \frac{4,14 + 48}{2,36} : \times 62,5$

$= \frac{52,14}{2,36} : \times 62,5 = 22,09322 \times 62,5 = 1380,82625 =$

1380£. 16*s*. 6*d*. ,3, the Answer.

Theorem XIV. $\frac{tr+1}{tt\,r - t\,r + 2t} : \times 2p = U$, the Annuity.

(41) Here is given $t=6$, $r=,03$, and $p=1366,525$. To find U.

Then per Theo. $\frac{6\times,03+1}{6\times6\times,03-6\times,03+6\times2} : \times 1366,525 \times 2$

$= \frac{1,18}{12,9} \times 2733,05 = ,0914728 \times 2733,05 = 250$£. the Annuity required.

(42) Here is given $t=12$, $r=,015$, and $p=1376,25$. To find U.

Then per Theo. $\frac{12\times,015+1}{12\times12\times,015-12\times,015+12\times2} : \times$

$1376,25\times4 = \frac{1,18}{25,98} \times 5505 = ,0454195 \times 5505 = 250$£. 8½*d*. the Annuity required.

(43) Here is given $t=24$, $r=,0075$, and $p=1380,875$. To find U.

Then per Theo. $\frac{24\times,0075+1}{24\times24,0075\times12\times,0075+24\times2} \times$

$1380,875\times8 = \frac{1,18}{52,14} \times 11047 = ,022631 \times 11047 = 250$£. the Answer.

Theorem XV. $\frac{ut - p\times2}{2pt + ut - u\,t\,t} = R$, the Rate.

(44) Here is given, $u=250$, $t=6$, and $p=1366,525$. To find R.

Then per Theo. $\frac{250\times 6-1366,525\times 2}{1366\ 525\times 2\times 6+250\times 6-250\times 6\times 6}$

$=\frac{133.475\times 2}{17898\ 39-00}=\frac{266.95}{8898.3}=,03=3$ per Cent. the Answer.

(45) Here is given $u=125$, $t=12$, and $p=1376,25$. To find R.

Then per Theo. $\frac{125\times 12-1376.25\times 2}{1376.25\times 2\times 12+125\times 12-125\times 12\times 12}$

$=\frac{1500-1376,25\times 2}{33030+1500-18000}=\frac{123,75\times 2}{34520-18000}=\frac{2275}{165000}$

$=,015$ half the Rate, which $\times 2=,03$ or 3 per Cent. the Answer.

(46) Here is given $u=62,5$, $t=24$, and $p=1380.875$. Then per Theorem.

$\frac{62.5\times 24-1380,875\times 2}{1380.875\times 2\times 24+52.5\times 24-62.5\times 24\times 24}=$

$\frac{238.25}{31872}=,0075$, which $\times 4=,03$ or 3 per Cent. the Answer.

Theo. XVI. $\frac{2}{r}+\frac{2p}{u}\ 1=X$. Then $\sqrt{\frac{2p}{ur}+\frac{xx}{4+2}}=t$ the Time.

(47) Here is given $u=,250$, $p=1366.525$, and $r=,03$.

Then per Theo. first $\frac{2}{,03}\frac{1366,525\times 2}{250}1=66.6-$

$11,9322=54,73446=X$.

Then $\sqrt{\frac{1360\ 525\times 2}{200\times,03}+\frac{54.73446\times 54.73446}{4}}$

$\frac{54.73446}{2}=\sqrt{2732.05}+\frac{2005080.}{4}\ 27.3=\sqrt{1113,36}=$

$27,3=33.3-27\ 3=6$ Years, the Answer.

(48) Here is given $u=125$, $p=1376,25$, and $r=,015$.

Then per Theo. first $\frac{1376,25\times 2}{125}^2 \div ,015 - 1 = 133,3$

$23.02=110.313=X$.

Then $\sqrt{\frac{1376.25\times 2}{125\times,015}+\frac{110.313\times 110.313}{4}}-\frac{110.313}{2}=$

$\sqrt{4510.2486}-55.15=67.15-55.15=12$ half yearly Payments, or 6 Years the Time required.

(49) Here is given, $u=62.5$, $p=1380.875$, and $r=,0075$.

Then per Theo. first $\frac{1380.875\times 2}{62.5}^2 \div ,0075 - 1=266.6-$

$45.188=221.4786=X$.

Then $\sqrt{\frac{1380.875\times 2}{62.5\times,0075}+\frac{221,4786\times 221,4786}{4}}-\frac{221.4786}{2}=\sqrt{5891.733+12263.1961}-110.7393=$

$\sqrt{18154.92}-110.7=134.7-110.7=24$ quarterly Payments, or 6 Years, the Time required.

62. ANNUITIES, &c. TAKEN IN REVERSION.

Theo. XVII. $\frac{ttr-tr+2t}{2tr+2}:\times u=p$, which change to a.

Then $\frac{a}{tr+1}=p$, the present Worth.

(50) Here first $U=250$, $t=6$, and $R=,03$, then per Theo. I.

$\frac{ttr-tr+2t}{2tr+2}:\times U=\frac{6\times 6.03\times 6\times,03+6\times 2}{6\times,03\times 2+2}:\times 250=$

$\frac{1.08-,18+12}{,36+2}\times 250=\frac{,9+12}{2.36}\times 250=\frac{12.9}{2.36}:\times 250=$

$5.4661016\times 250=1366.5254=P$.

Now per Rule 2. $A=1366.5254$, $t=4$, and $r=,03$.

Then per Theo. II. $\frac{a}{tr+1}=\frac{1366.5254}{4\times,03\times 1}=\frac{1366.5254}{1.12}=$

1220.112, or 1220*l.* 2*s.* $3\frac{3}{4}$*d.* the present Worth required.

(51) Here is given first $r=,045$, $t=7.5$, and $u=80$.

Then per Theo. I. $\frac{7.5\times 7.5\times,045-7.5\times,045+7.5\times 2}{7.5\times 2\times,045+2} : \times 80 = \frac{2,53125-,3375+15}{,675+2}\times 80 = \frac{17,10375}{2,675}\times 80 =$

$6.42756\times 80 = 514.2048 = p$.

Now per Theo. II. $p=a$, and r, as before $=5$.

Then $\frac{514.2048}{5\times,045+1} = \frac{514.2046}{1.225} = 419.95759 = 419\text{£}.\ 19s.$

$1\frac{1}{2}d.$ the Answer.

(52) Here is given $u=40$, $t=10$, and $r=,05$.

Then per Theo. I. $\frac{10\times 10\times,05-10\times,05+10\times 2}{10\times 2\times,03+2}\times 40$

$= \frac{4.5+20}{1+2}\times 40 = \frac{24.5}{3}\times 40 = 80.1\text{ℓ}\times 40 = 326.\beta = p$.

Now per Theo II. $a=326,\beta$, and $t=7$.

Then $\frac{326.6}{7\times,05+1} = \frac{326.6}{1,35} = 241.9755 = 241\text{£}.\ 19s.\ 6d.$

$,12$, the present Worth.

Theorem XVIII. $ptr+p=A$, the Amount. Change A, and call it p, and t here will $=$ the Time of its Continuance.

Then $\frac{tr+1}{ttr-tr+2t} : \times 2p = U$, the Annuity, &c.

(53) Here is given $p=1220.111458\mathcal{Z}$, $t=4$, and $r=,03$.

Then per Theorem I. $1222.111458\mathcal{Z},\ \times\ 4\times,03.$

$1220.111458\mathcal{Z} = 146,413375 + 1220,111458\mathcal{Z} = 1366.52483 = A$.

Then per Theo. II. $\frac{6\times,03+1}{6\times 6\times,03-6\times,03+6\times 2} : -$

$1366,52483\times 2 = \frac{1,18}{12,9}\times 2733,049\beta = 2723,049\beta$

$\times,0914728 = 250\text{£}.$ the Annuity.

(54) Here is given $p=419,755203$, $t=5$, and $r=,045$.

Then per Theorem I. 419,755208×5×,045+419,755208
=94,4449218+419,755208=514,2001298=A.
Now A, changed becomes p, and t here=7,5.
Then per Theo. II. $\frac{7,5\times,04+1}{7,5\times7,5\times,045-7,5\times,045+7,5\times2}$
×514,2001298×2=$\frac{1,3375}{17,19375}$×1028,4002596=
,077789×1028,4002596=80£. the Answer.

(55) Here is given p=241,975308, t=7, and r=,05.
Then per Theorem I. 241,975308 × 7,05+241,975308
=326,6666658=A.
Now, A. changed becomes p=326,6666658, and t=10.
Then per Theo. II. $\frac{10\times,05+1}{10\times10\times,05-10\times,05+10\times2}$
× 326,6666658 × 2 = $\frac{1,5}{24,5}$ × 653,3333316 =
653,3333316×,06122448=80£. nearly the Answer.

63. REBATE OR DISCOUNT.

Theorem XIX. $\frac{s}{tr+1}$=P, the present Worth.

(56) Here s=150, t=,75 and R=,05; then per Theorem
$\frac{s}{tr+1}$=$\frac{150}{,75\times,05+1}$ ——=$\frac{150}{1,0375}$=144,5784+ or
144£. 11s 6¼d. the present Worth required.

(57) Here is given s=1000, t=5 mo.=,116 yr. and r=,045.
Then per Theo. $\frac{1000}{,416\times,045+1}$=$\frac{1000}{1,01875}$=981,595=
981£. 11s. 10¼d. the present Worth required.

(58) Here is given s=9342, r=,04, and t=10 mo.=,83 yr.
Then per Theo. $\frac{9342}{,83\times,04+1}$=$\frac{9342}{1,03}$=90,38,9 the present Worth.

And 9342—9038.9=303.1=303*£*. 2*s*. the Discount.
Theorem XX. P $tr+p$=S. the Sum due.

(59) Here is given p=144,578125, t=,75, and r=,05.
Then per Theo. 144.578.125×,75×,05+144 578125=
5.421679+144.578.125=149.9998, or rather 150*£*.
the Answer.

(60) Here is given p=981.52085, t=,416, and r=,045.
Then per Theorem, 981.52083×,416×,045+981.52=
18,403518+9981.52083=999.9243485=999*£*. 18*s*.
5¾*d*. the Sum due.

(61) Here is given p=9111.185416, t=,83, and r=,045
Then per Theo. 9111.185416×,83×,04=9111.185416
=303.70618+9111.185416=9414.891326=9414*£*.
17*s*. 9¾*d*. 6, the Debt.

Theorem XXI. $\frac{s-p}{pr.}$=T, the Time.

(62) Here is given s=150, p=144.578125, and r=,05.
Then per Theo. $\frac{150-144.578125}{144.578125\times,05}=\frac{5.42187500}{7.22890625}=,75$
yr. or 9 Months, the Time required.

(63) Here is given s=1000, p=981.52083, and r=,045.
Then per Theo. $\frac{1000-981.52083}{981.52083\times,045}=\frac{18,47916}{44.16845749}=$
,416 or 5 months the Answer.

(64) Here is given s=9342, p=9111.185416, and r=,04,
Then per Theo. $\frac{9342-9111.185416}{9111.185416\times,04}=\frac{230\,814584}{364.447416}=$
,63332=7mo. 18 Days the Answer.

Theorem XXII. $\frac{s-p}{pt}$=R, the Rate.

(65) Here is given s=150, p=144.578125, and t=,75.
Then per Theo. $\frac{150-144.578125}{144.578125\times,75}+\frac{5,421875}{108,43359375}=$
,05 or 5 per Cent. the Rate required.

(66) Here is given s=1000, p=981.52083, and t=,416.

Then per Theo. $\frac{1000-981,52085}{981.52083\times,416}=\frac{18.47916}{40.89570138}=$
,045 or $4\frac{1}{2}$ per Cent. the Answer.

(67) Here is given $s=9342$, $p=9111.185416$, and $t=,83$.
Then per Theo. $\frac{9342-9111,185416}{9111,185416\times,83}=\frac{230.814584}{769.26989582}=$
,03 or 3 per Cent. the Answer.

64. EQUATION OF PAYMENTS.

Theorem XXIII. $\frac{s}{tr+1}=P$, the present Worth.

(68) Here is given $r=,035$, the first $t=,5$, the second, and the third $=,83$.
Then per Theo. the present Worth will be as follows, viz.

	£.
Paid down	400
1. $\frac{500}{,5\times,035+1}=\frac{500}{1,075}=$	491.40049
2. $\frac{250}{,6\times,035+}=\frac{250}{1.0213}=$	244.78605
3. $\frac{250}{83\times.035+1}=\frac{250}{1.025}=$	243.002439
the Sum	$1380.089179=p.$
	$1400=s.$
	$s-p=$ Diff. $19.910821=d.$

Now per Theo. II. $\frac{d}{pt}=\frac{19.910821}{1380.8179\times,035}=\frac{19.9108210}{48.303121265}$
$=,41013$ or 5 Months nearly the true equatic Time required.

(69) As 12.3£. : 47 Days :: 949.5£. : 18.45 Days, the Answer.

(70) Here is given $a=542.4$, $p=384$, and $t=8.25$.
Then per Theo. VIII. $\frac{a-p}{pt}=\frac{542.4-384}{384\times8.25}=\frac{185.4}{3168}=$
,05 or 5 per Cent. the Answer.

65. COMPOUND INTEREST.

Theorem I. $p\,r^t$=A. the Amount.

(1) Here is given p=246.5, t=7, and r=1.05.
Then per Theorem.

246.5 × 1,05 × 1.05 × 1.05 × 1.05 × 1.05 × 1.05 × 1.05 = 246.5 × 1 407004 = 346.8502486 = 346*£*. 17*s*. the Amount required.

By Table I. thus,

Again, 7 yrs. under 3 per Cent. is	1.4071004
Which multiplied by the Principal	246.5

Gives the Amount as before, viz. 346.8502486=346*£*. 17*s*.

(2) Here is given p=500, t=30, and r=1,045, which raised to the 30th Power will give 3.7453181=r^t. (See Table I.)

Then 3.7453181 × 500 = 1772.65905 = 1772*£*. 13*s*. 2*d*. ,172, the Amount for 30 Years.

Again, suppose the Amount was required only for 30*d*.

Then r=1.0001206, which raised to the 30th Power, = 1.0036243. (See Table II.) the Amount of 1*£*. for 30 Days.

∴ 1.0036243 × 500 = 501.81215 = 501*£*. 16*s*. 3*d*. ,156, the Amount.

(3) Here is given p=523, t=5 yrs. 194*d*. and r=1.05.

Now the Amount of 1*£*. for 5 yrs. is 1.05 × 1.05 × 1.05 × 1.05 × 1.05 = 1.2762816. (See Table I.)

Then per Table II. the Amount of 1*£*. for 100*ds*. = 1.0134569; for 90 = 1.0121031; and for 4 = 10005348.

∴ 1.2762816 × 1.0134569 × 1.012031 × 1.0005348 = 1.3109760?7, the Amount of 1*£*. for 5 yrs. 194*d*.

Now 1,3109760?7 × 423 = 685.640485656 × 685*£*. 1.310976072, the Amount of 1*£*. for 5 yrs. 194*d*.

Therefore 1.310976072 × 523 = 685.640485656 = 685*£*. 12*s*. 9½*d*. ,8, the Answer.

Theorem II. $\frac{a}{R^t}$=P.

(4) Here is given a=243.10126, t=4, and r=1,05.

Then per Theo. $\frac{243.10126}{1.05 \times 1.05 \times 1.05 \times 1.05} = \frac{243.10126}{1.2155063} =$ 200*£*. the Answer.

(5) Here is given $a=346.85$, $t=7$, and $r=105$.
Then per Theorem,

$$\frac{346.85}{0.5 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05} = \frac{346.85}{1.4071004}$$

$=246.5=246$*£*. 10*s*. the Answer.

(6) Here is given $a=1872.65905$, $t=30$, and $r=1.045$, which raised to 30th Power $=3.7453181$. (See Table I.)

Then per Theo. $\frac{1872,65905}{3,7453181} = 500$*£*. the Answer.

(7) Here is given $a=685.64$, $t=5$ yrs. 194*d*. and $r=1.05$ which raised to the Power of $t=1.310976$.

Then per Theo. $\frac{685.64}{1.310976} = 523$*£*. the Answer.

Theo. III. $\frac{a}{p} = R^t$.

(8) Here is given $a=346.85$, $p=246.5$, and $r=10.5$.

Then per Theo. $\frac{346.85}{246.5} = 1.4701004 = R^t$

Which $\div 1.05$ till nothing remains, and the Number of Divisions will be 7, or 7 yrs. the Answer.

(9) Here is given $a=1872.65905$, $p=500$, and $r=1,045$.

Then per Theo. $\frac{1872.65905}{500} = 3,7453181 = R^t$.

Which Number look for in Table I. under $4\frac{1}{2}$ per Cent. and you will find it to be even with 30 yrs. the Answer.

(10) Here is given $a=685.64$, $p=523$, and $r=1.05$.

Then per Theo. $\frac{685.64}{523} = 1.310976 = R^t$. the nearest Number (less) to which under 5 per Cent. is even with 5 yrs. viz. 1.2762816.

Then $1.310976 \div 1.2762816 = 1.027184025$ the nearest Number to which, in Table II. is even with 100 Days, viz, 1.0134569.

Therefore $1.027184025 \div 1.0134569 = 1.01264437$ the

nearest Number less to which in Table II. is even with 90 ds. viz. 1,0121031.

So, 1,012643 ÷ 1,012103 = 1,0005348, which is even with 4 Days, in Table II.

Therefore the Answer is 5 yrs. 194 d.

Theorem IV. $\frac{a}{p} = R^t$.

(11) Here is given $a = 346,85$, $p = 246,5$, and $t = 7$.

Then per Theo. $\frac{346,85}{246,5} = 1,4071004 = r$, raised to the 7th Power.

Then per Rule in Page 238 $r^7 = \sqrt{1,4071004} = 1,05$ or 5 per Cent. the Answer.

(12) Here is given $a = 1872,65905$, $p = 500$, and $t = 30$.

Then per Theo. $\frac{1872,65905}{500} = 3,7453181 = r$, which Number look for in Table I. and even with 30 yrs. you will find it to be under 4½ per Cent. the Answer.

(13) Here is given $a = 685,64$, $p = 523$, and $t = 5$ yrs. 194 d.

Then per Theo. $\frac{685,64}{523} 1,310976 = r^t$. Now the nearest Number to which, even with 5 yrs. is under 5 per Cent. the Answer.

66. ANNUITIES, PENSIONS IN ARREARS, AT COMPOUND INTEREST.

Theorem V. $\frac{Ur^t - u}{r - 1} = A$, the Amount.

(14) Here is given, $u = 70$, $t = 4$, and $r = 1,04$.

Then per Theo. $\frac{Ur^t - u}{r - 1} = 70 \times \frac{10,4 \times 1,04 \times 1,04 \times 1,04 - 76}{1,04 - 1}$

$= \frac{70 \times 1,698586 - 70}{0,4} = \frac{11,8012}{,04} = 297,25255$ or 297 £. 5 s. 0½ d. the Amount.

By Table III. thus,

Against 4 yrs. and under 4 per Cent. is	4,246464
Which multiplied by the Annuity	70
Gives the present Worth as before, viz.	297,25248

(15) Here is given $u=100$, $t=6$, and $r=1,05$.

Then per Theorem $\frac{100\times 1.05\times 1.05\times 1.05\times 1.05\times 1.05\times 1.05-100}{1.05-1}=$

$\frac{100\times 1.3400956-100}{,05}=\frac{134.00956-100}{,05}=\frac{34.00956}{,05}$

$=680,192=680\mathcal{L}.$ 3*s.* 9$\frac{1}{2}$*d.* the Amount.

(16) Here is given $u=70$, $t=7$, and $r=1,03$; which raised to the 7th Power $=1,2298733$. (See Table I.

Then per Theo. $\frac{70\times 1.2298733-70}{1.03-1}=\frac{86,091131-70}{,03}=$

$\frac{16.091131}{,03}=536,37103=536\mathcal{L}.$ 7*s.* 5*d.* the Answer.

(17) Here is given $u=30$, $t=30$, and $r=1,035$; which raised to the 30th Power $=2.8067937$ per Table I.

Then per Theo. $\frac{30\times 2.8067937-30}{1.035-1}=\frac{84,20381-30}{,0.35}=$

$\frac{54,20381}{,035}=1548.680257=1548\mathcal{L}.$ 13*s.* 7$\frac{1}{4}$*d.* the Amount required.

Theorem VI. $\frac{ar-a}{r-1}=$U, the Annuity, &c.

(18) Here is given $a=680.1912$, $t=6$, and $r=1,05$; which raised to the 6th Power $=1.3400956$.

Then per Theo. $\frac{680.\ 912\times 1\ 05-880.1912}{1.3400956-1}=\frac{34.00956}{,3400956}$

$=100\mathcal{L}.$ the Answer.

(19) Here is given a 536.37103 (per Quest. 3.) $t=7$, and $r=1,03$, which raised to the seventn Power $=$ 1.229ᵒ733.

Then per Theo. $\frac{536\ 37103 \times 1{,}03 - 536{,}37103}{1.2298733 - 1} =$

$552{,}46219 - \frac{536\ 37103}{{,}2298733} = \frac{16{,}0911632}{{,}2298733} = 70$£. the Answer.

(20 Here $a = 1548.680257$, $t = 30$, and $r = 1{,}035$; which raised to the 30th Power $= 2{,}8067937$ per Table I.

Then per Theo. $\frac{1548.680257 \times 1{,}035 - 1548.680257}{2.8067937 - 1} =$

$\frac{1602{,}884066}{1.8067937} - \frac{1548.680257}{1.8067937} = \frac{54.203819}{1.8067937} = 30$£. the Answer.

Theorem VII. $\frac{\overline{ar+u}-a}{u} = R^t$.

(21) Here $a = 680.1912$, $u = 100$, and $r = 1.05$.

Then per Theo. $\frac{680.1011 \times 1.05 + 100 - 680{,}1912}{100} =$

$\frac{814.20076 - 680.1912}{100} = \frac{240{,}0956}{100} = 3{,}400956 = R^t$.

which proceed with as directed in Example 8, will give 7 yrs. the Answer.

(22) Here $a = 536.37103$. $u = 70$, and $r = 1.03$.

Then per Theo. $\frac{136.37103 \times 1{,}03 + 70 - 536.37103}{70} =$

$\frac{622{,}46219 - 536\ 37103}{70} = \frac{86{,}09113}{70} = 1{,}2298733 =$

R^t. which proceed with as before directed, will give 7 years the Answer

(23) Here $a = 1548{,}680257$, $u = 30$, and $r = 1.035$.

Then per Theo. $\frac{1548.68025 \times {,}053 + 30 - 1548{,}680257}{30} =$

$\frac{1622.884066 - 1548.680257}{30} = \frac{84{,}203809}{30} =$

$2{,}8067936 = R^t$. which Number look for under $3\frac{1}{2}$ per Cent. and you will find it to be even with 30 yrs. the Answer. (See Table I.)

Theorem VIII. $\frac{ar}{u}r = \frac{a-u}{u} = R^t$.

(24) Here $a=680.1912$, $u=100$, and $t=6$.
Then per Table 680,1912,÷100=6,801912=r^t, even with 6 yrs. you will find it to be under 5 per Cent.

(25 Here $a=536,37103$, $u=70$, and $t=7$.
Then 536,37103÷70=7,662729=R^t. then in Table III. even with 7 yrs. you will find it to be under 3 per Cent. the Answer.

(26) Here $a=1548,680257$, $u=30$, and $t=30$.
Then 1548,680257÷30=51,6226752=R^t. which even with 30 yrs. you will find it to be under $3\frac{1}{2}$ per Cent. the Answer.

67. PRESENT WORTH OF ANNUITIES.

Theo. IX $\frac{u}{r}-;\div r-1=P$, the present Worth.

(27) Here U=50, $t=8$, and $r=1,05$, which being involved to $R^t=1,4774554$ by the first Table.

Then per Theo. U$\frac{u}{r}\div r-1=50-\frac{50}{1,477454}\div 1,05-1=50-33,84196\div,05=16,15804\div,05=323,1608$, or 323*l*. 3*s*. $2\frac{1}{2}d$.

By Table V. thus,

Against 8 yrs. and under 5 per Cent.	6,4632128
Which multiplied by the Annuity, viz.	50
Gives the present Worth, viz.	323,16064

(28) Here is given $u=60$, $t=6$, and $r=1,04$. which being involved to the 6th Power=1,265319, per Table I.

Then per Theo. 60,$-\frac{60}{1,265319}:\div 1,04-1=\frac{60-47.41096}{,04}=\frac{12,58904}{,04}=314,726=314$*l*. 14*s*. $6\frac{1}{2}d$. the Answer.

(29) Here $u=1000$, $t=21$, and $r=1{,}045$, which being involved to the 21st Power $=2{,}5202411$, per Table I.

Then per Theorem $1000\frac{\quad 1000}{2{,}5202411} :\div 1{,}045-1 =$

$\frac{1000-396{,}787434}{{,}045} = \frac{603{,}21256\,57}{{,}045} = 1340472386 =$

13404£. 14s. $5\frac{1}{4}$d. the Answer.

Theorem X. $\frac{prt\times r-pr^t}{r-1} = U$, the Annuity.

(30) Here is given $p=323{,}1608$, $t=8$, and $r=1{,}05$; which being involved to the 8th Power $=1{,}4774554$, per Table I.

Then per Theorem,

$\frac{323{,}1608\times 1{,}4774554\times 1{,}05-323{,}1608\times 1{,}4774554}{1{,}4774554-1}$

$=\frac{501{,}338-477{,}46567}{{,}4774554} = 50$£. the Annuity.

(31) Here $p=314{,}726$, $t=6$ and $r=1{,}04$; which being involved to the 6th Power $=1{,}265319$.

Then per Theorem,

$\frac{314{,}726\times 1{,}26531\times 1{,}04-314{,}726\times 1{,}265319}{1{,}265319-1} =$

$\frac{414{,}157939-398{,}228785}{{,}265319} = \frac{15{,}92815}{{,}265319} =$ £60. the Annuity required.

(32) Here $p=13404{,}72386$, $t=21$, and $r=1{,}045$; which being involved to the 21st Power $=2{,}5202411$, per Table I.

Then per Theorem,

$\frac{13404.72386\times 2{,}5202411\times 1{,}045-13404{,}7236}{2{,}5202411-1}$

$\times 2{,}52\,\,411 = \frac{353{,}0337712--337{,}83136}{1{,}5202411} = \frac{1520{,}2411}{1{,}520{,}2411}$

$=1000$£. the Annuity.

Theo XI. $\frac{u}{p+u-p^t} - R^t$.

(33) Here is given $u=50$, $p=323,1608$, and $r=1,05$.

Then per Theorem, $\frac{50}{323,1608+50-323,1608\times 1,05}$

$=\frac{50}{33,84196}=1,4774554=R^t.$ which proceed with as in Example 8, and the Number of those Divisions will be 8, that is, 8 yrs. the Answer.

(34) Here $u=60$, $p=314,726$, and $r=1,04$.

Then per Theo. $\frac{60}{314,726+60-314,726\times 1,04}$

$\frac{60}{374,726-327,15}=\frac{60}{47,411}=1,265319=R^t.$ which proceed with as in the last Example, and the Number of Divisions will be 6, which is 6 yrs. the Answer.

(35) Here $u=1000$, $p=13404,72386$, and $r=1,045$.

Then per Theorem,

$$\frac{1000}{13404,72386+1000-13404,72386,\times 1,045}=$$

$$\frac{1000}{14404,72386-14007,936433}=\frac{1000}{396,87426}=$$

$2,5202411=R^t.$ which in Table I. under $4\frac{1}{2}$ per Cent. is even with 21 Years, the Answer.

Theorem XII. $\frac{u}{p}=\frac{u}{p}R^t.+R^t.-R^t.+1$

To perform the three following Examples by the Theorem, see the Appendix.

(36) Here is given $u=50$, $p=323,1608$, and $t=8$. Then by the fifth Table, thus,

First $323,1608\div 50=6,463216$. Then even with 8 yrs. you will find it under 5 per Cent. the Answer.

(37) Here $u=60$, $p=314,726$, and $t=6$.

Then $314,726\div 60=5,24548$, which Number look for in Table V. even with the given Time, and the nearest Number is under 4 per Cent. the Answer.

(38) Here $u=1000$, $p=13404,72386$.

Then 13404,72386 ÷ 1000 = 13,40472386, which Number even with the given Time, you will find under 4½ per Cent. the Answer.

68. ANNUITIES, LEASES, &c. TAKEN IN REVERSION.

Theorem I. $u\frac{u}{r}:\div r-1=p$, which change to A.

Then $A\frac{A}{R^t.}=p$

(39) Here $u=80$, $t=6$, and $r=1{,}05$, then per Rule 1.

$u-\frac{u}{r^t}\div r-1=80-\frac{80}{1{,}34000955}\div 1{,}05-1=80-$

59.6976 ÷ ,05 = 20.3028 ÷ ,05 = 406.056 = P. Then per Rule 2. A = 406.056, $t=2$, and r, as before, and per

Theorem 2, $\frac{a}{r^t}=\frac{406{,}056}{1.1025}=368{,}30419$, or 368£. 6s. 1d

the present Worth required.

Thus by Table V. 2 + 6 = 8 Years.

Then { 6.4632128 = Value of 1l. for 8 Years,
{ 1.8594104 = ditto for 2 Years.

Remains 4.6030024

Which × 80 Annuity.

Gives £.368.304192 as before.

(40) Here is given $u=1000$, $t=20$, and $r=1.05$, which involved to the 20th Power = 2.6532977 per Table I.

Then per Theo. $1000-\frac{1000}{2.6532977}\div 1.05-1=$

$1000-376.88948\div ,05=623{,}11052\div ,05=$

$1246{,}22104=p$.

Now per Rule II. $a=124622104$, $t=5$, and r as before; which being involved to the fifth Power = 1.276816.

Then per Theo. II. $\frac{1246\ 22104}{1.2762816}=9767{,}60175=9767$£.

12s. ,035.

Theorem, first $prt=A$. which change to P.

$\frac{pr\times r-frt}{}=U$, the Annuity.

(41) Here $p=368.30419$, $t=2$, and $r=1.05$; which being involved to the 2d Power $=1.1025$ per Table I.
Then per Theo. I. $368.30419 \times 1.1025 = 406.05536 = A$.
Now per Rule II. $p=406.05536$, $t=6$, and r as before; which being involved to the 6th Power $=1.3400956$.
Then per Theorem II.

$$\frac{406.05536 \times 1.3400956 \times 1.05 - 406.05536 \times 1.3400956}{1.3400956 - 1}$$

$$= \frac{571.360652 - 544.153002}{,3400956} = \frac{2720765}{,3400956} = 80\mathcal{L}.$$ the Annuity required.

(42) Here for 8132.14.8 read $9767\mathcal{L}.$ 12*s.* $0\frac{1}{4}$*d.* $78 = p = 9767.60175$, $t=5$, and $r=1,05$; which being involved to the 5th Power, $=1,2762816$.
Then per Theorem I. $9767.60175 \times 1,2762816 = 12466.2103896 = A$.
Now per Rule II. $p=12466.2103896$, $t=20$, and r as before; which being involved to the 20th Power $=2.6532977$ per Table I.
Then per Theorem II.

$$\frac{12466.2103896 \times ,6532977 + 1,05 - 12466,2103896}{2.6532977 - 1} \times 2,6532977 = \frac{34730.30572229 - 33076,56735457}{1.0532977}$$

$$= \frac{1,65382836}{1.6532977} = 1000\mathcal{L}.$$ the Annuity.

69. REBATE OR DISCOUNT, AT COMPOUND INTEREST.

Theorem XIII. $\frac{s}{r^t} = p$, the present Worth.

(43) Here $\frac{s}{r^t} = \frac{150}{1,2155063} = 123,40536$, or 123*l.* 8*s.* $1\frac{1}{4}$*d.* present Worth required.
By the Table, thus, $,8227025 \times 150 = 123.405375$, as before.

(44) Here s=743,2375, t=6, and r=1 04, which being involved into the 6th Power=1.265319.
Then per Theo. 743.2375 ÷ 1.265319=587,891=587£. 7*s* 9¾*d.* the Answer.
Theorem XIV. pr^t=S, the Sum owed.

(45) Here p=123.405208, t=4, and r=1,05; which being involved to the 4th Power=1,2155063=p^t.
Then per Theorem, 123.405208 × 1.2155063=150£. the Debt.

(46) Here p=587.391, t=6, and r=1,04; which being involved to the 6th Power=1.265319 per Table I.
Then per Theorem, 587,391 × 1.265319=743.237=743£. 4*s.* 9*d.* the Answer.

Theorem XV. $\frac{s}{p}=r^t$.

(47) Here p=123.405208, s=150, and r=1.05.
Then per Theorem, 150 ÷ 123 405208=1,2155063=r^t; which being continually divided by 1,05 till nothing remains, the Number of those Divisions will be 4, that is 4 Years the Answer.

(48) Here p=587,391, s=743.235, and r=1.04.
Then per Theorem, 743.235 ÷ 587,3391=1,2653191; which proceed with as directed in the last Example, and the Number of Divisions will give 6 Years, the Answer.

Theorem XVI. $\frac{s}{p}=R^t$.

(49) Here s=150, p=123.405208, and t=4.
Then per Theo $\frac{150}{123.405208}=1,2155063=rt$.
$\therefore \sqrt[4]{1,2155063}=1.1025$, and, $\sqrt[4]{1.1025}=1.05$ or 5 per Cent. the Answer.

(50) Here s=743.235, p=587.3391005, and t=6.
Then per Theo. 744.235 ÷ 587.3391005=1,265319=r^t.
Then $\sqrt[6]{1.265319}=1,124864$.
And $\sqrt[3]{1.124864}=1.04$ or 4 per Cent. the Answer. (See the Guide, Page 222.)

70. PURCHASING FREEHOLD OR REAL ESTATES.

(51) Here $u=500$, and $r=1.05$.

Then per Theorem, $\frac{1.05-1}{500}=\frac{500}{,05}=10000£$.

Theorem XVII. $\frac{u}{r-1}$ = the present Worth.

(52) Here $u=25$, and $r=1,045$.

Then per Theo. $\frac{25}{1,045-1}=\frac{25}{,045}=555,5=555£$. 11s. $1\frac{1}{4}d.$,28, the Answer.

Then per Theo. XVIII. $\frac{P+u}{p}=R$, the Rate per Cent.

(53) Here $p=10000$, and $u=500$

Then per Theo. $\frac{10000+500}{10000}=\frac{10500}{10000}=1,05$, or 5 per Cent. the Answer.

(44) Here $p=555.5$, and $u=25$.

Then per Theo. $\frac{555.5+25}{555,5}=\frac{580,5}{555.5}=1.045$ or $4\frac{1}{2}$ per Cent. the Answer.

Theo. XIX. $P\times r-1=U$, the Annuity.

(55) Here $p=10000$, and $r=1,05$.

Then per Theo. $100000\times 1.05-1=1000\times,05=500£$. the Answer.

(56) Here $p=555,5$, and $r=1,045$.

Then per Theo. $555.5+1.04-1=555,5\times,045.=25£$. the Answer.

71. PURCHASING FREEHOLD ESTATES IN REVERSION.

Theorem XX. First $\frac{u}{r-1}=P$. which change to A.

Then $\frac{a}{R^r}=P$. the present Worth.

(57) Here $a=500$, and $r=1{,}05$.

Then per Theo. I. $\frac{500}{1{,}05-1}=\frac{500}{{,}05}=10000=$ P. which change to a.

Then per Theo. II. $a=10000$, $t=4$, and r as before; which involved to the 4th Power $=1{,}2155063$.

Then per Theo. II. $\frac{10000}{1.2155063}=8227.024=8227l.\ 5\frac{3}{4}d.$ the Answer.

Theorem XXI. First $pr^t=A$. which change to P.

Then $\frac{prtr-pr}{r}=U$, the Annuity.

(58) Here $p=8227.024$, $t=4$, and $r=1.05$; which being involved to the 4th Power $=1\ 2155063$.

Then per Theo. I. $8227{,}024\times 1{,}2155063=10000,=A$.

Now (per Rule II. $p=10000$, and r as before.

Then per Theo II. $\frac{10000\times 1.05\times 1.05-1000\times 1.05}{1.05}=$

$\frac{11025-10500}{1.05}=\frac{525.00}{1{,}05}=500l.$ the Answer.

72. QUESTIONS FOR EXERCISE.

(59) First $486{,}5l.-94=392{,}5l.$ the Annuity.

Now the present Worth of $1l.$ for 7, 14, and 21 yrs. at 5 per Cent. is 5,7863737.9.8986409, and 12,18211427, (per Table V.)

Then

		£.	s.	d.		Yrs.
5.7863737		2271.1507	= 2271	3	0	7
9.8986409	×392.5—	3885.21655	= 3885	4	4	14
12,8211527		5032.30243	= 5032	6	0	21

P. W. for

(60) First $15+5=20$ yrs. and $186l.\ 7s.\ 6d.=186.375l.$

Then the Worth of $1l.$ for 20 yrs. = 13.5903253

And ditto for 15 yrs. = 11,1184858

Diff. 2.4718395

$\therefore$ $2{,}4719395\times 186{,}375=460.70753=460l.\ 14s.\ 1\frac{3}{4}d.$ the Fine required.

(61) First 7+14+10=31 Years.
Now the present Worth of 1£. at 5 per Cent. for 31.21, and 7 yrs. is 5.7863734, 12.8211527, and 15,5928104.
Then

15.5928104 12.8211527 5.7863734	50× =	779,64082 641,057635 289,31867	Present Worth. of 50£. for	*Years.* 31 21 7

£. s. d.
∴ 289.31867 = 289 6 4½
Also 641.057635—289.31867 = 351 14 9½ } Val. of A's B's
And 779.64052—641.057635 = 138 11 7¾ } C's

(62) By Table V. the present Worth of 300£. per Annum for 7 Years, at 4 per Cent. is 6,002547×300=1800.61641.
Then

£.	s	d.	
1800.61641 + 150 = 1950	12	4	
6.002547 + 250 × 400 = 1920	10	3¼	Value of A's B's C's D's Offer.
6.002547 × 200 + 650 = 1850	8	2½	
1800	0	0	

Hence it appears that A's Offer 50 100 150 than B's. C's. D's.
is better by above

(63) First 54.85—7.35=47.5£. the Annuity.
Now as the Payment is Half-yearly, we shall have $u=475 \div 2=23.75$, $t=6$, and $r=1,01$; *see the Guide, p. 237.* which raised to the 6th Power=5,0756921, per Table V.

∴ 5.0756291×23.75=120.54768735=120£. 10*s*. 11½*d*. the Answer.

(64) First 153—50=103£. for 12 yrs at 5 per Cent.
Then per Table VI. 1£. will purchase for 12 yrs. at 5 per Cent. an Annuity of ,1128254 per Annum.
Then ,1128255×103=11,6210162=11£. 12*s*. 5*d*. advance Rent

∴ 16£.+11£. 12,5=27£. 12*s*. 5*d*. the Answer.

(65 First 44—20=24£. and 1, 9487171=Amount of 1£. for 7 Years. =£.250 0 0
Then 1.9487171×24=46.9292104 = 46 18 7
At 10£. per Annum for 7 yrs. = 70 0 0

Answer 360 18 7

(66) First 33.3×50=166.6=£.166 13*s.* 4*d.* Value of the Annuity.
Then per Table VI. 1£. for 12 yrs. at 4 per Cent. will purchase, 1065512, per Annum.
∴ 166.6×,106512=177.587=177£. 11*s.* 8¼*d.* the Answer.

(67) First 237—10=227£. the Annuity.
Then the present Worth of 1£. for 4, 12, and 14 Years, is 3.5459505; 8,8632516, and 9.8986409, per Table V.
∴

		£.	*s.*	*d.*
3.5459505	804.9307635	804	18	7¼
8.8632516 × 227 =	2011.9581132	2011	19	1¾
9.8986409	2246,9914843	2246	19	9¾
	Sum =	5063	17	6¾
Discount 4+12+14×5=30×5 =		150	0	0
Answer, received		4913	17	6¾

73. SUPERFICIAL MEASURE.

EXAMPLES.

PROBLEM I.

		Ft.	′			*Ft.*	′	″
(1) Multiply		17	7	(2) Multiply		47	8	0
by			6	by			8	4
	Answer	105	6			15	10	8
						381	4	0
					Answer	397	2	8

(3) Multiply — Ft. 7 10 0 ′ ″
by 8 6

	3	11	0
62	8	0	
Answer 66	7	0	

(4) Multiply Ft. 64 7 0 ′ ″
× 4 8

43	0	8
258	4	0
Answer 301	4	8

(5) Multiply Ft. 12 8 9 0 0 ′ ″ ‴ ⁗
× 9 6 7

	7	5	1	3
6	4	4	6	0
114	6	9	0	0
Answer 121	6	6	7	3

(6) Multiply Ft. 9 11 6 0 ′ ″ ‴
× 11 8

	6	7	8 0
109	6	6	0
Answer 116	2	2	0

(7)

	Ft.			′	″	‴
6 ½	64	0	10	0		
			14	9		
½	32	0	5	0		
	16	0	2	6		
	896	11	8	0		
Answer	945	0	3	6		

(8)

	Ft.			′	″	‴
6 ½	124	4	0	0		
⅓		42	0	0		
½	5	2	2			
	2	7	1			
	14	0	0			
	248	0	0			
	496	0	0			
Answer	5229	9	3			

(9)

	Ft.	′	″	‴	⁗
	16	7	10	0	0
			6	5	7
	0	9	8	6	10
	6	11	3	2	0
	99	11	0	0	0
Answer	107	7	11	8	10

(10)

		Ft.	′	″
6′	1/2	474	6	8
6	1/2	186	7	4
1	1/6	237	3	4	0	0
4″	1/12	39	6	6	8	0
		13	2	2	2	8
		93	0	0	0	0
8	1/12	10	4	0	0	0
		2844	0	0	0	0
		8532	0	0	0	0
Answer		88557	4	0	1	0	8

(11)

		Ft.	′	″	‴	⁗	v
6′	1/2	24	11	8	6	7	0
					8	6	7
		1	2	6	9	10	1
		12	5	10	3	3	6
		199	9	8	4	8	0
Answ.		213	6	1	5	11	41

(12)

		Ft.	′	″	‴	⁗	v	vi
4	1/3	46	6	8	4	0	0	0
						6	4	8	6
″		15	6	2	9	4	0	0
6	1/8	1	11	3	4	2	0	0
2	1/3	0	7	9	1	4	8	0
		0	1	11	3	4	2	0
		279	4	2	0	0	0	0
Answ.		297	7	4	6	2	10	0

PROBLEM II.

(1) Here A B = 14.5 and A C = 9.75 Chains, per Fig. 2.
Then per Theo. 14.5 × 9.75 = 141.375 = 141*a.* 1*r.* 20 p. the Area required.

(2) Here A B = A C = 245 yds per Fig. 1.
Then 245 × 245 = 60025 Acres, or yds.
∴ 4840 60025 (12,4 = 12 A. 1*r.* 24p.

(3) Here A B = 64 ft. 6 inc. and A C = 47 ft. 8 inc. see Fig. 3
Then per Problem I. 64 ft. 6 inc. × 47 ft. 8 inc. = 3074 ft. 6 inc.
∴ 9)3074 ft. 6 inc. (= 341 yds. 5 ft. 6 inc. the Answer.

PROBLEM III.

(4) Here A B = 28 and C D = 20.5 Chains. See Fig 4. G.

Then per Theo. $\frac{28 \times 20.5}{2} = 14 \times 20.5 = 287$ Acres.

(5) Here A B = 738, and C D = 583 Links.

Then $\frac{738 \times 583}{2} = \frac{420254}{2} = 210127$ Links, or 2,15127 Acres.

∴ As 2,10127 : £. : : 1 : 5.7108 = 5£. 14*s*. 2½*d*. the Answer.

(6) Here $\frac{a+b+c}{2} = \frac{400+348+312}{2} = \frac{1060}{2} = 530 = s$.

Then per Rule.

$530 - \left\{\begin{matrix} 400 = 130 = e. \\ 348 = 182 = f. \\ 312 = 218 = g. \end{matrix}\right\}$ The three Remainders.

∴ $530 \times 130 \times 182 \times 218 = 2733676400$.

And $\sqrt[2]{2733676400} = 52284.5$

Then 4840 | 52294.5 | 10.8 = 10 acr. 3 r. 8 p. the Answer.

PROBLEM IV.

(7) Here A C = 45, B F = 17.25, and D E = 14. See Fig. 5. G.

Then per Theo. $\frac{17.25+14}{2} \times 45 = \frac{31.25}{2} \times 45 = 15{,}625 \times 45 = 703{,}125$ ft. these ÷ 9 = 78 yds. 1 ft. 18 inc. the Answer.

(8) Suppose A B = 15.6, B B = 13.2 C D = 10, and A D = 26.

Then per Rule, First, $\frac{15.6+13.2+10+26}{2} = \frac{64.8}{2} = 32.4$

$\therefore \sqrt[2]{\frac{32.4-15.6 \times 32.4-13.2 \times 32.4-10 \times 32.4-26}{16.8 \times 19.2 \times 22.4 \times 6.4}} = \sqrt[2]{462.42.216} = 215{,}0$

Area required. (See Quest. 29, in Algebra).

Or if the Trapezium can be inscribed in a Circle, that is

if the Sum of any two opposite Angles be equal to two right Angles, or 180°; then multiply any two adjacent Sides together, and the other two Sides together, and multiply by the Sum of these Products, half the Sine of the Angle included by either of the Pair of Sides which are multiplied together, so shall this last product be the Area.

Thus $\frac{AD \times DC + AB \times BC \times s.+A, \text{ or } s.\, C}{2}$ = the Area.

EXAMPLE.

If the Sides be AD=4, DC=7.5, AB=6, BC=5.5, and the Angle C=74° 40¼.

Therefore the Angle A=180°—74°40¼′=105° 19¾′.

Then by Rule $\frac{4 \times 6 + 7{,}5 \times 5{,}5}{2} \times 9644229$ (*s.* 74° 40¼′)

=31.46429+sq. Feet, the Area.

PROBLEM V.

(9) Here AD=48, CB=41.57, and *n*=5.

Then per Theo. $\frac{48 \times 41{,}57 \times 5}{2} = \frac{9976{,}8}{2}$ =4988,4 ft. the Area required.

(10) First the Multiplier for an Hexagon is 2,598076 by the Table.

Then per Rule II. thus 2,598076×30×30=2338,2684, the Area.

(11) Here the Multiplier for an Octagon is 4.828427 by the Table.

Then 4,828427×24×24=2781,173952, the Area.

PROBLEM VI.

(12) Thus 3,1416×7=21,9912, or rather 22, the Circumference required, (per Rule I).

(13) Thus 22÷3.1416=7 (nearly) the Diameter, per Rule II.

(14) Thus 3.1416×8000=25132,8 Miles, the Circumference required.

PROBLEM VII.

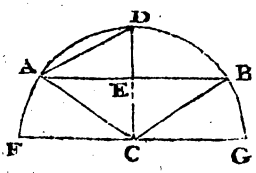
(15) $C=6.2832$, $D=\frac{6.2832}{3,1416}=2$. the Diameter.

Then per Theo. $\frac{6,2832}{2}\times\frac{2}{2}=\frac{6.2832}{2}=3.1416$ the Area (per Rule II.)

16) First $12\times12=144$, Square of the Diameter.
Then $,7854\times144=113,0976$, the Area, (per Rule II.)

PROBLEM VIII.

) $DE=9$, $ABD=29.5$, and $b=,01745329$.
Then per Theorem, $,01745329\times29,5\times9=4,6338484895$, the Length of the Arch required.



PROBLEM IX.

(18) Here $AC=55$, and $AB=59$; see Fig. 9. G.

Then per Theo. $55\times\frac{59}{2}=55\times26,5=1622,5$ the Area.

First $42-30=12=EC$.

(19) Then by Rule II. p. 188. G. $^2BC-^2CE=^2AE=1764-144=1620$. $\therefore\sqrt[2]{1620}=40,249$ (nearly) $=AE$, which $\times2=80.498$ the Base AB.
Now $80.498\div2=40,249=EB$.
Again, $40,249\times40,249=1619.982=^2EB$.
And, $30\times30=900=^2DE$.
$\therefore\sqrt[2]{1619.982+900}=\sqrt[2]{2519.982}=\sqrt{EB\times ED}=52.2$ nearly, the Chord of half the Arch AB. See Fig. 8. in the G.
Therefore $\overline{80.498\times6}+\overline{50.2\times8}=482.988+401.6=884.588$, Sum of the Product.

$\therefore$ $884.588\times\frac{30}{15}=884.588\times2=1769.176$ the Area.

(20) First, $,7854\times18\times18=254,4696$, the Area of the whole Circle, (per Prob. VII.)

$\therefore \frac{254,4696}{4} = 63,6174$, Area of the Quadrant C A B D; see the last Figure.

And $\frac{9 \times 9}{2} = \frac{81}{2} = 40,5$, Area of the Triangle, ABC, (per Prob. III.)

Then from 63,6174 take 40,5 remains 23,1174, the Area of the Segment ABD.

PROBLEM XI.

(21) Here AB=84, CD=72.5, and AC=3.5 see Fig. 10.

Then per Theo. $\frac{84+72.5}{2} \times 3,5 = \frac{156,5}{2} + 3,5 = 78,25 \times 3,5 = 273.875$, the Area required.

(22) First, as 7 : 22 : : 24 : 75.4 which $\div 2 = 37.7$, the greatest Semicircle AB.

Again, as 7 : 22 : : 16 : 50.2, which $\div 2 = 25.1$, lesser CD.

Also $\frac{24-16}{2} = \frac{8}{2} = 4 =$ A B; see Fig. 10.

Then $\frac{37.7+25.1}{2} \times 4 = \frac{62.8}{2} \times 4 = 31.4 \times 4 = 125.6$.

PROBLEM XII.

(23) Here AB=24, CD=18, and b=,7854.

Then per Theo. ,7854×24×18=339.2928, the Area.

Of ARTIFICERS' WORK.

Ft.	′	″
3	10	0
	×2	8
2	6	8
7	8	0
10	2	8
	× 12	
122	8	0 Content.

	Ft.	′	″	
6*d.* $\frac{1}{10}$	122	8	0	at 8½*d.*
2 ½	3	1	0	
½ ¼	1	0	4	
	0	5	1	
6 ½	0	0	4¼	= ½ of 8*d.*
2 ⅓	0	0	1¼	
	£.4	6	10¼	Value.

(2)

Ft.	′
6	6
5	3
4	9
Sum 16	6
×4	
66	0
3	9
247	6 Content.

4*d.*	⅓	247 6 at 1*s.* 4*d.*
		82 4
6	½	0 8 = ½ of 16*d.*
2,0)		33,0 0
		£.16 10 Answer.

By Decimals, thus 6.5+5.25+4.75=16.5, Sum of their Heights; which × 4, (the Number of Windows in a Tier)=66 Heights together.
Then 66×3.75 (the Breadth)=247,ℊ the Area.
Now 26*d.*=,0ϕ£. ∴ 247,5×,0ϕ=16.499, or 16£. 10*s.* as before.

(3)

Ft.	′	″
6	6	0
× 3	3	
1	7	6
19	6	0
21	1	6 Content.

Ft.	′	″		
21	1	6		at 8*s.*
×4				
8	8	0		
1′	1/12	0	0	8 = 1/12 of 8*s.*
6″	½	0	0	4
£.8	9	0	Answer.	

By Decimals thus, 3,25×6.5=21.125 ft. at 8*s.* or 4£.
∴ 21.125×.4=8.45 or £.8 9*s.* as before.

(4) First 16×2=32 the Length; and 9×3=27 the Breadth.
Then 32×27=864, Square Inches.
∴ 144)864(6 ft. the Content required.

(5)

Ft.	′	″
176	6	0
×56	9	0
88	3	0
44	1	6
28	0	0
1056	0	0
880	0	0
9)100	16	4 6

For the odd Parts thus,

3 Ft. | 1/3 | 6d.

3		2
		2
		0½ 2/3
4′	1/3	0½ 2/3
6″	1/6	2/3
		5½

6d. 1/10 1112 yds. 8 ft. 4 inc. 6 sec. at 6d. per Yard.

27 16 0
+ 5½

£27 16 5½ the Answer.

By Decimals, thus 176,5×56,75=10016.375 Feet.
Then 9)10016.375(1112.93 yds. at 6d. or, 025£. per yd.
∴ 1112.93×,025=27.823, or 27£. 16s. 5½d. the Answer as before.

(6)

Ft.	′	″
9	6	0
× 8		3
2	4	6
76	0	0
9)78	4	6

yds. 8 6 4 6 at 6s.
×3

2	8	0
0	4	3
£.2	12	3

For the odd Parts, thus,

3 Ft. | 1/3 | 6s.

		2
3	1/3	2
4′	1/8	0 2½ 2/3
6″	1/8	0 0¼ 1/3
		4 3

By Decimals, thus 9,5×8,25=78,375 Feet.
Then 9)78.375(8,7083 yds. at 6s. or ,3£.
∴ 8,7083×,3=2,6125 or 2£. 12s. 3d. the Answer.

(7) First 84 ft. × 9 ft. 6 inc. = 798 0 Room.

And $\left\{\begin{matrix} 6\times3\times3,5=61,6 \\ 4\times4,=16 \end{matrix}\right\}$ = 77 6 Win. and Chim.

9)720 6 Diff.

80 yds. 6 inc.

∴ 80 yds. × 2 + 6 inc. = 160 yds. 6 inc. Answer.

(8) First 47 ft. 7 inch. — 4 ft. = 43 7 Breadth.
× 47 7 Length.

9 2073 10 1

6*d.* | $\frac{1}{20}$ yds. 230,3 10 1 at 6*d.* per yd.

3 ft. | $\frac{1}{3}$ 5 15 0
6 in. | $\frac{1}{6}$ 2 = $\frac{1}{3}$ of 6*d.*
4 | $\frac{1}{9}$ $\frac{1}{4}$ $\frac{1}{2}$ = $\frac{1}{9}$ of 2*d.*
$\frac{8}{9}$ = $\frac{1}{9}$ of ditto.

£.5 15 $2\frac{1}{2}$ $\frac{7}{18}$ the Answer.

(9)

Ft.	*In.*		*Ft.*	*In.*		*Ft.*	*In.*	*ll*	
129	6	×	16	3	=	2104	4	6	Room.
7		×	3	9	=	13	1	6	½ Doors and Shutters.
2									
7		3×4	6		=	32	7	6	

8,6+4,6×2=26,×1,2,×2 = 60 8 0 Cheek-board.

14+3.9×1,4=17,9,×1,4= 23 8 0 Door Linings.

Ft.	*In.*	*Ft.*			
To be deducted 3	9	= 3	2234	5	6
			11	3	0 Chimney.

9 2223 2 6 Wood Work.

s. *d.*

2 6 | $\frac{1}{8}$ | yds. 247 2 6″ at 3*s.* 6*d.*

1 0 | $\frac{1}{10}$ | 30 17 6 } = 43*l.* 4*s.* 6*d.*
12 7 0 } the Cost.

(10)

Ft.	Inc.
34	6
×	20
9)690	0
76	6

d.		Yds.	Ft.	
6	$\frac{1}{10}$	76	6	at 10½ per yd.
3	½	1	18	
1½	½		19	
			9	6
3 ft.	⅓		0	3½ = ⅓ of 10½d.
3	do.			3½

£.3 7 1

(11)

Ft.	In.
6¼ 34	8
× 14	6
17	4
485	4
9 502	8

$\frac{2}{3}$ 55 yds. 7 ft. 8 inc. the whole Area.

				£.	s.	d.
	11	1	6⅔			
Plastered	44	6	1½ at 8d. per yd. =	1	9	9¼
Whited	67	0	2⅔ at 2d. per yd. =		11	2
			Answer £.	2	0	11¼

(12)

		Ft.	In.	
6	½	120	6	
		12	9	
		60	3	0
3	½	30	1	6
		1446	0	0

	Sq.	Ft.	In.	
1,00	15,36	4	6	(15, 36 4 6 the Answ.

(13) First 28×20 = 560
And 14×10×2 = 280 } = 280 the Diff.

1,00)840 Sum.

8,4 sq. at 2*l*. 5*s*. or 2,25*l*.

∴ 2,25 × 8,4 = 18,9 or £.18 18*s*. the Answer.

(14)

Ft.	*Ft.*		*Ft.*
30,5 × 20,5			2501 Area of the 4 Floors.
5,25 × 6	× 4 =		126 = dit. of 4 Fire places.
4,25 × 4			68 = dit. of ditto.
8,5 × 10			340 = dit. of a Well-hole.

The whole Deductions = 534

1,00)19,67 sq. at 8,5*£*.

∴ 19,67 × 85, = 167,195 = *£*.167 3*s*. 10½*d*. the Answer.

(15) First 1,25 × 15 = 18,75 Area of one Plank.
And 33,5 × 60,5 = 2026,75 ditto of the Room.
∴ 18,75)2026,75(108$\frac{7}{75}$ Planks (nearly.)

(16) First $36 + \frac{36}{2} = 36 + 18 = 54$ Width of the Roof.

Then 64 × 54 = 3456 Feet the Area.
And 1,00 34,56(34,56 sqrs. at 12*s*. 6*d*. or ,625*£*.
∴ 34,56 × ,625 = 21,6 or *£*.21 12*s*. the Answer.

(17) First, 30 × 2 = 60, and 70 × 60 = 4200 ft. the Area.
Then 1,00)42,00(42 sqr. at 10*s*. 8*d*. per sqr.

s.	*d.*
10	8
	× 7 × 6 = 42.
3	14 8
	6

Answer *£*.22 8 0.

(18)

In.	*Ft.*	
	22,75	Flat and Half.
10 × 2 =	11,375	Eves Boards.
	1,6664	

35,7916 Whole Width.
× 32,75 Length.

1,00)1172,177083(11,72177083 sqr. at ,75*£*.

∴ 11,72177 × ,75 = 8,7913275 = 8*£*. 15*s*. 10*d*. Answer.

(19) First 470 ft. ×9 ft. 6 inc. =4465 ft.
And 4465 × 6 half Brick. =26790.
Then per Rule I. 816,26790 32 rds. $226\frac{1}{3}$ ft. the Answ.

(20) First 840×9=7560 Feet.
× 5 Half-Bricks thick.

Per Rule II. 3,37800
Rds. Ft.
272)12600(46 88

Rds. Ft.
46 88 at £.4 19*s.* 6*d.* per Rod.
×4

£.	£.	*s.*		
46	— 1	3 =	44	17 at 19*s.* 6*d.*
68	$\frac{1}{4}$		1	4 $10\frac{1}{2}$ = $\frac{1}{4}$ of £.4 19*s.* 6*d.*
17	$\frac{1}{4}$			6 $2\frac{1}{2}$
3	$\frac{1}{6}$		1	$0\frac{1}{4}$ nearly.
		£.230	9	$1\frac{1}{2}$ the Answer.

(21) First 38+4=9.5 Height of the Gable.
And 40÷3=13.3

24.5×13.3× } 4 = 1306,66β at 2 } Bricks thick.
3 = 980 at $1\frac{1}{2}$
2 = 653,333 at 1
24.5× 9.5 ÷ 2 = 116,375 at $\frac{1}{2}$

816 3056,375(3,74β Rods.

∴ 3.7454+β×5.5=20.6 or £.20 12*s.* the Answer.

QUESTIONS.

(1) First 14 ft. 3 inc. =171 inc. and 3 square yds. =36×36 =1296 Inches.
∴ 171 1296($7\frac{1}{19}$ Inches, the Answer.

(2) First a Foot Square=144 Inches.
Then 144÷27=5,3 Inches, Breadth of the first Cut.

And 58—5,3=52,6 remaining Length.
Also 144÷52,6=2,734 Breadth of the 2d Cut.
Then 27—2,734=24,266 remaining Breadth.
144÷24,206=5,934 Breadth of the 3d Cut.
52,6—5,934=46,732 remaining Length.
144÷46,732=3,0814 Breadth of the 4th Cut.
24,266—3,0814=21,1846 remaining Breadth.
144÷21,1846=6,7974 Breadth of the 5th Cut.
46,732—6,7974=39,9346 remaining Length.
144÷39,9346=3,6059 Breadth of the 6th Cut.
21,1846—3,6059=17,5787 remaining Breadth.
144÷17,5787=8,1917 Breadth of the 7th Section.
39,9346—8,1917=31,7429 remaining Length.
144÷31,7429=4,5364 Breadth of the 8th Cut.
17,5787—4,5364=13,0423 remaining Breadth.
144÷13,0423=11,0409 Breadth of the 9th Section.
Then 31,7429—11,0409=20,702 remaining Length.
Also 144÷20,702=6,956 Breadth of the 10th Section.
∴ 13,0425—6,956=6,0863 Breadth remaining at the last.
Q. E. F.

(3) First 10584÷6=1764.
Then $\sqrt{1765}$=42 long Rows.
∴ 42×6=252 short Rows, 42 in a Row.

(4) First 7×2,5=17,5 Inches, Area of an End.
And 17,5×2=35, double Area.
∴ 35÷3=11⅔ Inches deep, the Answer.

(5) First 19×11=209, Area of an End.
Then 209÷4=52,25 Area of an End of the Piece wanting.
∴ 52,25÷9=5,805 Width required.

(6) First a yd.=36×36=1296 Inches. And 3*s.* 2*d.*=38*d.*
Then as 6*d.* : 1296 :: 38*d.* : 8208 inc. Area of the whole Trough.
Also 102×21×2= 4284 Area of the two Sides.
∴ 8208—4284=3924 Area of the Bot. and Ends.

Then $\overline{102+21}\times 2$=144.
∴ 3924÷144=27¼ Inches, the Breadth required.

(7) First $16+\overline{40\times 2}=106$ Breadth of the Bot. and Sides.
Then $106\times 74 = 7844$ Area of Bottom and Sides.
And $40\times 26\times 2 = 2080$ ditto of both Ends.
Also $26\times 16\times 3 = 1248$ ditto of the Stays.

144 11172(77,583 sqr. Feet.

28 lb. $\frac{1}{4}$ 77,583(19,39583 cwt. at 22*s.* or 1,1 £.
∴ 19,39583 × 1,1 = 21,33541 *p* = £.21 6*s.* 8½*d.* Value of the Cistern.
Now 3*s.* 6*d.* = ,175 £. Therefore 21,33541 *p* — ,175 = 21,1604 *p* Mason's Bill.
7*d.* = ,02916)21,160215(725,5 sqr. Feet, Shop.
22 ft. 10 inc. = 22,83)725,(31778 = 31 ft. 9¼ Inches, the Answer.

(8) First 27 ft. 112 inc. = 4000 sqr. Inches.
Then 20 × 16 × 2 = 640 Area of the two Ends.

∴ 20×2 + 16 = 56)3360 ditto of Bot. and Sides.

1260 Inches.

Answer 5 Feet.

Yds.

(9) First 16½ ft. = 5,5
Then 5,5 × 5,5 = 30,25
And 6×6 = 36
Also 7×7 = 49
} = 1 Per. { Statute / Cheshire / Yorkshire } Measure.
Receip. as 30,25 : 110 :: 36 : 92 1 28 Chesh. } Measure,
30,25 : 110 :: 49 : 67 3 25 York. } the Ans.

(10) First 7 × 4 = 28 sqr. Feet, Area that each Plant takes.
∴ 3584 × 28 = 100352 sqr. Feet, in all.
Square Feet in an Acre = 43560)100352(2,303764 or 2 acr. 1r. 8½ pol. the Answer.

(11) First 18*d.* = ,075)100(1333,3 Feet the Area.
Now suppose Fig. 4, in the Guide, to represent the Court.
Then $133,3, \frac{88}{2} = 1333,3 \div 44 = 30,303 = CD$, the Perpendicular.

Then $44 \times 44 = 1936$ Square of A D, or B D.
Also $30,303 \times 30,303 = 918,27$ Square of D C.

$\sqrt{2854,27} = 53,425 =$ A C, or B C.

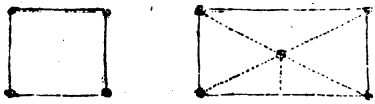
And $53,425 \times 2 = 106,85$ the Sum required.

(12) Here the last mentioned Fig. may represent the Form of the Bath.
Then $3)125(41,\emptyset =$ A B or A C.
And $2)41,6)20,83 =$ A D or B D.
Then $41,\emptyset \times 41,\emptyset = 1736,0\emptyset$
Also $20,83 \times 20,83 = 434,025$

$\sqrt{1302,052} = 36,084 =$ D C.

$\therefore \frac{36,084}{2} \times 41,\emptyset = 751,75$ the Area per Prob. III.

(13) Square Order. Quincunx Order.

Fig. 23. 

First $6 \times 6 = 36$ sqr. Feet each in the Square Order.
And $3 \times 3 = 9$
$\sqrt{27} = 5,19615$
Then $5,19615 \times 6 = 31,1769$ sqr. Feet each Plant in the Quincunx Order.
Now an Acre $= 43560$ Feet.
$\therefore 43560 \times 10 = 435600$ sqr. Feet in 10 Acres.

Plants.

Therefore $31,1769)435600(13972$ Quincunx
$36)435600\ 12100$ Square } Order.

Answer 1872

(14) First $10+2\frac{1}{2}+1=13\frac{1}{2}$ or $\frac{27}{2}$ whole Breadth.
Then $\frac{27}{2} \times \frac{1}{5}=3$ Feet, Rise of the Roof.
And $13,5 \div 2=6,75$ Half the Width.
Also $6,76 \times 6,75=45,5625$
Also $3 \times 3 \quad =9$

$\sqrt[2]{54,5625}=7,38664$ Length.

Then $13,5 \times 2 \times 7,38664=199,43928$ Feet, the Area at $3\frac{1}{2}d.$ or ,01458$\frac{3}{4}$£.
$\therefore$ $199,43928 \times ,01458\frac{3}{4}=2,90849$ or 2£. 18*s.* 2*d.* Answ.

(15) First, as 4,5 : 8 :: 9 A. to 16 B.
And 4 : 8,5 :: 16 : 34=A's+C's.
Then $34-9=25$=C's.
$\therefore$ $9+16+25=50$=A's+B's+C's Area.
Now $\sqrt[2]{9}=3$ A's, $\sqrt[2]{16}=4$ B's, and $\sqrt[2]{25}=5$ C's Side.
Also $\frac{3+4+5}{2}=\frac{12}{2}=6$ Poles, the Area of the Triangle.
Therefore $272,25 \times 144=39204$ Inches in a Pole.
$\times$ 6 Poles.

235224 Inches in six Poles.
6 Feet= $\times 72$ Inches, the Depth.

A gal.=231 inc.) 16936128 (73316,5$\frac{1}{2}$ galls.

$\therefore$ 63) 73316,$\frac{1}{2}$ (1163,755=1163 hhds. 47$\frac{1}{2}$ galls. the Answer.

(16) Here suppose Fig. 5, in the Guide, to represent the Orchard; which contains 3,75 Acres.
Then AC=the Diagonal; and 430+360=790=B F + D E, the Sum of the Perpendiculars; half of which is 395 Links, and 3,75 Acres=375000 Links.
$\therefore$ 395) 375000 (949$\frac{25}{79}$ Links A C, the Diagonal.

(17) First 3,1416×16=50,2656 Poles, the Circumference, (per Prob. VI. Rule I.)
Then $\frac{50,2656}{2} \times \frac{16}{2}=25,1328 \times 8=201,0624$ Poles, the Area, (per Prob. VII.)
$\therefore$ 160) 201,0624 (1,2 Acre, the Answer.

(18) First 16,5 ÷ 2 = 8,25 one Round.
∴ 3,1416)8,2500(2,665 ft. or 2 ft. 7½ inches, the Answer.

(19) First 3,1416)130,0000(41,38 ft. the Diameter.

Then $\frac{41,38}{2} \times \frac{130}{2}$ = 20,69 × 65 = 1344,85 Area.

9)1344,85(149,428 yds. at 4*d.* or, 01 ℔ £.
∴ 149,428 × ,01 ℔ = 2,490466 or £.2 9*s.* 9½*d.* ,8, Answ.

(20) First 4840 ÷ 2 = 2420 yds. half an Acre.
Then ,7854)2420(3081, 23 yds. Square of the Diameter, (per Prob. VII. Rule II.)
∴ $\sqrt[2]{}$ 3081,23 = 55,5 Diameter.
Therefore 55,5 ÷ 2 = 27,75 or 27¾ yds. the Answer.

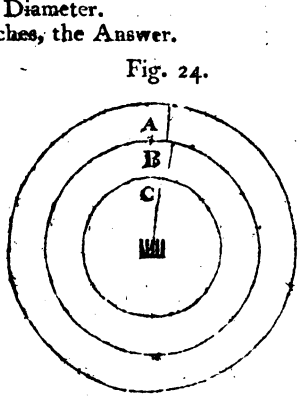
(21) First 42 + 14,5 × 2 = 71, greater Diameter.
Then 71 × 71 = 5041; also 5041 × ,7854 = 3959,2014
And 42 × 42 = 1764; — 1764 × ,7854 = 1385,4456
Diff. of their Areas, are 2573,7558

Then 144)2573,7558(17,8733 ft. at 8*d.* or ℔*s.*
∴ 17,8733 × ,℔ = 11,9155 or 11*s.* 11*d.* (nearly) the Ans.

(22) First, as 3,1416 : 5 :: 60 : 92,307 Circumference.
∴ 92,307 ÷ 3,1416 = 2938 Diameter.
And 29,38 ÷ 2 = 14,69 Inches, the Answer.

(23) First 36 × 36 × ,7854 = 1017,8784, Area of the whole Stone and Spindle-hole. (Per Prob. VII. Rule II.)
And 5 × 5 × 2 = 50 Square of the Diameter of the Circle, circumscribing the Spindle-hole; also, 7854 × 50 = 39,27 Area of ditto.
∴ 1017,8784 — 39,27 = 978, 6084, Area of the Stone to be divided.

Fig. 24.



Therefore,

$$\text{As } 20s. : 978{,}6084 :: \left\{\begin{array}{l} 7 : 324{,}5129 \text{ A's} \\ 8 : 391{,}4433 \text{ B's} \\ 5 : 244{,}6521 \text{ C's} \end{array}\right\} \text{Area.}$$

Then $244{,}6521 + 39{,}27 = 283{,}9221$, this $\div {,}7854 = 361{,}5$, whose Square Root $= 19{,}03$ inc. the Diameter where C begins to grind.

And $283{,}9221 + 391{,}4433 = 675{,}3654$, this $\div {,}7854 = 859{,}9$, whose Square Root $= 29{,}324$ inc. the Diameter where B begins to grind.

(24) First $10 \times 10 = 100$; also $100 \times 2 = 200$, Area of the inside Square.

Then $\sqrt[2]{200} = 14{,}142135$ Side of the inscribed Square.

Again $20 \times 20 = 400$ Area of the circumscribed Square.

Lastly $,7854 \times 400 = 314{,}16$ Area of the circular Section.

Hence $314{,}16 - 200 = 114{,}16$ Inches too little.

And $400 - 314{,}16 = 85{,}84$ ditto too much.

(25) First $2s.\ 4d. = {,}11\text{ℓ}\mathcal{L}$. As $11\text{ℓ} : 1$ ft. :: $10\mathcal{L} : 85{,}7143$ ft. Area of the Semicircle, which $\times 2 = 171{,}4286$, Area of the whole Circle.

Then $,7854)171{,}4286(218{,}269$ Square of the Diameter, (per Prob. VII. Rule II.)

$\therefore \sqrt[2]{218{,}269} = 14{,}7739 = 14$ ft. $9\frac{1}{4}$ inc. the Answer.

(26) First $,18 \times 18 \times 100 = 32400$ French Feet, the Arpent.

And $16{,}5 \times 16{,}5 \times 160 = 43560$ English Feet, in an Arp.

Then $16 \times 16 = 256$; also $15 \times 15 = 225$.

$\therefore$ Recipe as, $256 : 32400 :: 225 : 36864$ English Feet in an Arpent.

So that the English Acre is to the Arpent of France, as $43560 : 36864$, or $: 605 : 512$, nearly $13 : 11$, or $: 1 : {,}84628$, the Answer.

(27) First $3{,}1416 \times 4 = 12{,}5664$ Circumference of the Wheel.

$\therefore 12{,}5664 \times 5 = 62{,}832$ by the greater.

And $62{,}832 \div 2 = 31{,}416$ by the less.

(28) Here (per Prob. IX. Fig. 9, in the Guide) is given C A $= 30$, and A B $= 63$ ft. $= 21$ yds.

Then per Theo. XXX. $30 + \frac{21}{2} = 30 \times 10{,}5 = 315$ yds. Ans.

(29) First 15 Inches=,41⅋ Yards.
Then 21—,41⅋=20,583=AB, (see Fig. 22.)
And 20,583÷2=10,291⅋=AE.
Also 30×30=900=²AC.
10,291⅋×10,291⅋=105,92=²AE.

Diff. 794,08=²CE.

∴ $\sqrt[2]{}$794,08=28,18=CE, the Perpendicular.

Then per Prob. X. $30\times\frac{21}{2}$=315, Area of the Sector.

A B C D.
And 10,291⅋×28,18=290,1091⅋, Area of the Triangle A B C.
∴ 315—290,01916=24,98083, Area of the Segment, A B D.

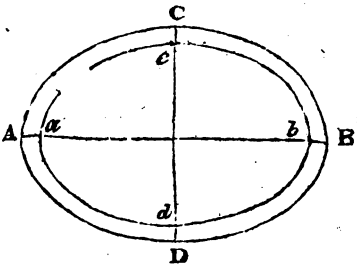


Fig. 25.

(30) First 14×2=28 Inches; or 2,3 Feet.

	Lin.	*Ft.*	*Lin.*	*Ft.*
Then, as	100	: 66 ::	840	: 554,4=*ab*.
	100	: 6⅋ ::	612	: 403,92=*cd*.

∴ 554,4+2,3=556,73=AB.
403,92+2,3=406,253=CD; then per Prob. XXVI.
556,73×406,253×,7854=177637,53=Area of ABCD.
554,4×403,92×,7854=175877,17=ditto *a b c d*.

Covered by the Wall, 1760,36 Diff.

Now an Acre=4840 yds.×9=43560 ft.
∴ 43560)175877,17(4,0375 or 4 Acres, 6 Perches, Area of what they inclose. (See Prob. XII.)

(31) First 4×10=40 Feet, and 7×7=49 square Side.
Then, as 4 : 49 :: 40 : 490 Square of the Diameter.
∴ $\sqrt[2]{490}$=22,136 Inches, nearly the Diameter required.

(32) First 6×6=36. And 3×3×4×2=72.
Then, as 36 : 3 :: 72 : 6 Hours, the Answer.

(33) First 9×9=81. And 12×12=288.
Then, as 81 : 22lb. :: 288 : 78 $\frac{2}{9}$lb. the Answer.

(34) First 1,25×1,25+20=31,25 Feet.
And $\frac{1}{8}$=,875; then, ,875×875×50=38,28125 Feet.
∴ As 31,25 : $\frac{1}{2}$ Ton, or 1120lb. :: 38,28125 : 1372lb. at 3$\frac{1}{2}$*d.* per lb.

d.	*lb.*		
3 | $\frac{2}{80}$ |	1372		
$\frac{1}{2}$ | $\frac{1}{20}$ |	17	3	
	2	17	2

£. Answer 20 0 2

(35) First 22,5×22,5=506,25. And 22,5×2=45.
Also 45×45=2025.
Then, as 506,25 : 3 bush. :: 2025 : 12 bush. the Answ.

(36) First 64,3×144=9259,2 square Inches.
And $\frac{1}{8}$=,375 : ×9259,2=3472, 2 solid Inches.
Then 3472,2×437=1515141,8 Ounces, which÷1792 oz. (in 1 Cwt.)=8,455 Cwt.
∴ As 19,5 : 21£. :: 8,455 : 9,10538 or £.9 2*s.* 1$\frac{1}{4}$*d.* 648 the Answer.

74. MEASUREMENT OF SOLIDS.

PROBLEM XIII.

(1) Thus 2,5×2,5×2,5=15,625 Feet, the Solidity.

(2) First 57×42×34=81396 cubic Inches.
A Gal. of Ale=282,81396(288,638 Gal. the Answer.

(3) First $4,5 \times 4,5 \times ,7854 = 15,90435$ Area of the Base (per Prob. VII. Rule II.)
Then $15,90435 \times 8 = 127,2348$ Feet, the Content required.

PROBLEM XIV.

(4) Thus $10,5 \times 7,75 = 81,375$, the Surface required.

PROBLEM XV.

(5) First $12 \times 12 = 144$ Area of the Base.

Then $144 \times \frac{25}{3} = 144 \times 8,3 = 1200$ Feet, the Answer.

(6) Now (per Prob. III. Rule III.) $5,5 \times 3 = 16,5$ Sum of the three Sides, half of which is $= 8,25$.
Then, $8,25 - 5,5 = 2,75$ Difference.
And $8,25 \times 2,75 \times 2,75 \times 2,75 = 171,57421875$.
Then $\sqrt[2]{171,57421875} = 13,0986 =$ Area of the Base.
$\therefore 13,0986 \times \frac{30}{3} = 13,0986 \times 10 = 130,0986$, the solid Content required.

(7) First $3,5 \times 3,5 \times ,7854 = 9,62115$ Area of the Base, (per Prob. VII. Rule II.)

Then $9,62115 \times \frac{6}{3} = 9,62115 \times 2 = 19,2423$, the Solidity required.

PROBLEM XVI.

(8) Thus $\frac{3,25 \times 20}{2} \times = \frac{65}{2} \times 3 = 32,5 \times 3 = 97,5$, Area of the three Sides.

Now $3,25 \times 3,25 = 10,5625$
And $1,625 \times 1,625 = 2,640625$

Diff. $7,921875$

$\therefore \sqrt[2]{7,921875} = 2,81458$

Therefore $\frac{3,25 \times 2,81458}{2} = 4,5737$ Area of the Base.

Now, $97,5 \times 4,5737 = 102,0737$, the Area required.

(9) Thus $\frac{45 \times 20}{2} = \frac{900}{2} = 450$ Feet, the convex Surface required.

PROBLEM XVII.

(10) First $15 \times 15 = 225 = A$. And $6 \times 6 = 36 = a$. Also $b = 24$.

Then per Theorem $225 \times 36 = 8100$.

And $\sqrt[2]{8100} = 90$ the mean Prop.

$\therefore 225 + 36 + 90 \times \frac{24}{3} = 351 \times 8 = 2808$ Inches.

Therefore $144)2808(= 19,5$ Feet, the Solidity.

Or thus, to the Squares of the $\left\{\begin{matrix}\text{Diameters} \\ \text{Circumference}\end{matrix}\right\}$ of the Ends, add their Product; multiply the Sum by the Altitude of the Frustrum, and the Product which arises by $\left\{\begin{matrix},2618 \\ ,0265\end{matrix}\right\}$ viz. $\left\{\begin{matrix},7854 \\ ,0795\end{matrix}\right\} \div 3$

And this last Product will be the Content.

(11) Now by the preceding Rule $20 \times 20 = 400$.

And $3 \times 3 = 9$. Also $20 \times 3 = 60$, which are the Squares of the Diameters and the Product.

Then $400 + 9 + 60 = 469$ their Sum.

$\therefore 469 \times 60 = 28140$, which $\times ,2618 = 736,7052$, the Content.

(12) First $66 \times 66 = 4356$. And $56 \times 56 = 3136$.

Also $66 \cdot 56 = 3696$, which are the Square of the Circumferences and their Product.

Then $4356 + 3136 + 3696 = 11188$, their Sum.

$\therefore 11188 \times 4 \times ,0265 = 1185,928$ the Content.

PROBLEM XVIII.

(13) First 3 Feet 4 inc. $+ 2$ Feet 2 inc. $\times 4 = 40 + 26 \times 4 = 66 \times 4 = 264$ Inches, Sum of the Perimeters.

Then $\frac{264 \times 10}{2} : \frac{2640}{2} = 1320$ Inches.

∴ 12)1320(=110 Feet, the Surface.

Or thus, $\frac{3.3\times2,1\beta\times4\times10}{2}=\frac{5,5\times4\times10}{2}=220\div2=110$ Feet.

(14) First 32+8=40, Sum of the Circumference.

Then $\frac{40\times7}{2}=\frac{280}{2}=140$ Feet, the Content.

(15) First, as 30 : 10 :: 6 : 2, Circumference of the Top of the Frustrum.

And 30—6=24 the slant Height.

Then $\frac{10+2}{2}\times\frac{24}{2}=\frac{288}{2}=144$ Feet, the Surface required.

PROBLEM XIX.

(16) First $30\times16\times\frac{12}{2}=480\times6=2880$ ft. the Content.

PROBLEM XX.

(17) First $16+36\times2=16+72=88$, Sum.

Then $88\times29=1760$.

Also $1760\times\frac{12}{6}=1760\times2=3520$ cubic Feet.

PROBLEM XXI.

(18) First $16\times16=256$. And $13\times13=169$.

Also $16\times13=208$, which are the Areas of the Ends, and the Product of their Sides.

Then 256+169+208=633 Sum.

∴ $633\times\frac{60}{3}=633\times20=12660$, the Content required.

PROBLEM XXII.

(19) First $304\times20=6080$. And $300\times16=4800$.

Also $304+300\times20\times16=604\times36=21744$.

Now 6080+4800+21744,=32624 their Sum.

Then $32624\times\frac{5}{6}=32624\times,83=27186,\beta$, the Content required.

PROBLEM XXIII.

(20) First (per Prob. VI.) 3,1416×7=21,9912 the Circumference.
Then 21,9912×7=153,9384, the Surface required.

(21) Thus 3,1416×12×12=452,3904, the Surface.

(22) First 7957,75×3,1416=25000,0674 Miles (nearly) the Circumference, (per Prob. VI. Rule I.)
Then 7957,75×25000,0674=198944286,35235 square Miles, the whole Surface required.

(23) First 3,1416×42=131,9472 the Circumference.
Then 131,9472×9=1187,5248 the Surface required.

PROBLEM XXIV.

(24) Thus 7×7×7×,5236=179,5948, the Solidity required, (per Rule II.)

(25) First, the Surface is 198944286,35235 Miles, (per Prob. XXIII.) (See Ex. 22.)

Then $198944286,35235 \times \frac{7957,75}{6} = 198944286,35235 \times$
1326,2916=263858149120,06886875 Miles, the Solidity required, (per Rule I.)

By Rule II. thus $\overline{7957,75}^{3} \times ,5236 = 263857624944$ the Solidity by this Rule.

The Difference arises by taking the Number ,5236 a little too great.

PROBLEM XXV.

(26) First $\overline{18 \times 3} - \overline{4 \times 2} = 54 - 8 = 46$.
Then 46×4×4×,5236,=385,3696, the Content.

PROBLEM XXVI.

(27) For an Oblate, thus
,5235×33×55×55=52258,3875 solid Inches.
Then 1728)52258,3875(30,242+ solid Feet.

For an Oblong, thus, ,5236×55×33×33=31355,0325 solid Inches.

∴ 1728)31335,0325(=18,1452 solid Feet.

PROBLEM XXVII.

(28) First, 41888×36×36×99=53744 solid Inc. nearly.

Then 1728)53743,97952(311,0184 Feet, the Content required.

By Rule II. thus ,7854×36×36=1017,8784. Area of the greatest Circle, (per Prob. VII. Rule II.)

∴ 1017,8784×99=100769,9616.

Which ÷15=6717,99744=$\frac{2}{15}$.

Therefore 6717,99744×8=53744 solid Inches, as before.

PROBLEM XXVIII.

(29) By Rule I. Thus, first 42÷4=10,5 Quarter Girth.

And 10,5×10,5×16=1764 Inches.

∴ 144)1764(12$\frac{1}{4}$ Feet, the Content by this Rule.

By Rule II. Thus, first, 42÷5=8,4=$\frac{4}{5}$ of the Girth.

And 16×2=32 twice the Length.

Then 8,4×8,4×32=2257,92 Inches.

∴ 144)2257,92(15,68 Feet, the true Content.

(30) By Rule I. Thus 64÷4=16.

Then 16×16×30,5=7808 Inches.

These÷144, gives 54,2 Feet, the Content by this Rule.

By Rule II. Thus, 64÷5=12,8. And 30,5×2=61.

Then 12,8×61=9994,24 Inches.

∴ 144)9994,24(69,4 Feet (nearly) the true Content.

Note, the first Rule differs from the Truth about $\frac{1}{4}$ of its Contents; that is, when it produces 4 for the Content, it should be above 5. The second Rule is about 5 Times nearer to the Truth than the other, for it differs from the Truth only a Foot in 190; and it is full as easy in Practice; besides it hath in every Respect the Advantage of it. Therefore, I think it ought to be brought into general Use among the Measurers of Timber, who should certainly prefer Truth to such gross Errors as are always introduced by the other Method.

(31) First $42 \times 30 \times 40 = 50400$.
Then 144 50400(350 Feet, the Content.
$\therefore$ 5,0)35 0(7 Tons the Answer.

PROBLEM XXIX.

(32) First, 16 ft. 6 inc. = 198 inc. and 7 ft. 4 inc. = 88 ditto.
Then $198 \times 88 = 17424$ Area.
$\therefore \checkmark 17424 \div 282 = 61,78$ Gallons.
Or you may turn or change any Divisor into a Multiplicator; by dividing Unity or 1 by that Divisor.

Thus, $1,000000 \div \begin{cases} 282 = ,003546 \text{ Ale.} \\ 231 = ,004329 \text{ Wine.} \\ 268,8 = ,003722 \text{ Corn.} \end{cases}$

Therefore $17424 \times ,003546 = 61,78$ Gallons, as before.

(33) First, 58 ft. 6 inc. = 702 Inches.
Then $\overline{702 \times 04} \div 2 = 65988 \div 2 = 32994$ Inches Area.
$\therefore$ 32994 ÷ 282 117 Gallons.
Or thus, $32994 \times ,003546 = 117$ Gallons, as before.

(34) First $84,5 \times 50 = 4225$ Area.
Then $4225 \div 282 = 14,9+$ Gallons.

(35) Here by Prob. 3. the Area of the Top is 1772,8, and at the Bottom 4209,727.
Then by Prob. 7th, $4209,727 \times 1772,8 = 7463004,0256$, whose square Root = 2731,85, the mean Proportion.

Now $2731,85 + 1772,8 + 4209,727 \times \frac{36}{3} = 8714,377 \times 12 = 104572,524$ cubic Inches.

$\therefore$ $104572,524 \div \begin{cases} 282 = 370,824 \text{ Ale Gallons.} \\ 231 = 452,6 \text{ Wine ditto.} \end{cases}$

(36) First, by Rule II. $\overline{60,5 \times 60,3} \div 359,05 = 3660,25 \div 359,05 = 10,2$ Gallons nearly, Area of the Base.

Then by Prob. 15, $10,2 \times \frac{42,8}{3} = 10,2 \times 14,26 = 145,52$ Ale Gallons.
Again $3660,25 \div 294,12 = 12,5$.
$\therefore$ $12,5 \times \frac{42,8}{3} = 178$ Wine Gallons.

(37) First by Prob. 7th, $84\times84\times,7854=5541,7824$, Area at the Top.
And $62\times62\times,7854=3019,0776$ Ditto Bottom.
Now by Prob. 17th, $5541,7824\times3019,0776=16731071,10791424$.
$\therefore \sqrt[2]{16731071,10791424}=4090,3633$ mean Proportion.
Then $4090,3632+5541,7824+3019,0776=12651,2232$
And $12651,2232\times\frac{42}{3}=22651,2232\times14=177117,1248$ cubic Inches, which $\div 282=628,0749$ Ale Gallons.

By RULE I.

(38) Thus, $7854\times32\times32\times2=1608,4992$
$,7854\times26\times26=530,9304$
their Sum $=2139,4296$
which multiply by $\frac{4}{3}^{\circ}=13,3$
gives 28525,728 cubic Inc.

$\therefore$ 282)28525,728(101,155 Ale
231)28525,728(123,488 Wine } Gallons.

By RULE II.

(39) First $36\times36=1296$=Square of the Head.
And $40\times40\times2=3200=2$ Ditto of the Bung.
Their Sum $=4496$
Now $40-36=4$. And $4\times4=16$, $\frac{2}{5}$ of which $=6,4$.
Then $4496-6,4=4489,6$.
$\therefore 4489,6\times64=287334,4$.
Therefore
$287334,4\times\left\{\begin{array}{l},00092837=266,75\ \text{Ale}\\ ,00113333=325,64+\text{Wine}\end{array}\right\}$ Gallons.
Or 1077,157)287334,4(266,75 Ale
882,35)287334,4(325,64 Wine } Gallons as before.

QUESTIONS.

(1) First $6\times6\times6=216$ solid Inches in $\frac{1}{4}$ a Foot solid.

And 2)1728

∴ 216)864 solid Inches in $\frac{1}{4}$ a solid Foot.

Answer 4 Times as much as the first; or one is the $\frac{1}{4}$ of the other.

	Ft.	*In.*	*Ft.*	*In.*	*Ft.*	*In.*	*Ft.*	*In.*	
(2) First	25	6 ×	20	2 ×	14	=	7199	6	0
$\frac{1}{2}$ =	12	9 ×	10	1 ×	7 ×	2 =	7199	10	6 which is just $\frac{1}{4}$ of the first; or as 4 is to 1, the Answer.

	Ft.	*In.*	*Ft.*	*In.*	*Ft.*	*In.*	*Ft.*	*In.*	
(3) First	17	7 ×	13	10 ×	9	6 =	2310	8	11
And	5	6 ×	2	1 ×	9	6 =	108	10	3
						Remains	2201	10	8
Fire-place and Windows						27 × 4 =	108	0	0
						Answer	2309	10	8

(4) First 112,5 × 32 × 5,5 = 19800 in the whole Hold.
Also 112,5 × 4,5 × 55 = 2784,375 Gang-Way.

Remaining Concavity, 17015,625

Then 3,3 × 2,1 β, × 3 = 21, β, Content of 1 Bale.

∴ 21, β)17015,625(735,3365 Bales, the Answer.

(5) First a Fodder of Lead = 2184lb.
Then 16)2184(136,5 square Feet.
∴ 4,25 × 8 = 34 Area of the Bottom.
Also 136,5 — 34 = 102,5 = Area of the Sides and Ends.
8 × 2 + 4,25 × 2 = 16 + 8,5 = 24,5 round.
Then 24,5)102,5(4,183673 Feet, = 50,204 Inches.
∴ 50,204 — ,75 = 49,454 Depth taken at.
Now 8 ft. = 96 inc. Length, and 4½ ft. = 54 inc. Breadth.
Therefore 96 × 54 × 49,454 = 242126,7846 cubic Inc.
∴ 231)242126,7846(1048,1679 Gallons (nearly) which ÷ 63 = 16 Hhds. 40 Galls. (nearly) Answer.

(6) First 18 inc = 1,5 ft. Also 14 = 1,1 β ft.
Then 1,1 β × 1,5 = 1,75 Breadth and Depth.

∴ 1,75)2,5(1,42857 Length of the Piece cut off.
Therefore 18,5—2,42857=17,07143 Feet, the Answer.

(7) First, 8,5)126,25(14,853 ft.=2138,8234 Inches, Area of an End.
∴ 38,5)2138,8234(55,55 Inches deep, the Answer.

(8) First 18,5×18,5×,7854×8=2150,4252 cubic Inches in a Bushel.
Now 7 ft. 10 inc.=94 inc. Length; 3 ft. 10 inc.=46 inc. in Breadth; and 4 ft. 2 inc.=50 inc. Depth.
Then 94×46×50=216200 cubic Inches in the Bin.
∴ 2150,4252)216200,0000(100,5 Bush, or 12 Qrs. 4 Bush. the Answer.

(9) By the last Question a Bushel=2150,4252 Inches.
Then 2150,4252×9=19353,8268 Inches, the Content.
∴ 19353,8268÷12=1612,8189 Area of the Circle.
Which÷,7854=2053,49 Square of the Diameter.
And $\sqrt[2]{}$ 2053,49=45,3 Inches, the Answer.

(10) First 28÷2=14 Radius. And 14×14=196.
Also 7×7=49. Then 196—49=147.
Then $\sqrt[2]{}$ 147=12,1243557 Perpendicular.
∴ 12,1243557×7=84,8705, Area of one Triangle.
And 84,8705×6=509,223, Area of the Base.
Also 14×6×134=11256, ditto Sides.
Other Base 509,223.

Yds. Ft Inc.
A yd.=Inc. 1296)12274,446(9 4 34 Superf.

Again 509,223×134=68235,88 solid Inches.
∴ 1728)68235,88(39 solid Feet, 84 Inc. the Answer.

(11) First 3 cubic Feet=5184 cubic Inches.
As the Circumference is 44, the Diameter is 14, (per Prob. VI.)
Then $\frac{44}{2}\times\frac{14}{2}=22\times7=154$, Area of an End (per Prob. VII. Rule I.)
∴ 154)5184(33,66 Inches, the Answer.

(12) First 1,25×1,25×,7854=1,2271875. Area of the Circle, (per Prob. VII. Rule II.)
And a Pint contains 28,875 cubic Inches.
∴ 1,227187)28,875(23,5294 Inches the Answer.

(13) A Bushel contains 2150,4252 cubic Inches, (per Quest. VIII.)
And 13,5×8=108 Bushels in 13½ qrs.
Then 2156,4252×108=232245,9216 cubic Inches.
∴ $\sqrt[3]{}$ 232245,9216=61,468+, the Answer.

(14) First 20×15×8=2400 cubic Inches in the Stone.
And 290 Tons=649600 lb.
∴ As 220 : 2400 :: 469600 : 7086545 Inches.
Then 1728)7086545(4101 Feet the Answer.

(15) A Piece of Timber a Foot long, and 4 Feet round, is a solid Foot, customary Measure.
Also if a Circle be 4 Feet round, its diameter will be 4÷3.1416=1,2732, (per Prob. VI.)
Then a circular Piece of Timber 1 Foot in Length, will contain 1,2732 Foot.
Therefore 1,2732×50 (Feet in a Load)=636615 ft.
∴ 63,6615—50=13,6615 Feet in a Load, the Answer.

(16) *Sol. Inc.*

20×20×20× { ,7854=6283,2 Cycl. / ,5236=4188,8 Globe. / ,2618=2094,4 Cone. } See Prob. XIII. XV. and XXIV.

Now if the Diameter of a Circle be 20, the Circumference will be 3,1416×20=62,832, (per Prob. VI.)
Also if the Height (C D) of a Cone=20, and the Diameter of the Base A B=20.
Then A D=$\frac{20}{2}$= 10 ∴ $\frac{62,832\times 10}{2}$=314,16 Area of the Cone's Base. (See Fig. 16, in the Guide.)
Then $\overline{20\times 20+10\times 10}$=400+100=500=²A C.
∴ $\sqrt[2]{}$ 500=22,36067614=A C, the slope Side.
Then 314,16×2= 628,32 Area of the two Bases.

Cylinders 628,32 × 20 = 1884,96 Superficial Con.
1256,64 Globe's ditto
62,832 / 2 × 22,36067 + 314,16 } = 1006,6428 Cone's ditto
See Pr. XVI. and XXIII.
their Sum = 4158,2428 Inches, their Area.

A Yard = 1296 : 8 :: 4158,2428 : 25,6 or 2*s*. 1½*d*. the Answer.

(17) First 2170 × 2170 × 2170 × ,5236 = 5350308686,8 solid Miles, (per Prob. XXIV.)
Then a Mile = 1760 × 1760 × 1760 = 5451776000 solid Yards.
∴ 5350308686,8 × 5451776000 = 29168684491287756800 solid Yards in the Moon.
Now in a solid Yard are 36 × 36 × 36 = 46656 solid Inches.
Therefore 29168684491287756800 × 46656 = 1360894143625521581260800 solid Inc. in the Moon.
Then per Quest. VIII. a Quarter = 2150,425 × 8 = 17203,4) 1360894143625521581260800 (79106115281021285 40,06 Quarters of Wheat the Moon would hold if hollow. *Q. E. F.*
Again 2170 × 2,1416 = 6817,272 Circumference of the Moon, (per Prob. VI.)
Then per Prob. XXIII. 6817,272 × 2170 = 14793480,24 square Miles, the Surface of the Moon.
A Mile = 1760 × 1760 = 3097600 square Yards.
∴ 14793480,25 × 3097600 = 45824284391424 square Yards of Stuff. *Q. E. F.*

(18) First 7970 + 60 × 2 = 7970 + 120 = 8090 Diameter of the Earth and Atmosphere.
Then 7970 × 7970 × 7970 × ,5236 = 265078559622 sol. Miles in the Globe of the Earth, (per Prob. XXIII.)
Also 8090 × 8090 × 8090 × ,5236 = 2772331775444 sol. Miles in the Earth and Atmosphere.
∴ 2772331775444 — 265078559622,8 = 12154617921,6 solid Miles in the Atmosphere.
And per the last Quest. a Mile = 5451776000 solid Yards. Then
12154617921,6 × 5451776000 = 66264254274148761600 solid Yards in the Atmosphere. *Q. E. F.*

(19) First 30 In. $=2,5$ Feet.
Then $2,5 \times 2,5 = 6,25$ Area of the Base.

$\therefore 6,25 \times \frac{21}{3} = 6,25, \times 7 = 43,75$ solid Feet, (per Prob. XV.)

Then $21 \times \frac{2,5}{2} = 21 \times 1,25 = 26,25$ Area of one triangular Side, (per Prob III.)
$\therefore 26,25 \times 4 = 105$ Area of the four Sides.

	£.	s.	d.
Now 43,75 at 7s. =	15	6	3
And 105 at 8d. =	3	10	0
Answer, Cost	18	16	3

(20) In order to complete the Cone, use this Analogy: as half the Difference of the Top and Bottom are to the Depth, so is half the greater Diameter to the Altitude of the whole Cone.

Thus $\frac{72-54}{2} = \frac{18}{2} = 9$. Also $\frac{72}{2} = 36$

Then as 9 : 42 :: 36 : 168 Altitude.

$\therefore 72 \times 72 \times ,7854 \times \frac{168}{3} = 228004,7616$ Area of the whole Pyramid, (per Prob. XV.)

Again $\frac{168-42}{3} = \frac{126}{3} = 42 = \frac{1}{4}$ Altitude of the Piece wanting.

Then $54 \times 54 \times ,7854 \times 42 = 96189,5088$ Area of the Piece wanting.
Therefore $228004,7616 - 96189,5088 = 131815,2528$ cubic Inches.
$\therefore 282)133815,2528(467$ Gals. $3\ 1\frac{61}{141}$ pts. the Answer.

(21) First $16 + \frac{16}{5} = 16 + 3,2 = 19,2$ Bottom Diameter.

And $19,2 \times 8 = 153,6$ the Height.
Also $153,6 \div 3 = 51,2$ cylindrical.
$153,6 - 51,2 = 102,4$ a conical Frustrum.
Now $19,2 \times 19,2 \times ,7854 = 289,529856$ Area of the gr.

Also 16 × 16 ×,7854=201,0624 ditto lesser Base.

Sum of their Areas 490,592256

Again 289,529856 × 201,0624=58213,567719, whose square Root=241,274; which added to 490,592256=731,866256.

∴ 731,866256, $\times \frac{102,4}{3}$=24981,034874546 Conical.

And 201,0624×51,2= 10294,39488 Cylinder.

Solid Content of the Pillar 35275,42975 Inches.

∴ 1728)35275,42975(20,41988 solid Feet, at 3*s.* 6*d.* or ,175*£*.

Then 20,41988×,175=3,573479 or 3*£*. 11*s.* 5½*d.* Ans.

Farther

16×3,1416=50,2656 Circumference of the Cylinder.

19,2×3,1416=60,3187 ditto Base.

2)110,5843'55,29215

Then $\frac{19,2-16}{2}$=1,6, and 1,6 × 1,6= 2,56

Also 102,4×102,4 = 10485,76

their Sum 10488,32

∴ ✓ 10488,32=102,4125 Slope Height.

Then 102,412×55,29215=5662,6534 Con. Superf.

Also 51,2×50,2656 =2573,5987 Cylinder.

289,526 Bottom Area.

201,0624 Top ditto.

their Sum 8726,8405 Inches.

∴ 144)8726,8405(60,8 Feet, the Superficial Content.

(22) First 19,5—13,5=6 Diff. of the Sides.

And 6×6=36; ÷3=12 the ⅓ of the sq. of their Diff.

Now 16 ft. 6 inc.=198 Inches, the Length.

Then 19,5×13,5×12×198=5499,5 cubic Inches.

1728)5499,5(3 1,539 solid Feet, at 2*s.* 6*d.* or ,125*£*.

∴ 31,539×,125=3,942375 or *£*.3 18*s.* 10*d.* the Answer.

(23) First 3,1416×26×26×2 = 4247,4432
Also 3,1416×20×20 = 1256,64

Sum 5504,0832

Then $5504,0832 \times \frac{100}{3} = 183469,44$ cubic Inches, (per Prob. XXIX.)

∴ 282)183469,44(650½ Ale=2010½ Gal. nearly Barrel London Ale.

231)183469,44(794¼ Brandy. *Q. E. F.*

(24) First 6 Feet=72 Inches.

Then 3,1416×72=226,19568, the Circumference.

And 226,19568×72=16286,0544 Inches, superficial Content, (per Prob. XXIII.) at 3½*d.* per Inc.=237*£*, 10*s.* 1*d.* 2, nearly, the Answer.

(25) First 21×21×,7854=346,3614 Area of the Base.

Then 346,3614×31=10737,2034 solid Feet.

∴ 10737,2034×1728=18553887,4752 cubic Inches.

231)18553887,4752(80319,8 Gallons.

A Tun=Galls. 252)80319,8(318 Tuns, 183,8 Gallons, the Answer.

75. SPECIFIC GRAVITY OF METALS.

CASE I.

(1) Thus, ,03309446×12096=400,3122816lb.
Adv.÷112=3,5742+Cwt. the Answer.

(2) First $\frac{20}{5}=4$. Also 4 × 4=16 sqr. of ¼ Girth.

And 40 Feet=480 Inches.

Then 480×16=15360 cubic Inches, (per Prob. XXVIII. in Mensuration.)

By Rule II. ,023763×15360=356lb. nearly, or 3 Cwt. 29 lb. the Weight required.

(3) First 7×7×7×,5236=179,5948 solid Inches, (per Prob. XXIV.)

Then 179,5948×,2582647=46,347 lb. the Answer.

(4) First 2580647)42,0000000(162,75 the Solidity nearly.
Then ,5236)162,75(310,828677 Cube of the Diameter, (by Prob. XXIV.)
∴ $\sqrt[3]{}$ 310,828677=6,77, the Diameter required.

(5) First 16—6=10, Diameter of the Concavity.
Then ,16×16×16×5236=2144,6656 Cont. of the whole.
And 10×10×10,×,5236=523,6 Content of the Concavity, (per Prob. XXIV.)
Solidity of the Shell.=1621,0656 Inches.

∴ 1621,0656×,2580647=418,3398 lb. the Weight req.

(6) First 76+5=81, the greater Diameter.
Also 3 ft. 4 inc.=3,3 the greater Breadth.
∴ As 81 : 3,3 :: 76 : 3,127572, its least Breadth.
Here the Chords and their Arches being nearly equal, in so small a Part of so large a Circle, differs very little from a right Line: the Figure of the Key-stone may be reckoned a Prismoid, and measured accordingly. (See Prob. XXII.)
Now 3,3×4=13,3. And 3,127572×4=12,510288.
Also 6,460965×8=51,68772 which are the Numbers to be added.
Then 13,3+12,510288+51,68772=75,313413 Sum.
∴ $75,313413 \times \frac{5}{6} = \frac{3876567}{6} = 64,6094$ solid Feet.
Then 64,6094×1728=111645 cubic Inches (nearly.)
∴ 111645÷,0929543=12017,4 lb.
Therefore a Ton=lb. 2240)12017,4(5,3649 + or 1 Ton 5 cwt. 1 qr. 5 lb. the Answer.

(7) First 63×12×12=9072 solid Feet.
Then 9072×1728=15676416 cubic Inches.
∴ 15676416×,0977286=1532034,1887 lb.
∴ And 2240)1532034,1885(683 Tons, 18 cwt. 98 lb. the Answer.

CASE II.

(8) First 8 Tons 14 cwt.=19488 lb.
Then ,0977286,19488,0000000,199409,4 Inches.

Now 1728)199409,4(115,4 cubic Feet, at 6*s.* or 3*£*.
Then 115,4 × 3 = 34,62 = 34*£*. 12*s.* 4½*d.* the Answer.

(9) First 1300 × 4 = 5200 hhds. and 1 hhd = 282 × 54 = 15228 cubic Inches.
∴ 15228 × 5200 = 79185600 cubic Inches, displaced.
Then 79185600 × ,037253 = 2949901 lb. Avoird.
∴ 2240)2949901(1316 Tons, 18 Cwt. 1 qr. 17lb. the Weight required.

(16) First 3 × 3 = 9 solid Inches in the Gold Chain.
Then 9,682625 × 9 = 89663,25 its Weight in Air.
And ,527458 × 9 = 4,747122 wt. of its Bulk of Water.

Weight of the Gold 84,916503 in Water. *Q. E. F.*

Now a solid Inch of Silver = 5,556769 oz. Troy.
Then as 5,556769 : 1 :: 14,5 : 2,6094 inc. of Silver.
∴ 89,663(625—14,5 = 75,163625 oz. of Gold.
And 9,962625)75,163625(7,54456 Space taken up by the 2,6094 dit. of Silver. [Gold

Sum 10,154 by both.
9, ditto of Gold.

1,514 ditto of Silver.

∴ 9)1,154(,1281 inc. higher. *Q. E. F.*

(11) First 1036)63,00(6,08108, had it been Gold.
Also 5,85)63,00(10,76923, if all Silver.
Then by Sect. XXIX.

$$8,2245 \left\{ \begin{matrix} 6,08108 \\ 10,76923 \end{matrix} \right\} \quad \left\{ \begin{matrix} 2,54473 \\ 2,14342 \end{matrix} \right.$$

Sum 4,68815

4,68815)2,54473(,5428, Part Gold, per oz.
4,68815)2,14342(,4572, Part Silver.

$$\text{Then } \left\{ \begin{matrix} ,5428 \\ ,4572 \end{matrix} \right\} \times 63 = \left\{ \begin{matrix} 341884 = 34 & 3 & 22\frac{1}{4} \text{ Gold.} \\ 28,8036 = 28 & 16 & 1\frac{3}{4} \text{ Silver.} \end{matrix} \right.$$

oz. dwts. gr.

76. CHRONOLOGY.

(1) Thus, $1795 \div 4 = 448$ and 3 remains, so it is the 3d Year after Leap Year.

(2) Thus, $1796 \div 4 = 449$ and 0 remains, so it is Leap Year.

(3)
4)1795
448
7)2243
320—3 Remains.
∴ 7—3=4=D
So D is the Dominical Letter.

(4)
4)1796
449
7)2245
320—5 Remains.
∴ 7—5=2=B.
So C B are the Dominical Letters.

(5) The Dominical Letter (by the last Prob.) is D. And per the Verse, the 1st of May is B, viz. on Friday. Then B 1, B 8, B 15, C 16, D 17, E 18, F 19; so the 19th of May happens on Tuesday.

(6) The Dominical Letter is B, (per Ex. IV.) and per Verse, the 1st of June is E, viz. on Wednesday.
Then E 1, F 2, G 3, A 4; consequently the 4th of June must be on Saturday.

(7)

1795	1795	1795
9	1	3
28)1804	19)1796	15)1798

Rem. sol. Cyc.=12 lun. Cyc.=10 Indiction=13.

1796	1796	1796
9	1	3
28)1797(64	19)1789(94	15)1791 119

Rem. sol. Cyc.=3 lun. Cyc.=11 Indiction=14

So that 4 and 3 are the solar Cycles; 10 and 11 the lunar Cycles, or Golden Numb.; and 12 and 14 the Indic. Cyc.

(8) The Golden Number is 10 (per last Ex.)
Then 11 × 10=110 which ÷30=3, and 20 remains.
∴ 20—11=9 the Epact.

(9) The Golden Number is 11, then 11 × 11=121, which ÷30=4 and 1 remains. ∴ 19+1=20, the Epact.

(10) The Epact is 9, (per Ex. VIII) Number of the Month 3, and Day of the Month 21.
Then 9+3+21=33 and 33—30=3, the Moon's Age.
Also 30—3=27 after 21st of May, or June 17th, the Day of the next new Moon.

(11) The Epact 20, (per Ex. IX.) the Number of the Month 1, and the Day of the Month is 24.
Then 20+1+24=45.
∴ 45—30=25 Days is the Moon's Age.
Now 30—25=5 Days after the 24th of March, or the 29th of March is the Day of the next new Moon.

(12) The Dominical Letter is D, (per Prob. IV.)
On the 20th of March the Moon's Age is 30, (by Prob. VI.); so that the nearest new Moon to March 20th falls on Feb. 12, and the 15th Day of that Moon is Feb. 18.
Now March 1st is D, or Sunday, (per the two Lines Prob. III.) Then Easter Sunday will be on the 5th of April.

(13) The Dominical Letter is B.
And on the 21st of March the Moon's Age is 22, so that the nearest new Moon to March the 21st falls on March 11th, and the 15th Day of that Moon is March 16th.
Now March 1st is D, consequently Easter Sunday will be on the 27th of March; from which the other moveable Feasts may be found.

Or by knowing what Day of the Month Shrove Sunday is on, all the rest may be found.
Shrove Sunday is always the first Sunday after the second Change of the Moon, which happeneth after New Year's Day: and if that Day of the Second Change be Sunday, then that Sunday is Shrove Sunday.

EXAMPLE.

What Day of the Month is Shrove Sunday in the Year 1786, being Feb. 24th?

First the Moon's Age is 14 Days old,* (per Prob. VI.)
Then, by running back, I find the Day of her second Change will be Feb. 10th.
Now by the Lines in Prob. III. the first of Feb. is D. or Wednesday, as the Dominical Letter is A, (per Prob. II.)
Therefore D being the first, D is 8, E 9, F. 10, G 11, A 12; so that Shrove Sunday will be on Feb. 26th, being the first Sunday after the second Change.
Now Shrove Sunday being found as above, Feb. 26th.

Quadragessima, or the first Sunday in Lent, must be	April 5.
Easter Day, being 7 Weeks after Shrove Sunday, must be	April 16.
Easter Term, beginning a Fortnight after Easter Wednesday, must be	May 3.
Rogation Sunday, being 6 Weeks after Easter Wednesday, must be	May 21.
Ascension Day, being 3,9 Days after Easter, must be	May 25.
Easter Term ends the Monday after Ascension Day,	May 29.
Whit Sunday is 7 Weeks after Easter, must be	June 4.
Trinity Sunday, the next after Whit Sunday, must be	June 11.
Trinity Term begins Friday after Trinity Sunday,	June 16.
Trinity Term ends Wednesday Fortnight afterwards,	July 3.

And the Beginning and End of Hilary and Michaelmas Terms are fixed.
Note: The Septuagesima is a Fortnight, and Sexagesima a Week, before Shrove Sunday.
Advent Sunday is always that Sunday next the 30th of November, whether before or after it.
Epiphany is reckoned among the moveable Feasts; but it is always the 6th of January.

* In this Case the Epact must always be used for the Year before; as it is for 1786, use the Epact for 1785, which is 18, (per Prob. V.)

(14) 3 Moon's Age.
4
5 12
2 hrs. 24 m. after Midday.

(15) 25 Moon's Age.
4
5 100
20
12
8 h. after Midnight.

(16) Per Ex. IV. the Moon south at — *h.* 2 *m.* 24 p. m. D.
At London ☽ bears N. E. or S. E. — 3 0
Sum 5 24
Answer 24 Min. past 5 at Night.

(17) The Moon souths at — *h.* 8 *m.* af. M. N.
At Bristol Key ☽ bears E. by S. and W. by N. — 6 45
Sum 14 45
Answer 45 Min past 2 M.

QUESTIONS.

(1) In the Reign of William I. were 5 intercalary Days, (by Prob. I.)
Between Sept. 9, and Oct. 4, are 25 Days.
And 1087—1066=21 Years, wanting 25 Days.
Then $21 \times 365 + 5 - 25 = 1645$ Days, William I. reigned.
In the Reign of William Rufus were 4 intercalary Days.
Between August 2, and Sept. 9, are 38 Days.
And 1100—1087=13 Years, all but 38 Days.
Then $365 \times 13 - 38 + 4 = 4711$ Days, William II. reigned.
Lastly, in the Reign of William III. were 3 intercalary Days.
From Feb. 3 to March 8, are 33 Days.
And 1701—1689=12 Years and 33 Days.
Then $305 \times 12 + 33 + 3 = 4416$ Days, Will. III. reigned.

(2) In the Reign of Richard I. were 2 intercalary Days.
Between April 6, and July 7, are 92 Days.
And 1199—1189=10 Years, wanting 92 Days.
Then 365×10+2—92=3560 Days, Richard I. reigned.
In the Reign of Richard II. were 5 intercalary Days.
From June 21, to Sept. 30, are 101 Days.
And 1399—1377=22 Years and 101 Days, he reigned.
Then 365×22+101+5=8136 Days, Rich. II. reigned.
In the Reign of Richard III. was 1 intercalary Day.
From June 18, to August 22, are 65 Days.
And 1485—1483=2 Years and 65 Days, he reigned.
Then 365×2+65+1=796 Days, Richard III. reigned.
Therefore 3560+8136+796=12492 Days, the Answer.

(3) In the Reign of Queen Mary was only 1 intercalary Day.
But from 1553, till 1602, were 12 intercalary Days.
So that in the Reign of Q. Elizabeth were 11 intercalary Days.
From July 8, to Nov. being 4m. 9d., are 132 Days.
Then 365×5+132+1=1958 Days, Mary reigned.
Now 1602—1553=49 Years.
And from March 14, to July 8, are 115 Days.
Also 365—115=250 Days.
Then 365×49×250+12=18147 Days, to the Beginning of James's Reign.
∴ 18147—1958=16189 Days, Elizabeth reigned.
In the Reign of James I. were 6 intercalary Days.
From March 14, to the 27th, is 13 Days.
And 1625—1602=23 Years; but the Date altering at our Lady Day, so that the intercalary was only 22 Years.
Therefore 365×22×13+6=8049 Days, James I. reigned.
In the Reign of Charles I. were 6 intercalary Days.
From March 27, to Jan. 30, are 309 Days.
And 1648—1625=23 Years.
Then 365×23+309+6=8710 Days, Charles I. reigned.
∴ 16189+8049+8710=32948 Days, the Answer.

(4) The Grant begun December 14th, 1109, and resumed Nov. 19th, 1219.
From Nov. 19, to Dec. 14, are 25 Days.

And 1219—1109=110 Years, wanting 25 Days.
In which Time are 27 intercalary Days.
Then $\overline{365 \times 110} + 27 - 25 = 40152$ Days, the first Continuance.
The second Grant revived July 16th, 1497, and ended May 10th, 1524.
Then from May 10, to July 16, are 67 Days.
And 1524—1497=27 Years, wanting 67 Days, in which Time there are 7 intercalary Days.
Therefore $\overline{365 \times 27} + 7 - 67 = 9795$ Days, last in Force.
$\therefore$ 40152+9795=49947 Days, the Grant was in Force.
The first Grant resumed Nov. 19th, 1219, and revived July 16, 1498.
From Nov. 19, to July 16, are 239 Days.
And 365—239=126 Days.
Also 1498—1219=279 Years, wanting 126 Days; in which Period are 69 intercalary Days.
Therefore $\overline{365 \times 279} + 69 - 126 = 101778$ Days, superseded.
Q. E. F.

77. GEOGRAPHY.

(1) Lat. 39° 54″N. 57°S. and Long. 116°. 28 E.
(2) Jamaica and Cape of Good Hope.
(3) Diff. 10° 30′ and 48″.
(4) 4691¼ Miles, Buda, in Hungary, Madrid, &c.
(5) Decl. 19° 15′N. Right Asc. 60° 45′, Mer. Alt. 57° 15′
(6) R. 20′ P. 4, sets 37′ P. 7, Amp. 30° N. Twil. beg. 21′ P 1, ends 30′ P 10.
(7) Azm. 78° from the N. Alt. 12° 30′.
(8) Port Royal bears W. from London, and London bears N. E. from Port Royal.
(9) 30′ P. 7, P. M. at Pekin, Breakfast at Port Royal, Dinner at Rome, and Supper at the Ladrone Isles.
(10) Antigua, Pagan, one of the Ladrone Isles, &c.
(11) May 10th, and Sept. 2d.
(12) All those Places whose Lat. is equal to 72° 45′ N.
(13) Begins May 4th, and leaves them Aug. 9th, following.
(14) Rising to N. Mexico, Ter. Fuego, &c. setting to Candy

in the Isle of Ceylon, and part of Asia, &c. Noon at Porto, St. Vincent, &c.

(15) Begin in N. and S. America, the S. and Pacifick Seas, Part of Tartary, the Land of Jesso, &c. Mid. All the great S. Sea, the E. and W. Indies, Part of Asia, &c. End of all Asia, and the East Indies, New Holland, the S. Sea, Part of N. America, &c.

(16) 63°, (nearly.)

(17) Antoeci, Part of the S. Ocean, Perioeci, Part of the Great Pacifick Ocean, Antip. Part of the S. Ocean, near New Zealand.

(18) Sun's Declination 23°.

(19) Sun's Place 21.30.8. Right Ascension 29° E.

(20) Rises 5¾. Sets 6¼. Amp. 5° N.

(21) Altitude 44° 30′. Azimuth 43 E.

(22) Hour 7¾. Azimuth 76½.

(23) Course 60°. Distance sailed 41°. 2460 Miles.

(24) Diff. of Long. 40°. Distance sailed 63, or 3780 Miles.

(25) Diff. of Long. 18°. Course 41°.

(26) Diff. of Lat. 23° 30′. Distance sailed 39°, or 2340 M.

(27) Diff. of Lat. 29° 30′. Long. 19°.

CELESTIAL GLOBE.

(1) At ¾ p. 3, sets 20′ p. 4, no Night.

(2, 3, 4,) For these Examples, the best will be to answer them for the Year you are in; as you will then (by the Ephemerides) have it in your Power to find the Node of the Moon, and her place at Noon, likewise the Places of any of the Planets.

(5) Decl. 29° N. Right Asc. 112, Lat. 6° 30′, Long. 90° 45 Leo.

(6) Aldebaran rises at 1 A. M. sets at 4 P. M. comes to the Mer. ¼ p. 8, Amp. 26°. N.

(7) R. cosmically Sept. 24th, sets July 6th.

(8) R. acronically Jan. 25th, sets December 18th.

(9) Jan. 13th.

(10) Nov. 16th.

(11) Obl. Decl. 120°, continues above the Horiz. 9 hrs.

(12 Azim. 10°, from the N. Alt. 8°.

(13) ½ p. 8.

(14) 14° 30′ N.

(15) 7′ p. 12.
(16) $\frac{1}{4}$ p. 3. A. M.
(17) Lat. 44° 30′ N.
(18) Lat. 22° 30′ N.
(19) 44° 30′ S.
(20) Arcturus on the Mer. nearly, Canis Minor setting, and the Stars Lyra, Altayr, Cor Hydræ, Cor Leonis, Castr. Capella, &c. will be visible.
(21) Lat. 3° 30′, Long. 26° 36′ of ♒, and Lat. 25° S. Long. 12° 30′ ♏.
(22) R. 3 h. 45 m. A. M. Amp. 14°, sets 2 h. 25 m. P. M. Decl. 9° 15′ S. Right Asc. 227° 30′.
(23) Through the Tail of Capricorn, the Head of Indus, the Neck and Body of Pavo, the Neck of Apus, Tail of Centaurus, between the two Stars in the Back of Hydra, and then to the Ecliptic near Cor Leonis: Velocity 4° $\frac{1}{6}$ per Day.

ALGEBRA.

78. ADDITION.

EXAMPLES.

(1)

To	$6a+7b-3c$
Add	$10a+b-7c$
Sum	$16a+8b-10c$

(2)

	$ab-6b+4x+10y-15z+6$
Add	$6ab-b+x+4y-9z+3$
Sum	$7ab-7b+5x+14y-24z+9$

(3)

To	$-6b-7c-8x$
Add	$4b+9c+5x$
Sum	$-2b+2c-3x$

(4)

	$6a-6x+7y-10$
Add	$-6a+6x-4y+13$
Sum	* * $3y+3$

(5)

To	$4a+6b+4c-6$
Add	$4x-7y+4z$
Sum	$4a+6b+4c-6+4x-7y+4z$

(6)

	$2a-6bc$
Add	$6x+10$
Sum	$2a-6bc+6x+10$

	(7)	(8)	(9)
To	$10a\sqrt{bc}$	$a\sqrt{bb+cc}$	$b+3\sqrt{aa-u}$
Add	$6a\sqrt{bc}$	$4a\sqrt{bb+cc}$	$b-3\sqrt{aa-u}$
Sum	$16a\sqrt{bc}$	$5a\sqrt{bb+cc}$	$2b$ * *

80. SUBTRACTION.

EXAMPLES.

	(1)	(2)	(3)
From	$3a$	$2a-4c+7y-7$	$6b-4c+4x$
Take	$-3a$	$6a+4x+7y+4$	$7b+7c-9x$
Rem.	$6a$	$-4a-8x$ * -11	$-b-11c+13x$

	(4)	(5)	(6)
From	$10a\sqrt{bc}$	$6b\sqrt{aa+xx}$	$a+b$
Take	$6a\sqrt{bc}$	$4b\sqrt{aa+xx}$	$a-\sqrt{cc-ae}$
Rem.	$4a\sqrt{bc}$	$2b\sqrt{aa+xx}$	$*b+\sqrt{cc-aa}$

81. MULTIPLICATION.

EXAMPLES IN CASE I.

	(1)	(2)	(3)
Mul.	$a+c$	$-a-b-c$	$x+y+z$
by	b	$-d$	a
Prod.	$ab+bc$	$ad+bd+dc$	$ax+ay+az$

	(4)	(5)
Mul.	$-8x$	$12x+6y$
by	$-4x$	$4a$
Prod.	$32ax$	$48ax+24ay$

(6)

Mul. $-6d$
by $7b$

Prod. $-42bd$

(7)

$4x-5y+z$
$-6f$

$-24fx+30fy-6fz$

(8)

Mul. $2a-4b$
by $2a+4b$

$4aa-8ab$
$+8ab-16bb$

Prod. $4aa * -16bb$

(9)

$aa+ab+bb$
$a-b$

$aaa+aab+abb$
$-aab-abb-bbb$

Prod. $aaa * * -bbb$

(10) Mul. $xx-ax$
by $a+x$

$axx-aax$
$xxx-axx$

Prod. $axx-xxx *$

(11) Mul. $xx+xy+yy$
by $xx-xy+yy$

$x^4+x^3y+x^2y^2$
$-x^3y-xy^2+xy^3$
$x^2y^2+xy^3+y^4$

Prod. $x4 * + x^2y^2y+ *y^4$

(12)

M. $aaa-3aab+3ab^2-bbb$
by $aa-2ab+bb$

$a^5-3a^4b+3a^3b^2-a^2b^3$
$-2a^4b+6a^3b^2-6a^2b^3+2ab^4$
$a^3b^2-3a^2b^3+3ab^4-b^5$

P. $a^5-5a^4b+9a^3b^2-10a^2b^3+5ab^4-b^5$

	(13)	(14)	(15)
Mul.	$\surd bc+dcc$	$\surd xx+zz$	$6cd\surd b+ad$
by	$\surd ac$	$\surd xx+zz$	$3a\surd ca$
Prod.	$\surd abcc+adccc$	$\surd x^4-z^4$	$18acd\surd abc-aadc$

82. DIVISION.

EXAMPLES IN CASE I.

(1) $d)ad+6d(a+6$ (2) $-d)-ad-bd(a+b$

(3) $a)aa+ab(a+b$ (4) $-a)ab(-b$

(5) $b)-ab-bd(-a-d$

(6 $-bc(abc+bcd+bcf(-a-d-f$

(7) $7b)42db(6d$ (8) $2bx)8abx-18bxc(4a-9c$

(9) $2b)ab+bb(=\frac{ab+bb}{2b}=\frac{a+b}{2}$, here Unity, or $1b$, is the common Measure. (See Sect. XXXVIII. CASE I.)

(10) $20a)\ 10ab+\ 15ac(=\frac{10ab+15ac}{20a}=\frac{2b+3c}{4}$, here 5 is the common Measure.

(11.) $a-b)aaa-3aab+3abb-bbb(aa-2ab+bb$

$aaa-aab$

$*\ -2aab+3abb$

$-2aab+2abb$

$*\ abb-bbb$

$abb-bbb$

$*\ *$

(12) $a+b)aa+2ab+bb(a+b$

$aa+ab$

$*\quad ab+bb$

$ab+bb$

$*\ *$

(13) $ab+)aa-bb(a-b$

$aa+ab$

$*\quad -ab-bb$

$ab-bb$

$*\ *$

(14) $3a-6)\ 6a-96(2a^3+4a^2+8a+16$

$6a-12a^3-$

$*\quad 12a^3-96$

$12a^3-24a^2$

$*\quad 24a^2-96$

$24a^2-48a$

$*\quad 48a-96$

$48a-96$

$*\ *$

It often happens that the Operation may be continued without End, and then you have an infinite Series for the Quotient; but by comparing the first three or four Terms, you may find what Law the Terms observe; by which means, without any more Divisor, you may continue the Quotient as far as you please.

(15)

$3x^2-4x+5)\,18x^4-45x^3+82x^2-67x+40\,(6x^2-7x+8$

$18x^4-24x^3+30x^2$

$*-21x^3+52x^2-67x$

$-21x^3-28x^2-35x$

$*-24x^2-32x+40$

$-21x^2-32x+40$

$*\quad *\quad *$

(16)

$4x-5a)48x^3-76ax^2-64a^2x+105a^3\,(12x^2-4ax-21a^2$

$48x^3-60ax^2$

$*-16ax^2-64a^2x$

$-16ax^2+20a^2x$

$*-84a^2x+105a^3$

$-84a^2x+105a^3$

$*\quad *$

(17)

$3x+4a)81x^4**-256a^4\,(27x^3-36ax^2+48a^2x+64a$

$81x^4+108ax^3$

$*-108ax^3$

$108ax^3-144a^2x$

$*\ 144a^2x$

$144a^2x+192a^3x$

$-192a^3x-256a^4$

$-192a^3x-256a^4$

$*\quad *$

(18)

$$2x-3a\overline{)16x^4 \quad *-72a^2x^2 \quad *+8a^4}\,(18x^3-12ax^2-18x^2a-27a^3$$
$$16x^4-24ax^3$$

$$*-24ax^3-72a^2x^2$$
$$-24ax^3-36a^2x^2$$

$$*\,36a^2x^2 \quad *$$
$$36a^2x^2+54a^3x$$

$$*\;-54a^3x+81a^4$$
$$-54a^3x+81a^4$$

$$* \qquad *$$

(19) $2xy\sqrt{z}\,)\,4xy\sqrt{xzz}\,(2\sqrt{xz}$ the Quotient.

(20) $20\sqrt{2cy}\,)\,60ab\sqrt{10acxy}\,(3ab\sqrt{5ax}$ the Quotient.

(21) $x^2)x^5(x^3$ the Quotient.

(22) $a+x.\,)\,a^4+a\,|\,(a+x^3$, the Quotient.

83. FRACTIONS.

(1) Thus $a-x+\frac{a^2-ax}{x}=\frac{ax-xx+a^2+ax}{x}=$
$\frac{a^2-x^2}{x}$ (by Sect. XXXVIII. Case III.)

(2) Thus $a+b+\frac{x}{z}=\frac{az+bz+x}{z}$

(3) Thus $a-x+\frac{aa-ax}{x}=\frac{ax-xx+aa-ax}{x}=$
$\frac{aa-xx}{x}$.

(4) Thus $\frac{a^2-x^2}{x}=-x+\frac{a^2}{x}$.

(5) Thus $\frac{az+bz+x}{z}=a+b+\frac{x}{z}$.

(6) Thus $\frac{ax-xx+aa-ax}{x} = -x+\frac{aa}{x}$.

(7) Thus $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}, = \frac{bce}{bdf}, \frac{dae}{bdf}$, and $\frac{fac}{bdf}$.

(8) Thus $\frac{b+c}{a+b}$ and $\frac{d-c}{b-d} = \frac{bb-bd+cb-cd}{ab-ad+bb-bd}$ and $\frac{ad-ac+bd-bc}{ab-ad+bb-bd}$.

(9) First

$a^2+{}^2ab+b^2)a^3+\ *\ -ab^2(a$

$a^3+2a^2b+ab^2$

Rem. $-2a^2b-2ab^2)a^2+2ab+b^2(-\frac{1}{2b}\ \frac{1}{2a}$

a^2+ab

$ab+b^2$

$ab+b^2$

* *

Hence it appears that $-2a^2b-2ab^2$, is the common Measure; by which a^3-2b^2, being divided:

viz. $-2a^2b-2ab^2)a^3-\ *\ -ab^2(\frac{a}{2b}+\frac{1}{2}$

$a^3+{}^2ab$

$-a^2b-ab^2$

$^2ab-ab^2$

* *

Then $\frac{a}{2b}+\frac{1}{2} = \frac{-a+b}{2ba}$ the new Numerator.

And $\frac{1}{2b}\ \frac{1}{2a} = \frac{-a-b}{2ba}$, the Denominator.

Let both be $\times$ $2ba$, and we shall have $\frac{-a^2+ab}{-a-b}$ New N. Denom.

Or change the Signs of all the Quantities, it will be $\frac{a-ab}{a+b}$, the new Fraction required.

That is, $\frac{a2-ab}{a+b}=\frac{a^2-ab^2}{a^2+2ab+b^2}$ the common Measure.

(10) Thus $\frac{25az}{5xz+15az}=\frac{5a}{x+3a}$, by expunging z, and dividing by 5.

Also $\frac{a^3+b^3}{a^2-a^2}=\frac{a^2-ab+b^2}{a-b}$, by dividing by $a+b$, the common Measure.

ADDITION.

EXAMPLES.

(1) Thus $\frac{a}{b}+\frac{c}{d}+\frac{d}{e}=\frac{bcd+dad+eac}{bde}$, the Sum.

(2) Thus $\frac{a-b+d}{d+a}+\frac{a+b-d}{d+a}=\frac{2a}{a+d}$, the Sum.

(3) Thus $\frac{2a-b}{a+c}+\frac{2b-a}{a+c}=\frac{a+b}{c+a}$, the Sum.

(4) Thus $\frac{a+b}{d}+\frac{2a+c}{d}=\frac{3a+b+c}{d}$, the Sum.

SUBTRACTION.

EXAMPLES.

(1) Thus $\frac{x}{2}-\frac{x}{3}=\frac{3x}{6}-\frac{2x}{6}=\frac{x}{6}$, the Difference.

(2) Thus $\frac{a+x}{b}-\frac{a-x}{c}=\frac{ac+cx}{bc}-\frac{ab-bx}{bc}=$

$\frac{ac+cx-ab+bx}{bc}$, the Difference.

(3) Thus $\frac{b^2+a^2}{c} - \frac{b^2}{c} = \frac{a^2}{c}$, the Difference.

(4) Thus $\frac{2b}{a+d} - \frac{a+b-d}{a+d} = \frac{b-a+d}{a+d}$, the Difference.

MULTIPLICATION.

EXAMPLES.

(1) Thus $\frac{a+b}{c} \times \frac{a-b}{d} = \frac{a^2-b^2}{cd}$, the Product.

(2) Thus $a + \frac{b}{c} \times \frac{d}{c} = \frac{ac+b}{c} \times \frac{d}{c} = \frac{acd+bd}{cc}$, the Product.

(3) Thus $\frac{3a-2b}{2d+c} \times \frac{4a+2b}{d} = \frac{12a^2-2ab-4b^2}{2dd-dc}$, the Product.

(4) Thus $2a + \frac{b}{c} - 25 \times 36 + 4c = \frac{2ac+b-25c}{c} \times \frac{36+4c}{1} = \frac{6bac+3b^2-75bc+8ac^2+4bc-100c^2}{c} =$

$6ba - 71b - 8ac - 100c + \frac{3b^2}{c}$, the Product.

DIVISION.

EXAMPLES.

(1) Thus $\frac{acd+bd}{cd} \div \frac{d}{c} = \frac{acde+bdc}{cdd}$, or $\frac{ace+bc}{cd}$, the Quotient.

(2) Thus $\frac{a-b}{a} \div \frac{a+b}{a-b} = \frac{a^2-2ab+b^2}{a^2+ba}$, the Quotient.

(3) Thus $a+\frac{b}{c}\div d+\frac{e}{f}=\frac{ac-b}{c}\div\frac{fd+e}{f}=\frac{fa-bf}{fac+ce}$, the Quotient.

(4) Thus $\frac{ab}{c}\div\frac{1}{c}=ab$, the Quotient.

84. INVOLUTION.

(1) Thus the fourth Power of $x=x^4$.
(2) The fifth Power of $\overline{ax+z}^2=\overline{ax+z}^{10}$.
(3) The third Power of $3x^3z^2=27x^9z^6$.
(4) The sixth Power of $\frac{5ab}{2c}=\frac{15625ab^6}{64c}$.
(5) Thus

$a+b$
$a+b$

$aa+ab$
$\quad ab+bb$

$a^2+2ab+b^2$, the Square or second Power.
$a+b$

$a^3+2a\cdot b+ab^2$
$\quad a2b+2ab^2+b^3$

$a3\ \ 3a^2b+3ab^2+b^3$=the Cube or third Power.
$a+b$

$a^4+3a^3b+3a2b^2\times ab^3$
$\quad a^3b+3a2b^2+3ab^3+b^4$

$a^4+4a^3b+6a^2b^2+4ab^3+b^4$, the fourth Power.
$a+b$

$a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4$
$\quad a^4b+4a^3b+6a^2b^3+4ab^4+b^5$

$a^5 5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$, the 5th Power.

$a+b$

$a^6+5a^5b+10a^4b^2+10a^3b^3+5a^2b^4+ab^5$

$ab+5a^4b^2+10a^3b^3+10a^2b^4+5a^5b+b^6$

$a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6=$ 6th P.

(6) Thus

$a-b$

$a-b$

a^2-ab

$ab+b^2$

$a^2-2ab+b^2$, second Power.

$a-b$

$a^3-2a^2b+ab^2$

$-a^2b+2ab^2-b^3$

$a^3-3a^2b+3ab^2-b^3$, third Term.

$a-b$

$a^4-3a^3b+3a^2b^2-ab^3$

$a^3b+3a^2b^2-3ab^3+b^4$

$a^4-4a^3b+6a^2b^2-4ab^3+b^4$, fourth Power.

$a-b$

$a^5-4a^4b+6a^3b^2-4a^2b^3+ab^4$

$-a^4b+4a\,b^2-6a^2b^3+4ab^4-b^5$

$a^5-5a4b+10a^3b^2-10a^2b^3+5ab^4-b^5$, fifth Power.

$a-b$

$a^6-5a^5b+10a^4b^2-10a^3b^3+5a^2b^4-ab^5$

$-a^5b+5a^4b^2-10a^3b^3+10a\,b^4-ab^5$

$a^6-6a^5b+15a^4b-20a\,b^3+15a^2b^4-6ab^5+b^6=$ 6th P.

These * Examples are performed by the Theorem as follows: Here, m, Index of the proposed Power, being 6; the first

* See the Note in the Tutor's Guide, under Involution, concerning Sir Isaac Newton's Theorem.

Term a^m, or the general Expression is equal to a^5; the second $ma^{m-1}b = 6a^5b$; the third $\frac{m \times m-1}{1\ 2} \times a^{m-2}b$ $= 15a^4b^2$; the fourth $\frac{m \times m-1 \times m-2}{1\ 2\ 3} \times a^{m-3}b =$ $20a^3b^3$; the fifth $\frac{m \times m-1 \times m2 \times m-3}{1\ 2\ 3\ 4} \times a^{m-4}b$ $= 15a^2b^4$; the sixth $\frac{m \times m-1 \times m-2 \times m-3 \times m-4}{1\ 2\ 3\ 4\ 5}$ $\times a^{m-5}b = 6ab^5$; the seventh $\frac{m \times m-1 \times m-2 \times m-3}{1\ 2\ 3\ 4}$ $\times \frac{m-4 \times m-5}{5\ 6} \times a^{m-6}b = b^6$; and the eighth $\frac{m \times m-1}{1\ 2}$ $\times \frac{m-2 \times m-3 \times m-4 \times m-5 \times m-6}{3\ 4\ 5\ 6\ 7} \times a^{m-7}b =$ nothing.

Therefore the sixth Power of $a+b = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$, as before.

85. EVOLUTION.

(1) $x^2 + 2xy + y^2 (x+y$, the Root required.
$1x) 2xy$
$x^2 + 2xy + y^2 =$ second Power of $x+y$.

* * *

Or thus by Sect. 52.

$x^2 + 2xy + y^2 (x+y$ Root as before.
x^2

$2x+y) * \quad 2xy + y^2$
$2xy + y^2$

* *

(2) $x^2 - 2xy + y^2 (x-y$, the Root.
x^2

$2x-y)\ *-2xy+y^2$
$\quad -2xy+y$

$\quad * \quad *$

(3) $4^4-2x^3y+3x^2y^2-2xy^3+y^4(x^2-xy+y^2$, the Rt.
4^4

$2x^2-xy)\ *-2x^3y+3x^2y^2$
$\quad -2x^3y+x^2y^2$

$2x^2-2xy+y^2)\ *\ 2x^2y^2-2xy^3+y^4$
$\quad 2x^2y^2-2xy^3+y^4$

$\quad * \quad * \quad *$

(4) $x^3-6x^2y+12xy^2+8y^3(x-2y$, the Root.
$3x^2)\ 6x^2y$

$x^3-6x^2y+12xy^2+8y^3$

$\quad * \quad * \quad * \quad *$

(5) $16x^4-96x^3y+216x^2y^2-216xy^3+81y^4(2x-3y$ Rt.
$32x^3)\ *\ 96^3$

$16x^4-96x^3y+216x^2x^2-216xy^3-81y^4$
$*\quad *\quad *\quad *\quad *$

87. REDUCTION OF SIMPLE EQUATIONS.

(1) Given $20-3x-8=60-7x$.
Then by transposing $7x$, we shall have $4x+12=60$.
Therefore $x=\frac{60-12}{4}=\frac{48}{4}=12$.
For $\overline{20-12}\times 3-8=60-\overline{12\times 7}=-24$.

(2) Given $5x-16=3x+12$.
Here by tranposing the $3x$ and 16 we get $2x=12+16$.
Therefore $x=\frac{12+16}{2}=\frac{28}{2}=14$.
For $\overline{14\times 5}-16=14\times 3+12=54$.

(3) Given $\frac{3x}{4}+5=\frac{5x}{6}+2$.

Then (per Rule III.) $18x+120=20x+48$.

Therefore $2x=72$, whence $x=72\div 2=36$.

For $\frac{36\times 3}{4}\times 5=\frac{36\times 5}{6}+2=32$.

(4) Given $\frac{7x}{8}-5=\frac{9x}{10}-8$

Then $70x-400=72x-640$, whence $2x=240$, and $x=120$.

For $\frac{120\times 7}{8}-5=\frac{120\times 9}{10}-8=100$.

(5) Given $\frac{5x}{9}-8=74-\frac{7x}{12}$.

Then by multiplying both Sides of the Equation by 9, and 12, we shall have $60x-864=7992-63x$.

Therefore $123x=7992+864=8856$, and $x=8856\div 123=72$.

(6) Given $56-\frac{3x}{4}=48-\frac{5x}{8}$,

Then $1792-24x=1536-20x$.

Therefore $4x=256$, and $x=256\div 4=64$.

(7) Given $36-\frac{4x}{9}=8$.

Then by multiplying both Sides of the Equation by 9, we have $324-4x=72$.

Therefore $4x=324-72=252$.

Therefore $x=252\div 4=63$.

(8) Given $\frac{2x}{3}=\frac{176-4x}{5}$.

Then $10x=528-12x$.

And $22x=528$.

Therefore $x=528\div 22=24$.

(9) Given $\frac{45}{2x+3}=\frac{57}{4x5}$

Then multiplying by $2x+3$, we shall have $45=\frac{114x+171}{4x-5}$; also multiplying by $4x-5$, we have $180x-225=114x+171$.

Therefore $180x-14x-225=171$, that is $66x=225+=396=171=396$, and $x=396\div 66=6$.

(10) Given $\frac{42x}{x-2}=\frac{35x}{x-3}$.

Then by multiplying by $x-2$, and $x-3$, we get $42x^2-126x=35xx-70x$.

Therefore $42x^2-35x^2=126x-70x$, that is $7x^2=56x$.

Therefore by dividing by $7x$, we have $x=8$.

(11) Given $\frac{xx-12}{3}=\frac{xx-4}{4}$.

Then by multiplying both Sides of the Equation by 3 and 4, we have $4x^2-48=3x^2-12$.

Therefore $4x^2-3x^2=48-12$, that is $x^2=36$.

Whence $x=\sqrt{36}=6$.

(12) Given $\frac{5xx}{16}=8+12$.

Then by multiplying by 16, we get $5x^2=20\times 16=320$.

Therefore $x^2=320\div 5=64$, and $x=\sqrt{64}=8$

(13) Given $\frac{x+1}{2}+\frac{x+2}{3}=16-\frac{x+3}{4}$.

Then by multiplying by 2, 3, and 4, we get $12x+12+8x+16=384-6x-18$, that is $20x+28=366-6x$.

Therefore $20x+6x=366-28$, that is $26x=338$.

Whence $x=338\div 26=13$.

(14) Given $ax+b^2=\frac{ax^2+ac^2}{a+x}$.

Then $\overline{ax+b^2}\times\overline{a+x}=ax^2+ac^2$, that is $a^2x+ab^2+ax^2+b^2x=ax^2+ac^2$.

Therefore $a^2x+b^2x=ac^2-ab^2$, and $x=\frac{ac^2-ab^2}{a^2+b^2}$.

(15) Given $\sqrt{\frac{5x}{3}}+12=17$.

Then $\sqrt{\frac{5x}{3}}=17-12=5$, and $\sqrt{5x}=5\times3=15$.

Therefore by Rule IV.) $5x=225$, and $x=225\div5=45$.

(16) Given $\sqrt{12+x}=2+\sqrt{x}$, then (by Rule IV.) $12+x=4+4\sqrt{x}+x$.

Whence by Transposition $4\sqrt{x}=8$, and by Division $\sqrt{x}=2$, consequently $x=4$.

(17) Given $\sqrt{x}+\sqrt{a+x}=\sqrt{\frac{2a}{a+x}}$

Then $\sqrt{ax}+xx+a+x=2a$, or $\sqrt{ax}-xx=a-x$.

Whence $ax+x^2=a^2-2ax+x^2$.

Therefore $3ax=a^2$.

$\therefore x=\frac{a^2}{3a}$.

(18 Given $615x-7x^2=48x$.

Then by Transposition $615x-48x=7x^2$, that is $7x^2=567x$.

Whence divided by $7x$, we get $x^2=81$.

$\therefore x=9$.

(19) Given $\sqrt{a^2+x^2}=\sqrt{b^4+x^4}$.

Then by raising both Sides to the fourth Power, we have $ac+xx)^2=b^4+x^4$, that is $a^4+2a^2x^2+x^4=b^4+x^4$.

Therefore $2a^2x^2=b^4-a^2$.

$\therefore x^2=\frac{b^4-a^4}{2a^4}=\frac{b^4}{a^4}-a^2$.

88. Of the EXTERMINATION of UNKNOWN QUANTITIES; or, the REDUCTION of Two or more EQUATIONS to a single one.

(1) Given $\begin{cases}5x+8y=106\\4x-5y=5.\end{cases}$

Here, by multiplying the first Equation by 4, and the second by 5, in order that the Coefficients of x may be the same in both, there arises,

$$25x+32y=434$$
$$20x-25y=25$$

By subtracting the latter from the former we have $57y=399$.

Hence $y=\frac{399}{57}=7$.

And so by the first Equation $x=\frac{5+5\times 7}{4}=\frac{40}{4}=10$.

(2) Given $\begin{cases} 5x-3y=150 \\ 10x+15y=825 \end{cases}$

Here the first Equation being multiplied by 2 (in order that the Coefficients of x in both Equations may be the same,) we have $10x-6y=300$. Let this Equation be subtracted from the second, and we shall have $21y=525$.

Hence $y=\frac{525}{21}=25$

Therefore by the first Equation $x=\frac{150+25\times 3}{5}=\frac{225}{5}=45$.

(3) Given $\begin{cases} \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62 \\ \frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47 \\ \frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38 \end{cases}$

Here the given Equations, cleared of the Fractions, become

$$12x+8y+6z=1488$$
$$20x+15y+12z=2820$$
$$30x+24y+20z=4560$$

Now, to exterminate z, let the second of these Equations be subtracted from the double of the first. And also, three Times the third, from four Times the second. When is had

$$4x+y=156$$
$$10x+3y=420$$

Again, to exterminate y, from three Times the first take the

second, and we shall then have $12x+3y=468$ and $10x+3y=420$. From which $x=\frac{48}{2}=24$

Therefore $y=156-24\times 4=60$.

And $z=\left(\frac{1488-8y-12x}{6}=120\right)$.

(4) Given $\begin{cases} \frac{x}{4}+\frac{y}{5}=15 \\ \frac{x}{6}+\frac{y}{9}=9 \end{cases}$

Here our Equations, cleared of Fractions, will be

$$5x+4y=300$$
$$9x+6y=486$$

And if from double of the latter we take triple of the former, we shall have $18x-15x=972-900$, that is $3x=72$.

Whence $x=\frac{72}{3}=24$

And $y=\frac{300-5x}{4}=45$

(5) Given $\begin{cases} \frac{x}{2}-12=\frac{y}{4}+8 \\ \frac{x+y}{5}+\frac{x}{3}-8=\frac{2y-x}{4}+27 \end{cases}$

Here the Equations, cleared of the Fractions, we shall have

$$4x-96=2y+64$$

And $12x+12y+20x-480=30y-15x+1620$

Which contracted, become

$$4x-2y=64+96=160$$
$$47x-18y=2100$$

From the last of which subtract nine Times the former, and we have $41x-36x=2100-1440$, that is $11x=660$.

Hence $x=\frac{660}{11}=60$

And $y=\frac{4x-160}{2}=\frac{240-160}{2}=\frac{80}{2}=40$

(6) Given $\begin{cases} x+y=80 \\ x+z=70 \\ y+z=60 \end{cases}$

Here by subtracting the second Equation from the first, (in order to exterminate x) we have $y-z=10$, to which the third Equation being added, z will likewise be exterminated, there coming out $2y=70$, or $y=35$.

Whence $z=y-10=25$, and $x=80-y=35$.

(7) Given $\begin{cases} x+100=y+z \\ y+100=2x+2z \\ z+190=3x+3y \end{cases}$

To the double of the first, let the second Equation be added; so shall the x's, on the contrary Sides, destroy each other, and we shall have $300+y=2y+4x$, or $y+4z=300$.

Moreover, to the triple of the first let the third Equation be added, whence will be had $z+400=6y+3z$ or $2z+6=400$.

Now if from the double of this last Equation which is $4z+12y=800$, the former (viz. $y+4x=300$) be subtracted, then will come out $11y=500$.

Therefore $y=\frac{500}{11}=45\frac{5}{11}$, and $z=\frac{300-y}{4}=75-\frac{y}{4}$

$=75-11\frac{4}{11}=63\frac{7}{11}$.

And $x=y+z-100=109\frac{1}{11}-100=9\frac{1}{11}$

(8) Given $x-y=2$, and $xy+5x-6y=120$.

Then to exterminate x.

By the first Equation $x=2+y$; which Value being substituted in the last (according to Rule II.) it becomes $y+2\times y+5\times y+2-6y=120$, that is $y^2+2y+5y+10-6y=120$, or $y^2+y=110$.

(9) Given $\left\{\begin{matrix} x+y=s \\ x^2-y^2=d \end{matrix}\right\}$ quere x and y.

Then $x=s-y$; the Square of which is $x^2=s^2-2sy+y^2$.

Also $^2x=d+y^2$.

Therefore $d+y^2=s^2-2sy+y^2$.
Hence $d=s^2-2sy$.
And $2sy=s^2-d$.
So $y=\frac{s^2-d}{2s}$.
And $x=\frac{s^2+d}{2s}$.

(10) Given $\left\{\begin{matrix} x+y+z=12 \\ x+2y+3z=20 \\ \frac{x}{3}+\frac{y}{2}+z=6 \end{matrix}\right\}$ To find x, y, and z.

Then we have $\left\{\begin{matrix} 1.\ x=12-y-z \\ 2.\ x=20-2y-3z \\ 3.\ x=18-\frac{3y}{2}-3z \end{matrix}\right.$

From whence we have these two Equations. $\left\{\begin{matrix} 12-y-z=20-2y-3z \\ 12-y-z=18-\frac{3y}{2}-3z \end{matrix}\right.$

Therefore we have (by Rule 1.) $y=4z=2$, and $x=6$.

Sometimes the Equations are such, that the same Quantities in different Equations may have contrary Signs, and destroy each other; or be otherwise affected, so as to shorten the common Process very much.

Thus $\left\{\begin{matrix} x+y+z=26 \\ x-y=4 \\ x-z=6 \end{matrix}\right.$

Then by Addition only, $3x=36$.
Hence $x=12$, $y=x-4=8$, and $z=x-6=6$.

104. QUADRATIC EQUATIONS.

(1) Given $x^2-4x=32$, to find the value of x.
Here $x^2-4x+4=32+4=36$, (by Rule III.)
And $x-2=\sqrt{36}=6$, (by Rule IV.)
Therefore $x=6+2=8$.

(2) Given $12x^2-420x=-1200$, to find x.
Here $x^2-35x=-100$, (by Rule II.)

And $x^2-35x+306,25=206,25$, (by Rule III.)
Also $x-17,5=\sqrt{209,25}=14,361406$, (by Rule IV.)
Therefore $x=31,861406,-28,722812$, or $3,138594$.

(3) Given $4x^2+60x=216$, to find x.
Here $x^2+15x=54$, (by Rule II.)
And $x^2+15x+56,25=110,25$.
Also $x+7,5=\sqrt{110,25}=10,5$.
Therefore $x=10,5-7,5=3$.

The fifteen following Equations, or Questions, are from Mr. WARD'S MATHEMATICIAN'S GUIDE.

(1) Given $\left\{\begin{matrix} a+e=s=240 \\ a-e=d=192 \end{matrix}\right\}$ To find the rest.
Here by Addition we get $2a=s+d$.
Therefore $a=\frac{s+d}{2}=\frac{240+192}{2}=\frac{432}{2}=216$.

Then by subtracting the second Equation from the first we get $2e=s-d$.
And $e=\frac{s-d}{2}=\frac{240-192}{2}=\frac{48}{2}=24$

Now, as a and e are found, the rest may be very easily found as follows:

$ae=\frac{ss-dd}{4}$, or $216\times 24=5184=p$.

And $\frac{a}{e}=\frac{s+d}{s-d}$, or $216\div 24=9=q$.

Also $a^2+e^2=\frac{s^2+d^2}{2}$, or $\overline{216\times 216}+\overline{24\times 24}=47232=z$.

Likewise $a^2-e^2=sd$, or $\overline{216\times 216}-\overline{24\times 24}=46080=x$.

(2) Given $\left\{\begin{matrix} a+e=s= 240 \\ ae\ \ =p=5184 \end{matrix}\right\}$ to find a, e, d, q, z, x.
Here by subtracting 4 Times the second Equation from the Square of the first, we have $a^2-2ae+e^2=s^2-4p$.

Therefore $a-e=\sqrt{s^2-4\frac{1}{2}}=d$. Now by adding this Equation to the first we get $2a=s+\sqrt{\frac{s^2-4p.}{2}}$

Hence $a=\frac{s+\sqrt{ss-4}}{2}=216$.

And by subtracting we have $2e=s-\sqrt{ss-4p}$.

Hence $e=\frac{s-\sqrt{s^2-4p}}{2}=24$.

Therefore the rest may be very easily found by proceeding as in the last Example.

(3) Given $\left\{\begin{matrix} a+e=s=240 \\ \frac{a}{e}=q=9 \end{matrix}\right\}$ to find the rest.

Here by subtracting the second Equation from the first, we have $e=s-qe$.

Therefore $e=\frac{s}{9+1}=\frac{240}{10}=24$. Which taken from the first Equation gives $a=s-\frac{s}{q+1}=240\frac{240}{9+1}=240-24=216$.

From hence the rest may be very easily found.

(4) Given $\left\{\begin{matrix} a+e=s=240 \\ a^2+e^2=z=47232 \end{matrix}\right\}$ to find the rest.

Here from the Square of the first Equation take the second; and we have $2ae=s^2-z$. Which taken from the second Equation gives $a^2-2ac+e2=2z-s^2$.

Therefore $a-e=\sqrt{2z-s^2}=d$.

This added to the first Equation gives $2a=s+\sqrt{2z-s}$.

Hence $a=\frac{s+\sqrt{2z-2}}{2}=216$.

And subtracted gives $2e=s-\sqrt{2z-s^2}$.

Whence $e=\frac{s-\sqrt{2z-s^2}.}{2}=24$.

(5) Given $\left\{\begin{matrix} a+e=s=240 \\ a^2-e^2=x=46080 \end{matrix}\right\}$ to find the rest.

Here the second Equation divided by the first gives

$$a-s=\frac{x}{s}=d.$$

Which added to the first, we have $2a=s+\frac{x}{s}=\frac{s^2+x}{s}$.

Hence $a=\frac{s^2+x}{2s}=216$.

And the third subtracted from the first gives

$$2e=s-\frac{x}{s^2}=\frac{s^2-x}{s}.$$

Hence $e=\frac{s^2-x}{2s}=24$.

(6) Given $\left\{\begin{matrix} a-e=d= & 192 \\ ae & =p=5184 \end{matrix}\right\}$ to find the rest.

Here to the Square of the first Equation add 4 Times the second, and we shall have $a^2+2ac+e^2=d^2+4p$.

Therefore $a+e=\sqrt{aa+4p}=s$.

This Equation added to the first gives $2a=d+\sqrt{dd+4p}$.

Therefore $a=\frac{d+\sqrt{dd+4p}}{2}=216$.

Then by taking the first Equation from the fourth, we have $2e=\sqrt{dd\times 4p}-d$.

Hence $e=\frac{\sqrt{dd+4p}-d}{2}=24$.

(7) Given $\left\{\begin{matrix} a-e=d= & 192 \\ \frac{a}{e}=q= & 9 \end{matrix}\right\}$ to find the rest.

Here by transposing e in the first Equation, and multiplying the second by e, we get $qe=d+e$

Then $e=\frac{d}{q-1}=24$

for $9-1\times e=qe-e$. And by adding this last Equation to the First, we have $a=d+\frac{d}{q+1}=216$.

(8) Given $\left\{\begin{matrix} a-e=d= & 192 \\ a^2+e^2=z= & 47232 \end{matrix}\right\}$ to find the rest.

Here, by taking the square of the first from the second, we have $2ae=z-dd$. This added to the second Equation gives $a^2+2ae+e^2=z+z-dd$. Therefore $a+e=\sqrt{2z-dd}$. Which added to the first Equation gives $2a=d+\sqrt{2z-dd}$.

Therefore $a=\frac{d+\sqrt{2z-dd}}{2}=216$.

Then by taking the first Equation from the fifth we get $2e=\sqrt{2z-dd}-d$. Hence $e=\frac{\sqrt{2z-dd}-d}{2}$.

(9) Given $\left\{\begin{matrix} a-e=d= & 192 \\ a^2-e^2=x= & 46080 \end{matrix}\right\}$ to find the rest.

Here, by dividing the second Equation by the first, we have $a+e=\frac{x}{d}$. Then by adding this Equation to the first, we get $2a=d+\frac{x}{d}$. Hence $a=\frac{dd+x}{2d}$.

And by taking this Equation from the third, we have $e=\frac{x-dd}{2d}$.

(10) Given $\left\{\begin{matrix} ae=p= & 5184 \\ \frac{a}{e}=q= & 9 \end{matrix}\right\}$ to find the rest.

Here, by multiplying the two Equations together, we have $aa=qp$. For $\frac{ae}{4}\times\frac{a}{e}=\frac{aae}{e}=aa$. Then $a=\sqrt{qp}=216$.

And by dividing the first Equation by the second, we shall have $ee=\frac{p}{q}$.

For $\frac{a}{1})\frac{ae}{1}(\frac{aee}{a}=ee$. Hence $e=\sqrt{\frac{p}{q}}=24$.

(11) Given $\left\{\begin{matrix} ae & =p= & 5184 \\ a^2+e^2 & =z= & 47232 \end{matrix}\right\}$ to find the rest.

Here to the second add twice the first, and we have $a^2 + 2ae + e^2 = z + 2p$.

Therefore $a + c = \sqrt{z + 2p} = s$.

Then by taking twice the first Equation from the second, we have $a^2 - 2ae + e^2 = x - 2p$.

Hence $a - e = \sqrt{z - 2p}$. This Equation, added to the fourth, gives $2a = \sqrt{z + 2p} + \sqrt{z - 2p}$.

Therefore $a = \frac{\sqrt{z + 2p} + \sqrt{z - 2p}}{2} = 216$.

Then, by taking the sixth Equation from the fourth, we have $2e = \sqrt{z + 2p} - \sqrt{z - 2p}$.

Hence $e = \frac{\sqrt{z + 2p} - \sqrt{z - 2p}}{2} = 24$.

(12) Given $\left\{ \begin{matrix} ae = p = 5184 \\ a^2 - e^2 = x = 46380 \end{matrix} \right\}$ to find the rest.

Here to the Square of the second Equation add 4 Times the Square of the first, and we shall have $a^4 + 2a^2e^2 + e^4 = xx + 3p^2$. Hence $a^2 + e^2 = \sqrt{x^2 + 4p^2} = z$.

This Equation added to the second will give $2a^2 = x + \sqrt{x^2 + 4p^2}$. Whence $a = \sqrt{\frac{x + \sqrt{x^2 + 4p^2}}{2}} = 216$.

Then by subtracting the second Equation from the fourth, we have $2e^2 = \sqrt{x^2 + 4p^2} - x$.

Hence $e = \sqrt{\frac{\sqrt{x^2 + 4p^2} - x}{2}} = 24$.

(13) Given $\left\{ \begin{matrix} \frac{a}{e} = q = 9 \\ a^2 + e^2 = z = 472321 \end{matrix} \right\}$ to find the rest.

Here from the second Equation take the Square of the first, and we shall have $ee = z - qqee$, or $qqee + ee = z$.

Hence $ee = \frac{z}{qq + 1}$. For $qq + 1 \times ee = qqee + ee$. Therefore $e = \sqrt{\frac{z}{qq + 1}} = 24$.

Then, by taking this last Equation from the second, we have $aa = z\,\frac{z}{qq+1} = \frac{qqz}{qq+1}$. Hence $a = \sqrt{\frac{qqz}{qq+1}}$ $= 216$.

(14) Given $\left\{\begin{matrix} \frac{a}{e} = q = \frac{9}{aa-ee=x=46080} \end{matrix}\right\}$ to find the rest.

Here by comparing the first, when multiplied by e, and squared with the second more ee, we have $qqee = x + ee$, or $qqee - ee = x$.

Hence $ee = \frac{x}{qq-1}$

Therefore $e = \sqrt{\frac{x}{qq-1}} = 24$.

Then, by adding the fourth Equation to the second, we get $aa = x + \frac{x}{qq-1} = \frac{qqx}{qq-1}$. Hence $a = \sqrt{\frac{qqx}{qq-1}}$

(15) Given $\left\{\begin{matrix} aa+ee=z=47232 \\ aa-ee=x=46080 \end{matrix}\right\}$ to find the rest.

Here by Addition only) we have $2aa = x = +z$.

Hence $a = \sqrt{\frac{x+z}{2}}$

Then, by subtracting the second Equation from the first, we get $2ee = z - x$.

Therefore $e = \sqrt{\frac{z-x}{2}}$

The Reason of my proceeding (only) with the first Question throughout, is being confined, not having Room; but I hope this will not be the Pupil's Case.

PROBLEMS.

(1) Here let x = the greater Number.
Then $x - 20$ = the lesser.
Which added together gives $2x - 20 = 70$.
Therefore $2x = 70 + 20 = 90$.

And $x=90\div 2=45$, the greater Number.
$\therefore$ $45-20=25$, the lesser.
For $45-25=20$.
And $45+25=70$.

(2) Here let x=the greater Number.
Then $x-14$=the lesser.

Therefore by the Quest. $\frac{x}{x-14}=3$.

Hence $x=3x-42$, and $2x=42$.
$\therefore$ $x=42\div 2=21$, the greater Number.
Likewise $21-14=7$, the lesser.
For $2\times 7=14$.
Also $21\div 7=3$.

(3) Suppose x to be the Number.

Then by the Quest. $\frac{x}{3}+\frac{x}{4}=21$.

Therefore $7x=252$.
Hence $x=36$, the Number required.

For $\frac{36}{3}+\frac{36}{4}=12+9=21$.

(4) Let x be the Number.

Then per Quest. $\frac{x}{3}-\frac{x}{4}=4$.

Hence $x=48$, the Number required.

For $\frac{48}{3}-\frac{48}{4}=16-12=4$.

(5) Suppose x=the Number required.

Then per Quest. $\frac{x}{3}-4=\frac{x}{4}-,25$.

Hence $4x-48=3x-3$.
Therefore by Transposition $x=45$.

For $\frac{45}{3}-4=\frac{45}{4}-,25=\overline{15-4}=\overline{11,25-,25}=11$.

(6) Let x represent the greater, and $x-8$ the lesser Number.
Then per Quest. $16x-64=208$.
Hence $16x=272$, and $x=17$, the greater.
Also $17-8=9$ the lesser Number.

For $17-9=8$, the Difference.

And $17\times17-9\times9=289-81=208$.

The above Question may be solved by making use of two Letters. See Mr. WARD's 9th Question.

(7) Let x=the greater Number, and y the lesser.

Then by Quest. we shall have $x+y=60$.

And $x : y :: 9 : 3$.

Therefore $9y=x$, or $9y-3x=0$.

To which add the first Equation multiplied by 3, and we shall have $12y=180$.

$\therefore y=15$, the lesser Number.

And $60-15=45$, the greater.

For $45+15=60$.

And as $45 : 15 :: 9 : 3$.

(8) Here let x=the greater Number, and y the lesser.

Then per Quest. $xy=108$.

And $\frac{3x}{y}=4$.

Therefore, by destroying x, we shall have $\frac{108}{y}=\frac{4y}{3}$.

Hence $4y^2=324$ and $y^2=81$.

$\therefore y=9$, the lesser Number.

And $\frac{108}{9}=12$, the greater.

For $12\times9=108$.

Also $\frac{12\times3}{9}=\frac{36}{9}=4$.

(9) Let x=the greater Number, and y the lesser.

Then by the Question we shall have,

1. $x+y+8=2x$

2. $x-y-4,5=\frac{y}{2}$.

Then, by multiplying the second Equation by 2, we get $2x-2y-9=y$.

And by the first Equation $x=y+8$.

Which substituted for x in the last Equation, we shall have $y=7$, the lesser Number.

Then $x=5+8=15$, the greater.

For $15+7+8=15\times=30$.

Also $15-7-4,5=\frac{7}{2}$ 3,5.

(10) For the Numbers sought, put x, y, and z.
Then by the Question we shall have

$$\left.\begin{array}{lll} 1. & x+\frac{y+z}{2}= & \\ 2. & y+\frac{x+z}{3}= & \\ 3. & z+\frac{x+y}{4}= & \end{array}\right\} 34=a.$$

Then, by clearing the Equations of the Fractions, we get

4. $2x+y+z=2a$
5. $3y+x+z=3a$
6. $4z+x+y=4a$

Now (to exterminate z) let the fourth Equation be subtracted from the fifth; also the sixth from four Times the fifth, whence is had

7. $2y-x=a$
8. $11y+3x=8a$

From which we have $2y-a=\frac{8a-11y}{3}$

Therefore $17y=11a=374$.

Hence $y=\frac{374}{17}=22$.

Then by the seventh, $x=44-34=10$.

Also by the fourth, $z=68-42=26$.

For $10+\frac{22+26}{2}=34$.

And $22+\frac{10+26}{3}=34$.

Also $26+\frac{10+22}{4}=34$.

(11) Let $3x$=the major Part of the first Division.
Then $100-3x$=the minor Part.
And by the Question we shall have x=the minor Part of the second Division.
Also $100-x$=the major Part.
Whence by the Question $100-x=200-6x$.
Hence $x=\frac{100}{5}=20$, the minor Part of the 2d Division.
And $100-20=80$, the major Part.

Again $20 \times 3 = 60$, major
And $100 - 60 = 40$, minor } Part of the first Division.
For $60 = 20 \times 3$, and $80 = 40 \times 2$.

(12 Let x represent A's Contribution.
Then $x + 10 =$ B's
And $2x + 10 =$ C's } by the Question.
Therefore $4x + 20 = 76$.
Hence $x = \frac{76 - 20}{4} = 14$ A's.
And $14 + 10 = 24$ B's.
Also $14 \times 2 + 10 = 38$£. C's Contribution.

(13 Let $x =$ the Length.
Then $52 - x =$ the Breadth.
Now $52x - xx = 480$, per Question.
Therefore $xx - 52x = -480$.
And $xx - 52x + 676 = 676 - 480 = 196$.
$\therefore x - 26 = 14$.
Hence $x = 40$ in Length.
And $52 - 40 = 12$, in Breadth.
This Question may be solved by making Use of two Letters. See Mr. WARD's second Question.

(14 Suppose $x =$ the Quantity sold at 12*d.* per lb.
Then as $\frac{3}{4} : \frac{2}{3} :: x : \frac{8x}{9} =$ the other at 15*d.* per lb.
Therefore, by multiplying each Quantity by its Price, we have $12x + \frac{40x}{3} = 19 \times 12 = 228$.
Hence $36x + 40x = 684$.
From which $x = \frac{684}{76} = 9$ lb. at 1*s.* and $\frac{8x}{9} = 8$ lb. at 1*s.* 3*d.* per lb.
For as $9 : 8 :: \frac{3}{4}$ to $\frac{2}{3}$.

(15 Suppose he had x Guineas.
Then by the Quest. $x - \frac{x}{4} + \frac{x}{5} = 66$.
$\therefore 20x - 9x = 1320$.
Hence $11x = 1320$
And $x = 120$ Guineas, had at first.

(16) Let x = the greater, and y the lesser Number.

Then $xx+xy=77$
And $xy-yy=12$ } by the Question.

Now by destroying xy, we get the following Equation,

$77-xx=12+yy$.

Hence $xx=65+yy$.

$\therefore x=\sqrt{65-yy}$.

Then by substituting this for x, in the second Equation, we have $y\sqrt{65-yy}-yy=12$.

Or $\sqrt{65-yy}=\frac{12+yy}{y}$.

Then by squaring each Side, $65-yy=\frac{144+24y^2+y^4}{y^2}$.

Or $2y^4-41y^2=-144$.

Therefore, by $\div 2$, we have $y^4-20{,}5\ y^2=-72$.

And by completing the Square, we get $y^4-20{,}5\ y^2+105{,}0625=33{,}0625$.

Or $y^2-10{,}25=\sqrt{33{,}0625}=5{,}75$.

Then $y^2=16$.

Hence $y=4$, and $65-16=49=x^2$.

Therefore $x=7$.

(17) Let x = the Number required.

Then by the Quest. $4xx-40x+100-1=7x$.

Or $4x^2-47x=-99$

Therefore by $\div 4$, we have $x^2-11{,}75x=-24{,}75$.

Or $x^2-11{,}75x+34{,}515625=9{,}765625$.

$\therefore x-5{,}875=\sqrt{9{,}7656}=3{,}125$.

Hence $x=3{,}125+5{,}875=9$, the Number required.

For $\sqrt{9\times 2-10}=8$.

Also $\sqrt{8\times 8-1}=9\times 7=63$.

(18) Here let x = his Part, and y her's.

Then $\left\{\begin{array}{l}\frac{x}{5}=\frac{y}{4}+10\\ x+y=1000\end{array}\right\}$ by the Question.

Therefore, by destroying the x's we get $\frac{5y+200}{4}=1000-y$,

Or $5y+200=4000-4y$.

Then $y=3810\div 9=422\frac{2}{9}$ her
And $1000-422\frac{2}{9}=577\frac{7}{9}$ his } Share.

(19) Let x=the least Number.
Then we shall have x, $x+2$, $x+4$, and $x+6$, for the Numbers.
But $x\times x+2\times x+4\times x+6=x^4+12x^3+44x^2+48x$.
Therefore $x^4+12x^3+44x^2+48x=945$, (by the Quest.)
Wherefore $105x^2=945$. Hence $x^2=9$. And $x=3$, the least.
Therefore the Numbers are 3, 5, 7, and 9.

(20) Put x=his Age in Months.
Then $x+\frac{x}{2}+\frac{x}{8}-1=21\times 21=441$, (by the Quest.)
Or $16x+8x+2x-16=7056$. Or $26x=7072$.
Hence $x=\frac{7072}{26}=272$ m. or 22 yrs. 8m. his Age.
For $272+\frac{272}{2}+\frac{272}{8}-1=21\times 21=441$.

(21) Let x, y, and z, be the Numbers required.
Then per Quest. $x-y=6$; or $y-6=x$, and $z-y=15$; or $z=15+y$.
But $x:y::y:z$; that is $y-6:y::y:15+y$.
Therefore $yy-90+9y=yy$. Hence $9y=90$. And $y=10$, the second.
Then $10-6=4$, the first.
And $15+10=25$, the third Number.

(22) Suppose x to be the Number.
Then $x^6-19x^3=216$, (by the Quest.)
Put $x^3=z$, $x^6=z^2$ and it will be $z^2-19z+\frac{361}{4}=$
$216+\frac{361}{4}=\frac{1225}{4}$
And $z-\frac{19}{2}=\frac{35}{2}$. Whence $z=\frac{19+35}{2}=27$,
or $=-8$. But $x=\sqrt[3]{z}$. Therefore $x=+3$, or -2.

(23) Let x=the Length.

Then per Quest. $3 : 2 :: x : \frac{2x}{3}$ the Breadth.

Therefore $\frac{x \times 2x}{3} = 60000$, or $2xx = 180000$.

Hence $xx = 90000$. And $x = 300$ Men in Rank.

Also $\frac{300 \times 2}{3} = 200$ Men in File.

Now $330 - 1 \times 2{,}75 = 822{,}25$ yds. Length.

And $200 - 1 \times 2{,}75 = 547{,}25$ ditto Breadth.

$4840)449976{,}3125(92$ ac. 3 rds. 35,2 p. the Ground they stand on.

(24) Put x the Side of the Square.

Then $xx + 284$ = the whole Army.

And $x + 1$ = the Number of Men designed for the Side of the Square.

Consequently $xx + 2x + 1 - 25 = xx + 284$. And by Transposition $2x = 308$. Hence $x = 154$ Men, the Side of the Square.

Wherefore $154 \times 154 + 284 = 24000$ Men, the whole Army required.

(25) Suppose x = what he received.

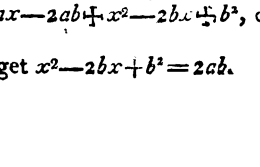
Then per Quest. $\frac{x-94}{2} + \frac{x-94}{5} + \frac{x}{10} + 94 = x$.

This Equation cleared of the Fractions, we shall have $50x - 4700 + 20x - 1880 + 10x + 9400 = 100x$ or $20x = 2820$.

Hence $x = 141$ £. the sum required.

(26) Put $DF = 9 = a$, $DE = 2 = b$, and $x = AD = DB$.

Then $DB = a + x$, and $AB = EB = a + x - b$.



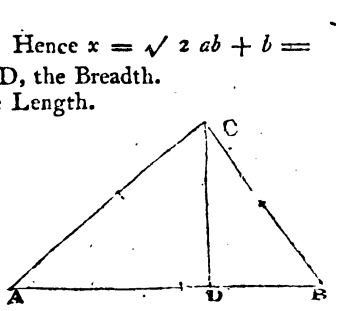
Therefore, by the 47th of Euclid's first book, we have $DB^2 = AB^2 + AD^2$, that is, $a^2 + 2ax + x^2 = x^2 + a^2 + 2ax - 2ab + x^2 - 2bx + b^2$, or $x^2 - 2bx = 2$.

Then by Comp. the Square we get $x^2 - 2bx + b^2 = 2ab$.

Therefore $x-b=\sqrt{2ab}$. Hence $x=\sqrt{2ab}+b=\sqrt{2\times9\times2}+2=8$,=AD, the Breadth.

Also $9+8=17$=AB, the Length.

(27) Here AB=15=a, AC=14=b, BC=13=c, and x=BD.



Then, by the 47th of Book I. of EUCLID, we shall have $AB^2-BD^2=AC^2-CD^2$. Or $a^2-x^2=b^2-c^2+2cx-x^2$. That is $a=b^2-c^2+2cx$.

And, by Transposition, we get $2cx=a-b^2+c^2$. Hence $x=\frac{a^2-b^2+c^2}{2c}=\frac{169-196+225}{30}=\frac{198}{30}=6{,}6$=BD.

And $15-6{,}6=8{,}4$=DA. *Q. E. F.*

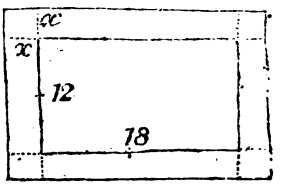
(28) Let x=the Breadth sought.

$18+2x$=Length AB or CD,

$12+2x$=AD or BC.

Hence $4x^2+60x+216=18\times12\times2=432$ Or $4x^2+60x=432-216=216$.

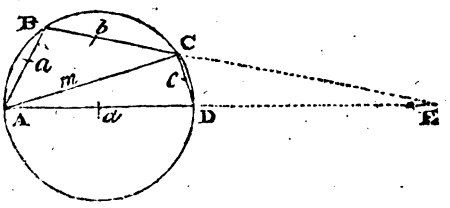
Which ÷ 4 gives $x^2+15x=54$.



Now by Comp. the Square, we have $x^2+15x+56{,}25=110{,}25$. Or $x+7{,}5=10{,}5$.

Therefore $x=10{,}5-7{,}5=3$, the Breadth required.

(29 Put a=, AB=15,60, b=BC=13,2 c=CD=10, and d=AD=26.



Also m=AC, and y the Co-sine of the Angle D, and $-y$ = the Co-sine of the Angle B.

Then per Theo. p. 12, in Mr. ANT. TACKER'S MISCEL.) we shall have $a^2+b^2+2ab=m^2$, and $d^2+c^2-2cd=a^2+b^2+2ab$. Hence $2aby+2dcy=a^2+c^2-a^2-b^2$.

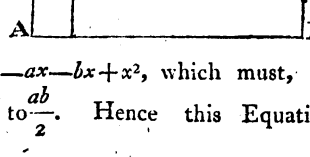
And $y = \frac{d^2+c^2-a^2-b^2}{2ab+2ac}$, whose Sine call s.

Then $\frac{abs+ds}{2}$ = 21,504, the Area of the Trapezium ABCD.

Or thus, suppose DE=x. Then by sim. Triangles $a : d+x :: c : \frac{d+x\times c}{a}$=50=CE. $\therefore$ BE $= \frac{ba+d+xc}{a}$

Again, $a : \frac{ba+d+x\times c}{a} : c : x$. Hence $x = \frac{ab+cd\times c}{a^2-c^2}$ = 32,5. And the Area of the Trapezium ABCD = 21,504 Acres as before (by Sect. 70. Prob. III. Rule III.)

(30) Put AB=100=a, BC=80=b, AF = CD, the Breadth of the Walk=x.

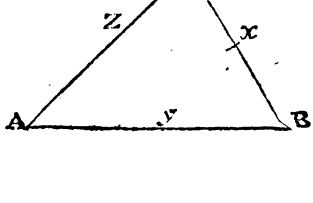


Then FG=$a-x$, GD =6—x. And FG× GD=$a-x\times b-x=ab-ax-bx+x^2$, which must, by the Question, be equal to $\frac{ab}{2}$. Hence this Equation

$ab-ax-bx+x^2=\frac{ab}{2}$. But $2d=a+b$. Then $xx-2dx=-\frac{ab}{2}$. Therefore $xx-2dx+da=dd-\frac{ab}{2}$.

Consequently, $x-d=\sqrt{dd-\frac{ab}{2}}$. Hence $x=d+\sqrt{dd-\frac{ab}{2}}$=90—64,031=25.969, the Breadth req.

(31) Here put s and c for the natural Sine and Co-sine of the Angle ABC, Radius=1, a for the Area of the Triangle, m for the Sum of the Cubes of the Sides, and x, y, and z, for the Sides, as in the Figure.



Then we shall have $sxy=2a$, and $x^2+y^2=z^2+2cxy$ (per Trigonometry).

Hence $xy=\frac{2a}{s}$ which call p, and $x^2+y^2=z^2+2cp$, for $2cp$ put b; then $x^2+y^2=z+b$.

Now $x=\frac{p}{y}$, $\therefore x^3=\frac{p^3}{y^3}$, and therefore the Equation $x^3+y^3+z^3=m$, as given by the Question, becomes $\frac{p^3}{y^3}+y^3+z^3=m$, whence $z^3=m-y^3-\frac{p^3}{y^3}$.

Again, $z^3=x^3+y^3-b$, whence $z^3=\frac{p^3}{y^3}+y^3-b$, consequently $z^6=m-y^3-\frac{p^3}{y^3}\Big]^2$, and $z^6=\frac{2p^3}{y^3}+2\Big]^2$ hence $m\sqrt{y^3-y^6-p^3}\Big]^2-p^2+y^4-by^2\Big]^2$.

This Equation, properly reduced, will give $y=8$, from whence x is found $=10$, and $z=6$, the Sides eq.

Suppose the Sum of the Squares of the Sides had been given, instead of the Sum of the Cubes; then we should have had $x^2+y^2+z^2=m$, and the other Equations as before, viz. $xy=p$, and $x^2+y^2=z^2+b$, hence $z^2+b+2z^2=m$; or $z=\sqrt{\frac{m-b}{2}}$, which being now known, put $z^2+b=n$, then $x^2+y^2=n$; but $xy=p$, therefore $x^2+2xy+y^2=n+2p$, and $x^2+2xy+y^2=n-2p$, consequently $x+y=\sqrt{n+2p}$, and $x-y=\sqrt{n-2p}$, therefore $x=\frac{\sqrt{n+2p}+\sqrt{n-2p}}{2}$.

QUESTIONS ANSWERED BOTH BY COMMON ARITHMETIC AND ALGEBRA.

(1) First $76\times56=4256$.

And $\sqrt[2]{4256}=65,238+$ lb. the true Weight required.

(2) First 1lb.=the first or least Weight.

lb.

1	= 1 first		
Then 2 + 1	= 3 second	}	Weight.
And 3 + 1 × 2 + 1	= 9 third		
Also 9 + 3 + 1 × 2 + 1	= 27 fourth		

Sum of which = 40 the whole Weight.

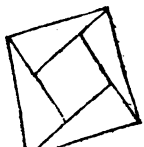
(3) First 6*s.* $\frac{1}{4}d.$ = 289 Farthings.
Then $\sqrt[2]{289} = 17$ Persons. Or 17 qrs. = $4\frac{1}{4}d.$ what each spent.

(4) Here x = the Number of Spots. $n = 7$, $r = 12$, and $w = 52$.
Then per Theo. $x = 13 \times n - w + r = \overline{13 \times 7 - 52 + 12}$.
Therefore $x = 103 - 52 = 51$ Spots.

(5) Supposing the Oranges to be laid in Rows, and upon each other, then the Solution will be thus; $2{,}5 \times 2{,}5 \times 2.5 \times 2000 = 31250$.
$\sqrt[3]{\quad}$
$\therefore$ 31250 = 31,498 Inches, inside of the Box.

Fig 1.

Fig. 2.



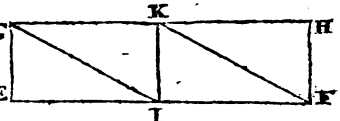
(6) Let Fig. 1. represent the Plank that is to be cut, and the second Fig. as they are placed to make the Square.

(7) The Hour Hand goes only $\frac{1}{12}$ the Circumference in an Hour; the other goes the whole, or $\frac{12}{12} = 1$.
Then $\frac{12}{12} - \frac{1}{12} = \frac{11}{12}$, Minute hand gains in an Hour.
Therefore as $\frac{11}{12}$ cir. : 1 h. :: 1 cir. : $\frac{12}{11}$ h. = 1 h. 5 $\frac{5}{11}$ m.
$\therefore$ 1 h. 5 $\frac{5}{11} \times 5 = 27\frac{3}{11}$ min. past 5, the Time required.
For when the Hands are in Opposition, the Minute must be $\frac{6}{11}$ a Head of the Hour-hand.
Hence as 11 : 1 h. or 60 min. :: 6 : 32 $\frac{3}{11}$ min. past 12.
And for the next Conjunction it will be as 11 : 1 h :: $6 \times 2\ (12) : = \frac{12}{11} = 1$ h. 5. $5\frac{3}{11}$ min. as above.

As 11 : 1 :: 12 + 12 (24) : $\frac{24}{11}$ = 2 h. $11\frac{9}{11}$ the next Conjunction.

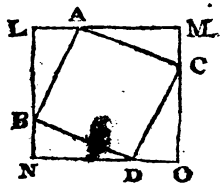
Also as 11 : 1 h. :: (12 × 5260) : $\frac{60}{11}$ = 5 h. $27\frac{3}{11}$ = $27\frac{3}{11}$ min. past 5, the Answer as above.

Fig. 1.



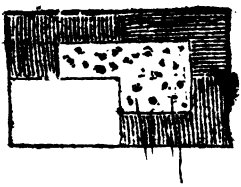
(8) Let ABCD, Fig. 2 represent the given Table = 27 Inches, and the Parallelogram EFGH, Fig. 1. represent the Plank = 4 ft. which as per Quest.) must be cut into four equal Parts, viz. EGI, KIG, IKE. and FHK.

Fig. 2.

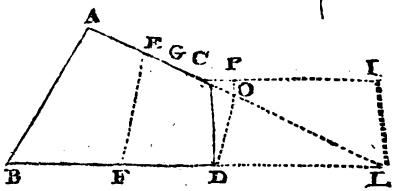


Then by properly applying the four equal Parts in the Parallelogram EFGH, to the given Table ABCD, then will EGI be = to NBD, KIG=LAB, IKF=MC A, and HFK will be=to ODC, which makes LMNO, a complete Square, each Side thereof=36 Inches, which was required.

(9) If the Remainder of the Land be divided into four Parts, as in the annexed Plan, these Parts will be equal and similar to each other.



(10) Let ABCD be the given Trapezium, and let AC and BD be produced to L; then upon L make the Parallelogram DPIL=the Triangle DCL+half the given Trapezium, and making the Angle LDO=LOD, produce IP to meet AL in G; then take EL a mean Proportional between PI, and

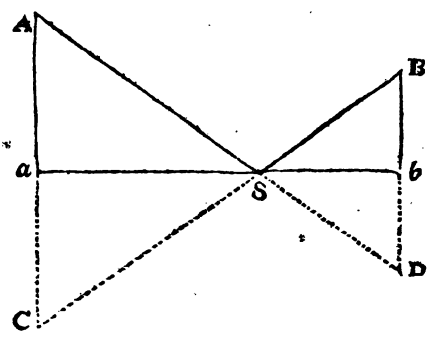


2 GL, and draw EF parallel to PD, so will EF be the Line required.

Demonstration from the Similarity of Triangles, we have $\frac{1}{2}$ GL × LI : $\overline{GL}^2$ :: PI × LI : $\overline{EL}^2$, consequently $\overline{EL}^2$ = 2 GL × PI.

Therefore when the Triangle LEF = the Parallelogram PDLI, then EL will be a mean Proportional between 2 GL, and PI, and EF is the shortest Line possible. *Q. E. D.* (See Theo. X. p. 112 of SIMPSON'S GEOM.)

(11) Thus let the Line A *a* be continued to C, so that C *a* = A *a*, Then draw CB, and (S) where intersect the Line *a b* is the Situation of the Place required.



∴ AS + BS = the whole Distance travelled.

For Example thus, suppose *ab* = 50, A*a* = 40, and B*b* = 30, required the Situation of a Place S, so that S A + SB, may be the least Distance possible.

Then the Theorem $\frac{Bb \times ab}{Aa + Bb} = Sb$, when AS + SB is the least possible, that is $\frac{30 \times 50}{40 + 30} = \frac{1500}{70} = 21{,}42857 = Sb$.

Therefore S*a* = 28,57143.

(12) Suppose x = the Number of Servants.

Then by the Appendix we have $\frac{x^x + 1 - x}{x - 1} = 960799$, all the possible Variations of the Servants; which Equation solved, will give $x = 7$, the Number of Servants required.

GENEALOGICAL PARADOXES.

(1) Lot, committing Incest with his two Daughters, and having by each of them a Son, produced that Paradoxical relation of the Sons to each other; for by that Means Lot became, at the same Time, their Father and Grand-Father; and they were Brothers and First Cousins to each other; also each Mother was Aunt to the other's Son. See Gen. xix. ver. 31, to the End.

(2) The Lord God formed the first Man, Adam, of the Dust of the Ground (Gen. ii. ver. 7.); and from Adam made he a Woman, and called her name Eve (Gen. ii. ver. 22.) Now Abel (who was the second Son of Adam and Eve his Wife) was murdered by his Brother Cain; therefore he got the Maidendead of his Grandmother (the Earth); and was got before his Father (Adam), who was made of the Earth, therefore was not begotten; and was born before his Mother (Eve), who was made of Man, therefore was not born.

APPENDIX.

1. CONCERNING DIVISORS.

It being often necessary, in Arithmetical Calculations, to find such Multipliers or Numbers which may be divided by any Number of given Divisors, without any Remainder or Remainders; by which means many pleasant Questions, not reducible to any other Rule in common Arithmetic, may be solved.

To find the least Number that can be divided by any Number of Divisors with a Remainder.

RULE.

Multiply all the Prime * Numbers, and the Root of such as are Square or Cube Numbers, continually; the Product will be the Number required.

* *A Prime* Number is such a Number as cannot be produced by the *Multiplication* of two or more Integers.

EXAMPLES.

(1) Required the three least Numbers, which, divided by 20, shall leave 19 for a Remainder; but, if divided by 19, shall leave 18; if divided by 18, shall leave 17; and so on, always leaving one less than the Divisor, to Unity.

G. DIARY, 1747.

By Algebra thus: suppose x=the least Number possible; and let a, b, c, d, &c. represent the Quotients respectively produced by dividing x, by 2, 3, 4, 5, &c. then will the Remainders be 1, 2, 3, 4, &c. Hence $x=2a+1=3b+2=4c+3=5d+4$, &c. to 20 A+19.

Now $a=b+\frac{b+1}{2}$. Hence $\frac{b+1}{2}=m\ a$, a whole Number.

$\therefore b=2m-1$, and $a=3m-1$.

Again $c=m-1+\frac{m}{2}$. Hence $\frac{m}{2}=n$. a whole Numb.

And therefore $m=2n$, $c=3n-1$, and proceeding in this Manner we get A=11639628 B−1; hence the Number required x=232792560 B−1, where B represents any whole positive Number; and when B=1, then the Number required $x=232792560-1=232792559$

2d. $232792560\times 2-1 = 465585119$

3d. $232792560\times 3-1 = 698377679$

And thus may be performed other Numbers, ad Infinitum.

Or thus by the Rule:

First 1, 2, 3, 5, 7, 11, 13, 17, and 19, are prime Numbers.

Also $\sqrt[2]{4}=2$, $\sqrt[3]{8}=2$, $\sqrt[2]{9}=3$, and $\sqrt[3]{16}=2$, and all the rest are composite Numbers.

$\therefore 1\times 2\times 3\times 2\times 5\times 7\times 2\times 3\times 11\times 13\times 2\times 17\times 19=$ 232792560, the least Number that can be divided by the given Divisors without a Remainder, as before.

(2) What's the least Number that can be divided by the Nine Digits, without a Remainder?

LADIES' DIARY, 1719.

Divisors, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Now $\sqrt{4}=2$; 6 may be cancelled, being composed 2×3; and 3, 5, and 7, are prime Numbers.

And $\sqrt[3]{8}=2$. Also $\sqrt[2]{9}=3$.
Then per Rule $1\times2\times3\times2\times5\times7\times2\times3=2520$.

(3) A Country Girl to Town did go,
Some Walnuts there to sell;
A Gentleman she chanc'd to meet,
And thus it her befel:
"My pretty Maid," says he to her,
"What Number have you here?"
'I can't tell, Sir,' said she to him,
'But this I'll make appear—
I told them o'er, ere I came out,
By sixes, fives, fours, threes, twos;
And every Time I number'd them,
One remain'd Overplus:
I told them o'er by sevens at last,
And there were no Remains:
If you can find the Number out,
Pray take it for your Pains.'

First, the least Number that can be divided by 1, 2, 3, 4, 5, 6, without a Remainder, will be $1\times2\times3\times2\times5=60$, per Rule.
Then $60+1=61$, will leave 1, when ÷ 2, 3, 4, 5, 6.
But 7)61(8, and 5 Remains.
Also $\overline{60\times2+}\ 1=121$
$\overline{60\times3+}\ 1=181$
$\overline{60\times4+}\ 1=241$
} None of which are divisible by 7 without a Remainder.
But $60\times5+1=301$, is the least Number which admits of the Conditions of the Question.
Then to find the next least Number which admits of the same Conditions, we shall find, by proceeding as above, to be $\overline{60\times12+1}=721$.
Also $721-301=420$, the common Difference of all the Numbers answering the Conditions of the Question.
∴ 301, 721, 1141, 1561, &c. ad Infinitum, will answer the Conditions of this Question.

(4) To find the least Number of Guineas, which, being divided by 6, 5, 4, 3, and 2, respectively, shall leave 5, 4, 3, 2, and 1, respectively remaining.

LADIES' DIARY, 1748.

First $1\times2\times3\times2\times560=$, the least Number; which will divide by 2, 3, 4, 5, and 6, as by the last Question.
$\therefore 60-1=59$, *Q. E. F.*

2. COMBINATIONS.

Combinations of Quantities is the Manner of finding how many different Ways they may be varied, being taken 1 and 1, 2 and 2, 3 and 3, &c. as the Number of Combination of three Quantities, viz. *a*, *b*, and *c*, viz. *ab*, *ac*, *bc*. If three Quantities are to be combined, and their Number is only three, as *a*, *b*, and *c*, then the Number of Combinations will be only one, viz. *abc*; and if there are four Quantities, *a*, *b*, *c*, and *d*, and three to be taken, then the Combinations will be four, viz. *abc*, *acd*, *bzd*, *acd*; and if the Number of Quantities to be combined be called *q*, and *u*, the Number of them to be taken, then the Number of Combinations will be $\frac{q-u+1}{1}\times\frac{q-u+2}{2}\times\frac{q-u+3}{3}\times\frac{q-u+4}{4}\times\frac{q-u+5}{5}$, &c. For suppose the Number of Quantities to be combined be 6, and the Number of them taken be 4, then the Number of Combinations will be $\frac{6-4+1}{1}\times\frac{6-4+2}{2}\times\frac{6-4+3}{3}\times\frac{6-4+4}{4}=\frac{1}{1}\times\frac{4}{2}\times\frac{5}{3}\times\frac{6}{4}=15$. The Number of all the possible Combinations, beginning from the Combination of every two, will be $2q-9-1$; as when the Number of Quantities be 5, then the Number of possible Combinations will be $2^5-6=26$. So that if *u* represents any Number of Quantities, then will $\frac{u^{u+1}-u}{u-1}$ express the possible Number of all the Variation, as if $u=7$, then $\frac{7^{7+1}}{7-1}7=\frac{5764801-7}{6}=\frac{5764794}{6}=960799$.

To find the different Combinations in any Number or Quantities.

RULE.

Having placed the given Quantity by itself decrease it gradually by an Unit, so often as there are Quantities in the Combination; placing one above another, with a Sign of Multiplication between them, which Numbers must be multiplied into one another for a Dividend: then placing an Unit with the like Number of Places, increasing by Unity till you arrive at the Number to be continued; which multiply continually for a Divisor, and the Quotient will be the Number of Combinations sought.

EXAMPLES.

(1) How many different Ways may 11 Halfpence, hustled in a Hat, turn up?

First, as a Halfpenny has two faces, $\therefore 2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$, or $2^{11}=2048$, the different Combinations required.

Now to find the different Chances for any Number of Heads or Tails, let a represent the Heads, and b the Tails; then by the Theorem in Page 323, we shall have $a^{11}+11a^{10}b+55a^9b^2+165a^8b^5+330a^7b^4+462a^6b^5+462a^5b^6+330a^4b^7+165a^3b^8+55a^2b^9+11ab^{10}+b^{11}$ are all the different Combinations or Ways 11 Halfpence can turn up, viz. $1+11+55+165+330+462+462+330+165+55+11+1=2048$, as before.

Here is to be observed, that a^{11}, or all Heads, hath but one Way of turning up; the same for b^{11} Tails: but 10 Heads and 1 Tail, and the contrary, may come up 11 different Ways each. Also nine Heads and two Tails, or the contrary may each come up $\frac{11\times10}{2}=53$ different Ways. Likewise eight Heads and three Tails, or the contrary, may come up $\frac{55\times9}{3}=165$ different Ways, &c.

It may be also observed, that, by this Theorem, the Unciæ, or Coefficients, do only increase until the Indices of the

two Letters become equal, or change Places, and then the rest decrease in the same Order.

(2) A famous General having serv'd his King
Long Time in Wars, and had victorious been;
For which his Service, with a pleasant Smile,
Ask'd of his King one Farthing for each File,
Of ten Men in a File, which he could then
Make with a Body of one hundred Men.
The King, considering his brave Actions past,
And seeming Modesty of his Request,
Gives his Consent. To what will it amount
In sterling Money? Take your Pen, and count.

Thus $\frac{100}{1} \times \frac{99}{2} \times \frac{98}{3} \times \frac{97}{4} \times \frac{96}{5} \times \frac{95}{6} \times \frac{94}{7} \times \frac{93}{8} \times \frac{92}{9} \times \frac{91}{10} = \frac{6281565095552047200}{3628800} =$ 173103094564440 Farthings, or 180315723505*l*. 9*s*. 2*d*. *Q. E. F.*

(3) Two Gamesters, one Day, at Dice they did play,
And being full merry with Wine;
Said B unto A, "What Odds will you lay,
I cast not the six Aces this Time?"
Says A then to B, 'Ten to one I'll lay thee,
With six Dice the six Aces you cast not.'
Pray Youths show, and here let them know,
For the Odds on the Cast, Sirs, they know not.

First $6\times6\times6\times6\times6\times6=46656=6^6$ different Combinations.
And $1\times2\times3\times4\times5\times6=720$ Variations.
Then $46656-720=45936$ Chances against A;
But as A laid 10 to 1. $\therefore$ 7200 Chances to B.
Therefore A's Chance to that of B, : : 45936 : 7200, or : 6,38 : 1. *Q. E. F.*

(4) Two Gamesters met, the other Day,
The one called B, the other A;
But having neither Cards nor Dice,
They got to Hotch-cap in a Trice;
With fifteen Halfpence, fair and flat,
All which they hustled in a Hat:

"Says A to B, all these are mine,
And I will lay a Pint of Wine,
That in two Trials there will be,
Nine Heads or Tails, as here you see"—
No matter which, but on they play'd,
Till Silver, Brass, and Gold were laid:
But as to B, his Chance was bad,
For he got broke of all he had.
What were the Odds?—I pray declare,
Ye ingenious Youths, and place it here.

First $2^{16}=65536$, Number of different Chances on 16 Halfpence.

Now let a represent the Heads, and b the Tails.

Then (by Theo. in p. 323) we shall have $a^{16}+16ab^{15}+120a^{14}b^{2}+560a^{13}b^{3}+1820a^{12}b^{4}+4368a^{11}b^{5}+8008a^{10}b^{6}+11440a^{9}b^{7}$, &c.

Then $11440\times2=22880$ Chances for nine Heads or Tails to come.

$\therefore$ $65536-22880=42656$ Chances, not come up the first Time, viz. 22880 to 42656, that they come up nine Heads or Tails the first Toss.

Therefore as $65536 : 22880 :: 42656 : 14892\frac{7}{64}$.

Then $22880+14892\frac{7}{64}=37772\frac{7}{64}$ for { Nine Heads,

Also $65536-37772\frac{7}{64}=27763\frac{57}{64}$ against { or 9 Tails.

MAGICAL SQUARES.

By a Magical we understand a Square divided into several other small equal Squares, filled with Terms of an Arithmetical Progression, so transplanted, that all the same Line or Rank, whether taken perpendicularly, horizontally, or diagonally, make the same Sum.

EXAMPLE.

(1) The Numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, being given, to form them in a magic Square; viz. counting each Rank perpendicularly, horizontally, or diagonally, that these Ranks may be equal to each other.

Suppose it done, and represented in its proper Form, by the following Symbols, placed as follows:

$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & m \end{matrix}$$

Then by the Quest. we have $\left\{\begin{matrix} a+e+m=15 \\ b+e+h=15 \\ c+e+g=15 \end{matrix}\right\}$ Required e the middle Number.

Their Sum $=a+b+c+3e+m+h+g=45$.

Again $a+b+c=15$ } added together, we get $a+b+c+m$
Also $m+h+g=15$ } $+h+g=30$; this taken from the fourth Equation gives $3e=15$.

Then $e=\frac{15}{3}=5$, the middle Number.

Or by Numbers, thus:

First the Sum of the progressional Numbers are $1+2+3+4+5+6+7+8+9=45$.

Then 3 = the Number of Rows.

$\therefore 45\div 3=15=$ the Sum of each Side or Rank.

And $15\div 3=5=$ the middle Number as before.

Again, to find the corner Figures; and first to find the Figure represented by a. Beginning with 1, I find the corner Letter a, or any other corner Letter cannot be 1; for if a was $=1$, then m must $=9, b+c=15-1=14$; as also $d+g=15-1=14$. But there remains no two Numbers after 5, 1, and 9, whose Sum is 14, but 6 and 8.

Then if any of these Figures were b, the other would be c; and then no Figures would remain for the Value of either d or g; wherefore a is not equal to 1, nor any corner Letter = to 1, or 9.

Now 3 cannot be $=a$; for, if it were, then m should be $=7$; $b+c=15-3=12$, as also $d+g=12$: but there remains no two Numbers after 5, 3, and 7, whose Sum is 12, but $8+4$, which cannot answer to b and c, and d and g: wherefore a, or any other corner Letter, is not $=3$; neither is m, nor another corner Letter $=7$; from what has been said, it is plain, that if the Question proposed is capable of being solved the corner Letters are all even Numbers; wherefore if $a=2$, m will be $=8$, and c must be either 4 or 6. Let $c=4$, then $g=6$, $b=9$, $d=7$, $f=3$,

HARVARD COLLEGE
LIBRARY

THE ESSEX INSTITUTE
TEXT-BOOK COLLECTION

GIFT OF

GEORGE ARTHUR PLIMPTON

OF NEW YORK

JANUARY 25, 1924